The Mathematics Educator
1997, Vol. 2, No. 2, 218-240

Problem Solving In Mathematics : Where Are We?

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Abstract

In 1993, a new national mathematics curriculum was introduced in New Zealand. This emphasised the importance of **problem solving**. Three years down the track there is evidence that a significant proportion of teachers believe problem solving to be important and many are making some effort to incorporate problem solving into their teaching. However, despite this evident commitment, problem solving does not seem to occupy a regular place in most classrooms. Teachers see the production of curriculum support material as being the most important single factor for increasing the amount of problem solving in schools. There appears to be no single view about how problem solving should, or might, be integrated with more traditional approaches to mathematics teaching. Lesson starters and homework tasks may be an easy way for teachers with no prior experience of problem solving to begin problem solving activities. However, in order for a substantial proportion of teachers to undertake problem solving on a regular basis, some form of professional development is required.

Introduction

The work in this paper is based upon two pieces of research. The first of these is a questionnaire that was given to eleven secondary school mathematics departments in a small provincial city of New Zealand. Some 53 teachers responded to the questionnaire (a 100% response rate from the schools) and the issues raised were discussed with seven experienced mathematics teachers.

The second study involved observations of problem solving sessions in five secondary classrooms at the Form 3 and 4 level. Each class was videotaped during a weekly problem solving lesson. The videotapes were watched by the teachers (and sometimes were also shown to the students) in private. Every fortnight the teachers and the researchers discussed the lessons and suggestions were made as to how teaching performances might be changed.

By problem solving here we mean the attempt to find the answer to a problem when the method of solution is not known. In problem solving situations the solver has to use strategic skills to find appropriate mathematical techniques which will settle the question.

In addition we also use the words *problem solving* to mean an approach to the teaching of mathematics. Such an approach is based on problems and open questioning. The solving of specific one-off problems is just the start, for teacher and student, of a wider approach to teaching and learning. When they use the words *problem solving*, we believe that curriculum documents are usually referring to a problem solving approach, while teachers often interpret the words to mean one-off events that are not integrated into the syllabus at all.

It is worth noting that what is a problem to one person may not be a problem to another. For a five year old determining the number of legs three cows have will almost certainly be a problem. However, it should not be a problem for most 15 year olds. The five year old may need to draw, use equipment or employ some other method to solve the problem. The 15 year old will just say "3 times 4 equals 12".

Word problems do not necessarily involve problem solving. If the word problems are introduced to practise a technique which has just been acquired, it should be reasonably obvious what technique has to be used. On the other hand, problem solving situations may be presented as wordy problems, though this is not always the case.

The 1980 Yearbook of the National Council of Teachers of Mathematics (Krulik, 1980), predicted that problem solving would be the theme of the 1980s. Also in 1980, the importance of problem solving in mathematics teaching and learning was underlined by the publication of the "Agenda for Action" (NCTM, 1980). This document had as its first recommendation

Problem solving must be the focus of school mathematics. (p. 2)

This was accentuated in 1989 with the publication of the "Curriculum and Evaluation Standards for School Mathematics" (NCTM, 1989), where there is an emphasis on mathematical processes and the integration of problem solving into all areas of the mathematics curriculum.

> *Problem solving should be the central focus of the mathematics curriculum. As such it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned.* (NCTM, 1989, p. 23)

We would like to underline the last sentence of the above quote as we find it difficult to differentiate between problem solving on the one hand and mathematics, or its essential nature, on the other. (This is expanded further in Holton, 1994).

But the movement toward problem solving was not confined to the USA. The Cockcroft Report (Mathematics Counts, 1982), released in the UK, promoted similar ideas.

> *Mathematics teaching at all levels should include opportunities for ... [among other things]*
> *- problem solving including the application of mathematics to every day situations;*
> *- investigational work.* (p.71)

In Australia, these views are reflected in "A National Statement on Mathematics for Australian Schools" (1990), where problem solving is one of four sub-headings in the Mathematical Inquiry Strand.

> *The mathematical processes described within this strand cannot be developed in isolation from the work of other strands. They should pervade the whole curriculum.* (p. 37)

In New Zealand, "Mathematics in the New Zealand Curriculum" (1992) was drafted in a similar vein to the document mentioned above. It lists seven aims, of which the third relates to problem solving.

> *[Mathematics education aims to] help students to develop a variety of approaches to solving problems involving mathematics, and to develop the ability to think and reason logically.* (p. 8)

Regarding problem solving, it goes on to say

> *A balanced mathematical programme includes concept learning, developing and maintaining skills and learning to tackle applications. These should be taught in such a way that students develop the ability to think mathematically ... The chance to look for problems as well as solve them, to create and produce rather than reproduce what already exists, is important for all students.* (p.11)

Problem solving could be used to enhance learning and understanding, generate more interest in students, and encourage students to transfer problem solving skills to areas outside mathematics. In fact problem solving could achieve all seven aims of the New Zealand mathematics curriculum (Holton, Spicer, Thomas and Young, 1996).

In New Zealand, the importance of problem solving has been underscored by the Ministry of Education publication "Implementing Mathematical Processes in Mathematics in the New Zealand Curriculum" (1995).

> *Problem solving is the first heading in the Mathematical Processes strand; unless the ability to solve problems is developed, there is little point in studying mathematics.* (p. 20)

The Mathematical Processes strand in the Mathematics in the New Zealand Curriculum document provides the umbrella for learning in the other five strands of Number, Measurement, Geometry, Algebra and Statistics. It incorporates three areas: problem solving, developing logic and reasoning, and communicating mathematical ideas. We have decided to focus our attention on problem solving because it can provide the medium for both developing mathematical processes and interlacing the learning of specific knowledge and skills. However, is problem solving valued in our classrooms?

The main questions in the questionnaire that we mentioned above were as follows:

1. In what ways are problem solving activities being used at present and how often are problem solving approaches being used?

2. What problem solving strategies are being promoted in the classroom?

3. How can teachers integrate skill and content learning with a problem solving approach to teaching and learning?

4. How do teachers want to use problem solving in their classrooms in the future?

5. What help do teachers need to implement problem solving approaches to their teaching of mathematics?

Each of the following sections of this paper will be based upon these questions. We will discuss the teachers' responses to the questions and amplify these by situations which arose in the other study discussed above. The final section brings together a general discussion of the results of the questionnaire and their ramifications.

1. How is problem solving being used and how often?

There are a variety of approaches to problem solving in the classroom. Sigurdson, Olson and Mason (1994), for example, outline five such approaches. The questionnaire avoided questions about specific types of activities (open ended tasks, investigations, short challenging tasks, projects or mathematical modelling) and concentrated on broader areas of usage that had been observed in classrooms. The five categories considered were homework tasks, lesson starters, throughout lesson, at the end of a unit of work and to cater for the needs of more able students. The lesson starter category was broken into two, the first where the starters were not directly related to the content of the lesson and the second where they were.

In the following table, we present the results of the questionnaire.

Table 1. Usage and Frequency of Problem Solving Areas
(% of respondents in each category)

Problem solving Activity	Daily	Weekly	Monthly	Sometimes	Never	No Reply
Homework Tasks	5.7	43.4	13.2	35.8	1.9	0
Lesson starter-not directly related to the content of the lesson	1.9	18.9	7.5	50.9	17.0	3.8
Lesson starter-related to the content of the lesson	3.8	24.5	9.4	54.7	5.7	1.9
Throughout all my teaching of a lesson	11.3	28.3	9.4	35.8	13.2	1.9
At the end of a unit of work	0	5.7	20.8	64.1	7.5	1.9
To cater to individual needs of extension students	7.5	22.6	13.2	52.8	1.9	1.9

Homework Tasks

Lovitt and Clarke (1988) acknowledge the need to bring together more closely the mathematical activity of the classroom and "the mathematical activity that occurs beyond the classroom walls". Homework provides one opportunity for doing this.

From Table 1, we see that problem solving abilities are being used as homework tasks on a regular basis. Nearly half of the respondents were using these at least weekly and approximately two-thirds at least monthly. Hence problem solving tasks were being used for homework on a more regular basis than any of the other five categories surveyed. However, of those teachers using homework tasks at least weekly, under half (47.8%) used problem solving through all of a lesson at least weekly. This suggests that homework tasks are often independent activities which are not supported by problem solving in the classroom.

Lesson Starters

One of the teaching approaches identified by Sigurdson et al (1994) is the **problem-process approach**. This involves using simple problems related to the mathematical content of the lesson, solving these problems through student-teacher interaction in a whole class setting and focusing on the processes being used. They saw this approach as being easier to implement because it is usually "teacher-led", "easier to identify and plan for, and the interactiveness of this approach makes it easier to involve the slower students". They suggest that teachers can go on from starter problems of this kind, and develop an approach where a problem becomes the basis for a whole lesson. Teachers do prefer problems to relate to content. They are also aware of the difficulties experienced by the less able and often initially try introducing problem solving with smaller tasks. For all these reasons using problems as lesson starters seems a good starting off point for the implementation of a problem solving approach.

Despite this, problem solving lesson starters are not being used to any great extent. Over half of the teachers surveyed never used problem solving starters or used them only sometimes. Less than 4% use lesson starters related to content on a daily basis. But, more surprisingly, stand alone problems are used as lesson starters at least once a week by only 20.8% of respondents. This suggests that teachers should be encouraged to use short challenging tasks to initiate problem solving in their classroom. This might be a useful point to begin teacher professional development in this area.

Throughout Lesson

Over one-third of teachers (39.6%) stated that they were using a problem solving approach throughout all of their teaching of a lesson, at least on a weekly basis. This suggests that a regular integrated approach to problem-solving is occurring in a number of classrooms. Some 11.3% (6 people) were committed to teaching problem solving in every lesson. However, in the written responses later in the questionnaire, one of these six teachers suggested that the need for skill and content learning should be the dominant component of classroom work. Yet another said that "less able students must learn key skills first before using problem solving techniques". Hence the percentage given above may be an overstatement of the actual situation.

The reason for the relatively small percentage of teachers using a problem solving approach regularly through all lessons may be found in Burkhardt (1988). He believes that standard teaching methods are largely single track and depend on the method being explained by the teacher. On the other hand, problem solving is multi-track and is led by the methods proposed by individual students. Burkhardt feels that problem solving is difficult for the teacher and he identifies three areas giving rise to this difficulty. These are **mathematical, pedagogical** and **personal**.

Mathematically the teacher must scan the different approaches that students are using and assess how useful each of these may be. The teacher then has to decide how best to complete the mathematical task from each of these starting points.

Pedagogically the teacher must decide when to help and when not to help, and what support and questioning to provide for each student or each group of students.

Personally there is the problem of teacher confidence. Because teachers may be confronted with approaches that they have not considered before, they need to believe that they can determine, possibly with student help, which approaches lead to solutions and which do not. This is a potentially non-trivial barrier to tackling problem solving in class.

Any one of Burkhardt's three areas of difficulty may lead to teachers delaying the introduction of problem solving through whole lessons.

End of Unit

The fifth category considered in the questionnaire is the end of a unit of work. In the past it has been the practice to include word problems at the end of a section in order for students to practise newly learned skills in potentially novel situations. Some 26.5% of the respondents are using problem solving in this way at least monthly. It is worth noting though that only one of the teachers who used problem solving at the end of a unit work was not also using problem solving throughout all of their teaching on a weekly basis. Applying skills to problems at the end of a unit of work seems to be only one component of the teachers' problem solving repertoire.

Extension Students

Many schools encourage their capable students to experience independent problem solving in a variety of ways. These include mathematics or computer clubs, small groups working apart from the normal class, through mathematics competitions or via individual extension material. In a survey of talented mathematics students, Curran, Holton, Daniel and Pek (1992) found that such students enjoyed the challenge and the open approach of problem solving tasks. Burkhardt (1988) highlights a common belief in the need for problem solving for the gifted child but is reserved about how best to balance this need in the gifted students' mathematical diet.

The survey shows that, at least monthly, individual students are being given specific extension opportunities in the area of problem solving by 43.3% of teachers. Other able students will benefit from being exposed to regular classroom problem solving activities. However, of the 56.6% of classrooms which are not providing specific problem solving experiences on a regular basis for their capable students, some 41.3% do not provide opportunities for problem solving in the regular classroom at least monthly. Hence in over 23% of classrooms, bright students are only occasionally being exposed to any form of problem solving.

At this stage it is worth making two points. The first relates to the three difficulties of mathematics, pedagogy and personality that we noted earlier (Burkhardt, 1988). It is often in the hands of bright students that the most creative solutions arise. Assessing the work of these solutions is likely to prove challenging to teachers and this may be one reason why teachers do not provide regular solving tasks for their more able students.

The second point is motivated by Kantowski (1977). This research suggests there is limited evidence to say which students gain most from problem solving. There is certainly much anecdotal evidence to support the view that bright students gain much by being exposed to problem solving through competitions and mathematical camps. But it may be that there are other critical factors in these activities apart from the problem solving.

There is also evidence in Holton et al (1996), and in our current research, which shows that there are students who perform well in problem solving situations who would not be regarded as among the top mathematics students in their particular class. For some reason there are students who do better in problem solving situations than in the standard mathematical activities.

It therefore seems important to try to find which students gain most from a problem solving environment, and what it is they actually gain. For instance, in this research, we have seen lower ability students gain confidence in mathematics as they learn to solve problems. They also are learning to work with a group of their peers on a common problem and to communicate their results to the whole class. In these situations problem solving is starting to span **all** of the aims of the curriculum.

2. What strategies are being promoted?

Pólya (1945) stresses the importance of strategies for solving problems. Strategies are just means to discover a solution. They are almost always not the method of solution nor a justification of the answer. They are, however, a mechanism by which the answer may be found and then justified, if necessary, by some other means.

The suggested learning experiences in the problem solving section of "Mathematics in the New Zealand Curriculum" include the "devising, using and modifying of problem solving strategies". It states

> *Teachers can create opportunities for students to develop these characteristics by encouraging them to practice and learn such simple strategies as guessing and checking, drawing a diagram, making lists, looking for patterns, classifying, substituting, rearranging, putting observations into words, making predictions and developing proofs.* (p. 11)

In the questionnaire, twelve strategies were summarised from the list on page 25 in "Mathematics in the New Zealand Curriculum". These were strategies that we thought would be commonly used by teachers. The results are summarised in Table 2.

Table 2. Problem Solving Strategies Used in the Classroom

Strategy	Percentage of respondents who use this strategy often when using a problem solving approach
Find a pattern/relationship/rule	77.4%
Make a list or table	71.7%
Guess and check	66.0%
Solve a simpler problem first	58.5%
Draw a diagram	54.7%
Work systematically	43.4%
Translate into algebraic or geometric form	37.7%
Think creatively/laterally	24.5%
Use a familiar procedure	22.6%
Work backwards	20.1%
Make a model	13.2%
Act out	13.2%

The predominance of "Find a pattern, etc." is perhaps to be expected. It is one of the easiest techniques to promote because of the natural occurrence of patterning in students' environments. In addition, children come to secondary school with a history of patterning experience from primary school for teachers to draw on. Further, the emphasis on patterning is encouraged by "Mathematics in the New Zealand Curriculum". Exploring patterns and relationships is an achievement objective for all age levels.

We believe that "Make a list" and "Guess and check" are also popular because they are easy to teach and can be applied to a range of problems. Equally it may be that these strategies are well suited to the types of problems that are frequently used in the early years of high school.

Surprisingly though, "Draw a diagram" was only rated as a commonly used strategy by just over half of the teachers. It is not clear whether this is because teachers feel that this is not "proper" mathematics at the secondary level or whether its value as a tool is not appreciated. In the research project we have seen diagrams being of great value, especially to weaker students. It provides a closer representation of the problem than, for instance, algebra does, and hence may make some problems more accessible to students.

A similar accessibility can be provided through the strategies "Make a model" or "Act out". These heuristics give more control of the problem to the students and can provide motivation. Lovitt and Clarke (1988) advocate these approaches, referring to them as "kinesthetic". It is now acknowledged that children learn in different ways and the tactile, physical, pictorial thinkers could well benefit if these strategies and "Draw a diagram" were used more often. It may be that some teachers avoid these strategies because they view them as being more appropriate for primary schools. They may also find the strategies too difficult to implement because of the resources needed and the time constraints on their lessons.

Hembree (1992) conducted an analysis of some 487 reports of mathematics problem solving that have appeared over the past seventy years. He says

> *... students instructed in using heuristic strategies displayed significantly higher scores than students who simply practised (problem solving) ...*

He found that "drawing a picture" and "translating into familiar terms" had by far the largest affect on problem solving performance. It seems likely that teachers should be giving more attention to such strategies than they are.

Only a quarter of respondents regularly discuss "Thinking creatively". It is difficult for a teacher to model with students that "flash of brilliance" which seems to come from nowhere. However, the fostering of such insights by discussion and brainstorming is vital if we are to extend the horizon of students' thinking and to enable them to strike out in new directions. "Mathematics in the New Zealand Curriculum" supports this.

> *Creativity in problem solving is recognised as one of the basic traits that must be developed if outstanding achievement is to result, and it plays a major role in innovation and scientific discovery ...* (p. 11)

Our research (see for example, Holton et al 1996) supports Hembree's conclusion that strategies need to be taught. They also need to be practised otherwise they become forgotten. This seems to be especially true for young students and students who are weak mathematically.

Generally a number of strategies can be used in one problem.

Example:

Greedygrimes Charlie ate a total of 100 jellybeans in 5 days, each day eating 6 more than the previous day. How many jellybeans did he eat on the third day?

This problem was solved by 14 year old students using the following:

(a) averages, then redistributing by adding and subtracting 6.

Day	1	2	3	4	5
Jellybeans	20	20	20	20	20
	8	14	20	26	32

(b) Guess check and improve

Day	1	2	3	4	5	Total
Jellybeans	10	16	22	28	34	110 / too big
	8	14	20	26	32	100 / correct

(c) Algebra

(i) By letting x = no. eaten on the first day

So $x + (x + 6) + (x + 12) + (x + 18) + (x + 24) = 100$

$5x + 60 = 100$

$x = 8$

Thus $x + 12 = 20$ so 20 jellybeans on the third day

and (ii) By letting x = no. eaten on the third day

so $(x - 12) + (x - 6) + x + (x + 6) + (x + 12) = 100$

$5x = 100$

$x = 20$

The "Australian National Statement on Mathematics" says that the methods of good problem solvers are likely to be "idiosyncratic" and that students need to discuss a variety of strategies to increase their "awareness of the range of techniques available". This is supported by Beagle (1979), Pólya (1945) and Schoenfeld (1992). The first of these authors states

> *... problem solving strategies are both problem and student specific often enough to suggest that finding one (or few) strategies which should be taught to all, or most students, is far too simplistic.* (pp. 145-146)

It is not clear, though, how heuristics should be taught. Holton (1994) and Neyland (1994) share a concern that teaching should not lead to strategies being employed like another set of algorithms or rules. They also say that students should be encouraged to see how to translate strategies to other problems and situations. Strategy usage and teaching is not straightforward and much more research is required in these areas.

3. What is the relative importance of skills and problem solving?

"Mathematics in the New Zealand Curriculum" states

> *While fluency with basic techniques is very important, such routines only become useful tools when students can apply them to realistic problems.* (p. 11)

The purpose of the question "How do you see skills and content learning and a problem solving approach knitting together in the classroom?" was to see whether in practice teachers agree with the curriculum and, if they do, how they interweave these two areas of teaching and learning.

In the literature there is evidence that both skills and problem solving are important aspects of mathematics, and that these two facets of the subject should work together. Holton (1993) says that

> *... mathematics encompasses both discovery and fact. We should provide the opportunity for our students to experience both these aspects of the subject.*

Earlier we quoted NCTM (1980) where problem solving is seen as "a process that should permeate the entire program"; "A National Statement on Mathematics for Australian Schools" (1990, p. 37) which says something similar; and "Mathematics in the New Zealand Curriculum", (1992, p. 11) where there is more on the same view. The question is how this should be put into practice. It is clear that skills must aid process. As Lovitt and Clarke (1988, p. 467) say "the most elaborate and extensive toolkit is of little use if it remains unopened". On the other hand process ability without skills is of little value. Siemon and Booker

(1990) say "the most elaborate and extensive set of keys is of little use if the toolbox is empty. But can skills aid the development of problem solving and can problem solving assist in the acquisition of skills? And are these two happening in classrooms?"

The teachers in the survey divide roughly equally in the way shown in Table 3.

Table 3. Skill and Content versus Problem Solving

	Category	Percentage Response
I.	Skill and content learning and problem solving should be integrated together	22.6%
II.	Skill and content learning and problem solving are linked but problem solving is used to motivate skills and separate skill learning should be promoted.	37.7%
III.	Skill and content learning must come first.	32.1%
IV.	No response	7.5%

The fact that there is a roughly equal split on the issue seems evidence of a real dilemma. One teacher said "I'm not sure if this is a simple chicken and egg situation."

That problem solving can be used to teach skills is argued both in Siemon and Booker (1990) and Holton et al (1996). In the latter reference, it was felt that some problem solving experience was necessary before mathematical skills could be taught effectively through problem solving.

In the research project that is part of the basis for this paper, we have used problem solving to teach probability, introductory algebra and general formula development, tessellations, angles in polygons, area concepts and trigonometry.

It may be that all the categories of Table 3 are appropriate and effective for different teachers and for different students. Research needs to establish the optimum conditions for the learning and teaching of mathematics. It is likely though that no single approach will be optimum all the time. Teachers will probably need to vary their teaching in order to maximise student learning.

4. How would you like to use problem solving in the future?

Just under 60% of the teachers surveyed want to use problem solving more often, suggesting that a receptive environment for future development does exist. They do identify a number of common difficulties though. Among these are

- the time necessary for resource collation and staff preparation;
- a feeling that problem solving takes more time;
- a feeling that assessing problem solving is difficult;
- a feeling that lessons are difficult to control;
- a perception that problem solving is difficult for weaker students.

Our experience is that problem solving does take more time, especially in the early stages. However, time can be saved later. One of the teachers in the research project commented that using a group investigative approach to construction tasks and locus problems enabled her students to develop a number of concepts in one lesson which usually took her two or three lessons of more structured teaching. This approach also freed her to help individuals who were struggling, resulting in all students achieving more than usual in the time.

There is a real concern for the needs of less able students. This relates in particular to their reading ability. One of the classes in our research project is a low ability class. They work effectively at problem solving. This may be because all the students are of comparable ability, because the teacher uses problems that can be completed in ten to fifteen minutes, because the reading level required by the problems is not too high or because the teacher chooses problems in a single lesson that require the same strategies.

Generally the teachers surveyed prefer to use a variety of techniques. They also want to use problem solving in a variety of ways including lesson starters, for extension work, as a means of providing variety to their lessons and to motivate skill learning.

5. What help do you need with problem solving?

Burkhardt (1988) lists four important areas in which teachers need support for change. These are as follows:

- a climate in which there is encouragement or pressure on them to introduce new ideas;
- appropriately supportive material for use in the classroom;
- in-service and pre-service training focusing on a broad range of teaching strategies;
- time for reflection and discussion.

Burkhardt goes on to say that he knows of no case where all of these have been instituted simultaneously. This has certainly been the case in the New Zealand situation. The pressure for change certainly exists because the new curriculum requiring the teaching of problem solving and other content areas is a legal document.

It is true that there is supportive material available but many teachers do not know how to access much of this material. This knowledge is slowly being circulated on teacher networks. But material alone will not lead to good problem solving in schools. Lovitt (1995) notes that originally it was thought that providing teachers with rich mathematical activities would naturally produce a problem solving approach in classrooms. He says that this has not happened in Australia. The situation is more complicated than this as we have noted earlier (see also Holton, Spicer and Thomas, 1995).

As far as pre-service training is concerned, this is left to individual teacher training institutions. No doubt as lecturers come to grips with problem solving themselves, the level of delivery will improve. On the other hand, especially with primary trainees, there may be important problems to tackle such as the students' poor mathematical knowledge. The time spent on bringing this to an acceptable level may significantly decrease the time available to spend on problem solving.

In the initial stages of the introduction of the New Zealand Mathematics curriculum, there was professional development for some teachers. However, the contracts that were let for this professional development were for the whole curriculum and not just problem solving, they involved less than 20 hours contact in one year and were not ongoing, they did not cover every teacher though they aimed to cover every school, and it is not clear that all contract deliverers had a sufficient overview of the requirements for the teaching of problem solving.

Finally there has been little time for reflection at all age levels. In primary school, teachers have had to deal with the almost simultaneous introduction of new curricula in several subject areas. In secondary schools, specialist mathematics teachers are having to come to grips with both a new mathematics curriculum and a new assessment process.

On the matter of change Shuard, Walsh, Goodwin and Worcester (1990, p. 27) make the following point

> *The process of change should be gradual and the teacher needs the freedom to experiment and make change. Although it is important that the teacher should be guided by advisors, advisory teachers and coordinators at the beginning, eventually this work and style of teaching should be self-initiated.*

The political climate that existed with the introduction of the new curricula in New Zealand militated against the gradual change advocated by Shuard.

The teachers in the survey overwhelmingly requested pre-prepared resources and problems that would fit in with content areas. Some 73.6% of the respondents asked for resources of this kind. Some typical comments regarding teacher needs were as follows:

- a scheme of work which fits in content/skills with a problem solving approach in the limited time we have available with students;
- ready to roll resources in connection with syllabus content/skill exercises;
- more time to make up or find problems that are relevant, open-ended, multi-level, etc;
- teachers need a lot of ideas and resources to build up a pile of relevant problems to fit in with the curriculum;
- heaps of problem solving activities catalogued into age levels and topics.

Problem solving activities by themselves are not enough to guarantee good problem solving. However a start in this direction cannot be made without these activities. There is an urgent need to produce them in a form which makes them easily used.

Other comments by teachers included the provision of appropriate physical environments for problem solving. This meant appropriate equipment as well as space. Teachers also felt they needed help and training in scaffolding, heuristics and metacognition (thinking about thinking – an important aspect of problem solving). Some assistance in extension work for bright students would also be appreciated.

Two valuable suggestions were made which could usefully be explored. These are as follows:

- watching other staff use problem solving techniques;
- the pooling of resources between schools.

With the change of the political environment, the Ministry of Education in New Zealand will no longer provide the continuing services it has done in the past. Resources for teachers are no longer a central responsibility and will be left to private enterprise to produce. Until these appear on the scene the next phase of teacher development will be largely left in the hands of the teachers themselves.

Discussion

In the short period since the implementation of the new curriculum, it appears that most teachers are making at least some effort to incorporate problem solving approaches into their mathematics classrooms. At present, problem solving is not a regular feature of most classrooms but it is clear that teachers, given appropriate assistance, are prepared to increase their use of problem solving. This increased use can be facilitated by the production of appropriate content-related problems. Indeed the teachers surveyed see this as the first priority to the further implementation of a problem solving approach to mathematics.

A common pedagogical approach regarding the relative importance of skill and content versus problem solving and how the two areas should be integrated, has not yet been reached. It is likely that it is in fact not necessary for a common approach to be formulated, as a variety of styles are possible and even desirable. Any professional development for teachers should then provide a range or options and allow teachers to choose the path which is best suited for them and their students at a particular time.

Perhaps an easy way for a teacher to introduce problem solving to their pedagogical repertoire is via homework problems or lesson starters which do not lead on to the content of the balance of the class. This puts problem solving into a more controlled environment. Marking homework questions which have used a novel approach should not be as threatening as trying to deal with such situations in front of a group or in front of the whole class. If such novel solutions come up through lesson starters the student providing that solution might be asked to explain the method to the rest of the class. The class could then be encouraged to judge whether the novel solution is valid. Having gained confidence in these relatively closed situations the teacher could develop into a more open style.

Professional development of teachers also needs to take into account the mathematical needs of teachers. This perhaps will involve looking at a range of mathematical problems from across the different content areas of the curriculum and exploring the many strategies which might be used in their solution.

The following questions have arisen from our research and it is clear that further work needs to be undertaken in order to find practical solutions.

- What is the appropriate amount of time to spend on problem solving in the classroom?
- To what type and variety of problem solving activities should students be exposed?
- What is the best way to provide problem solving experiences to less able students?
- What model of strategy teaching will encourage students to transfer problem solving skills to other problems and other subject areas?
- How can we measure the benefits of an integrated approach to problem solving compared to a skills first approach?
- Who should provide the resources necessary to support teachers in the implementation of change?

If the philosophical change to problem solving initiated in various national curricula is to be realised for students in the classroom, then there must be commitment to change from teachers and a commitment to provide support and resources from external agencies. Teachers generally are unable to go far down the road of change by themselves. If governments are unable to provide the support required, then it will have to come from groups of colleagues working together.

Acknowledgements

We would like to thank the Research Section, New Zealand Ministry of Education, for the financial support which allowed us to engage in the research on which this paper is based. Our thanks also go to the many teachers and students who allowed us into their classrooms.

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