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A Comparison Of How Textbooks Present Integer Addition And Subtraction In PRC And USA

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Abstract

The presentations of integer addition and subtraction in ten middle school mathematics textbooks, four from the Peoples' Republic of China (PRC) and six from the United States (US) were compared with respect to the worked-out examples, relevant illustrations, exercises, and irrelevant illustrations. All the PRC texts introduced integer addition by using models of commonly-applied situations - an approach that required the coordination of verbal, pictorial, and symbolic representations. US texts more often used pictorial and symbolic representations to introduce this topic. PRC books consistently presented a common vision of integer operations as a subspace of the rational number operations, a vision not evident in the US texts. Further analysis showed that PRC and US texts failed to (1) fully coordinate representations of integer addition problems with the steps required to solve such problems; and (2) use the same concrete analogy to introduce both integer operations. PRC textbooks also used an inductive approach to develop general addition rules based on absolute values, while only one of the US texts presented a comparable generalization.

International Comparisons

Recent cross-national comparisons of mathematics achievement show that Asian students exhibit superior performance to their US counterparts in computational as well as problem-solving assessment items (Stevenson, Lee, & Stigler, 1986; Travers et al., 1987; Robitaille & Garden, 1989). Particular current studies show Chinese students perform better than their American cohorts on mathematics achievement tests (Stevenson et al., 1990; Lapointe et al., 1992) and at least as well on various problem solving measures (Cai, 1995). Efforts to identify the contributing factors for these observed differences (Lapointe et al., 1992; McKnight et al., 1987) have led to the contention that the amount of mathematics students' study is a key factor, especially in the case of students from Japan and the US (Mayer et al., 1991; Mayer et al., 1995).

In further efforts to find underlying reasons for the observed differences in students' mathematical achievement, researchers have studied both students' inner ability and external influences. One group of studies has linked the origins of cross-national differences to mathematical competence. For example, English-speaking students may be disadvantaged in their early mathematical development by the names of the numbers in their numeration system, particularly when compared to those used in Asian languages (Miller & Stigler, 1987; Stigler et al., 1986). However, this disadvantage is evidently limited to specific deficiencies (e.g., the effect of language structure on children's difficulties in learning to count and account for quantitative change) but not apparently to general problem solving deficiencies (Miller et al., 1995). Another group of studies has attributed much of the evident variation in students' performance when solving word problems to the mode by which problems are presented (Cai, 1995; Mayer et al., 1991).

Convergent evidence has shifted the research focus from students' inner ability to external influences such as the time students spend in school learning mathematics and the different intensity and emphases of mathematical content in textbooks (Mayer et al., 1991; Stigler & Perry, 1988; McKnight et al., 1987). These studies increasingly support an exposure hypothesis, that is, cross-national differentials in students' mathematics achievements are due to differing social-cultural values that impact upon these students' exposure to schooling and mathematics.

These socio-cultural values contribute to differences in pedagogical approach and to the types of instructional materials employed in designing school mathematics curricula. Previous studies (Ferrucci & Evans, 1993; Stigler & Perry, 1988) have noted that cultural-specific beliefs about the nature of mathematical understanding influence the amount and level of abstraction as well as the concrete experiences provided to students during classroom instruction. Others (Ferrucci et al., 1991; Fuson et al., 1988) have found that US textbooks introduce many mathematical topics at a later stage and in much less depth than do Japanese, Chinese, or Russian textbooks. Since there has been no specific investigation of how the presentation of school mathematics content contributes to the observed differences between students from the People's Republic of China and the United States, this study investigated how these countries' school textbooks present particular mathematical content.

The Study

Lessons on addition and subtraction of signed whole numbers in four PRC textbooks and six US textbooks were selected for comparison. Nine of the ten texts were intended for use in what is grade 7 in the US educational system while one of the Chinese texts (Zhong et al., 1993) was intended for grade 6 in the distinctive (5-4) schooling system that operates within certain regions of the PRC. Grade 6 in this 5-4 system corresponds to grade 7 in the systems for which the other nine texts were intended. All Chinese mathematics textbooks were being used in different geographic areas of the PRC and bore the approval of the State Education Commission. The six American textbooks were commonly being used across that country in various settings and with diverse populations.

The selected portions of the Chinese textbooks were translated into English by a mathematics educator who is fluent in both languages. The pertinent lessons in the American and Chinese texts were then coded independently by three raters using criteria and procedures described in Mayer et al. (1995). The raters determined the number of exercises, the number of relevant and irrelevant illustrations, the number of worked examples, the number of words in the explanatory sections, and the models used to describe addition and subtraction of integers for each of the PRC and US texts. There were no unresolved differences between raters on any of these quantitative data.

Further codings identified four major categories of page space: (1) relevant illustrations, (2) irrelevant illustrations, (3) exercises, and (4) explanations and worked-out examples. The raters independently determined the area occupied by these major categories in each of the textbooks and there were no unresolved differences between raters on these measures.

Results and Discussion

The amount of instructional space devoted to the topics of addition and subtraction of integers varied considerably in the textbooks from the two countries. The number of pages for the lessons in the Chinese books ranged from 11 to 18 based on an average page size (without margins) of 9.7 by 15.1 cm. In contrast, the number of pages in the US books ranged from 4 to 21 based on an average page size (less margins) of 16.1 by 22.2 cm. The average number of pages for the lessons was 15.3 in the PRC books and 9.8 in the US books.

A comparison of the worked-out examples in the textbooks found the number to be about the same for both countries, but more practice exercises were included in the American textbooks. The number of exercises in the PRC textbooks ranged from 27 to 61 with an average of 47 per lesson, while the number of exercises in the American texts ranged from 31 to 92 with an average of 63 exercises per lesson. However, the number of worked examples, which were designed for the most part to illustrate the addition and subtraction algorithms for integers, was about the same in both sets of textbooks: the number of examples varied between 11 and 18 in the Chinese books with an average of 14 per text, whereas the US books exhibited a range of 6 to 23 examples with an average of 12 per textbook.

The amount of illustrations (both relevant and irrelevant) found in the American textbooks was larger than that in the Chinese texts. Relevant illustrations (illustrations that represented the steps to the solution of an integer arithmetic problem) numbered between 3 and 26 with an average of 11 illustrations per text in the US materials. The corresponding range for the PRC texts was 0 to 6 with an average of 4 relevant illustrations per text. No irrelevant illustrations were evident in the Chinese lessons, while in the American lessons the range was 0 to 3 with an average of 2 per text.

The sections of the lessons devoted to explanations and worked-out examples contained about the same number of words in both countries' textbooks. These sections contained from 340 to 1,351 words in the US texts for an average of 694 words and from 565 to 860 words in the PRC textbooks for an average of 712 words. Another measure of the instructional value of the texts may be the number of instructional words per exercise, that is, the number of words in the sections of the lessons allocated to explanations and worked-out examples divided by the number of exercises. In the Chinese texts this value ranged from 10 to 26 and the average was 16, while in the American texts the range was 4 to 44 with an average of 15.

Figure 1: Average Number of Worked-out Examples, Relevant Illustrations, Exercises, Irrelevant Illustrations, and Instructional Words per Exercise in US and PRC Textbooks

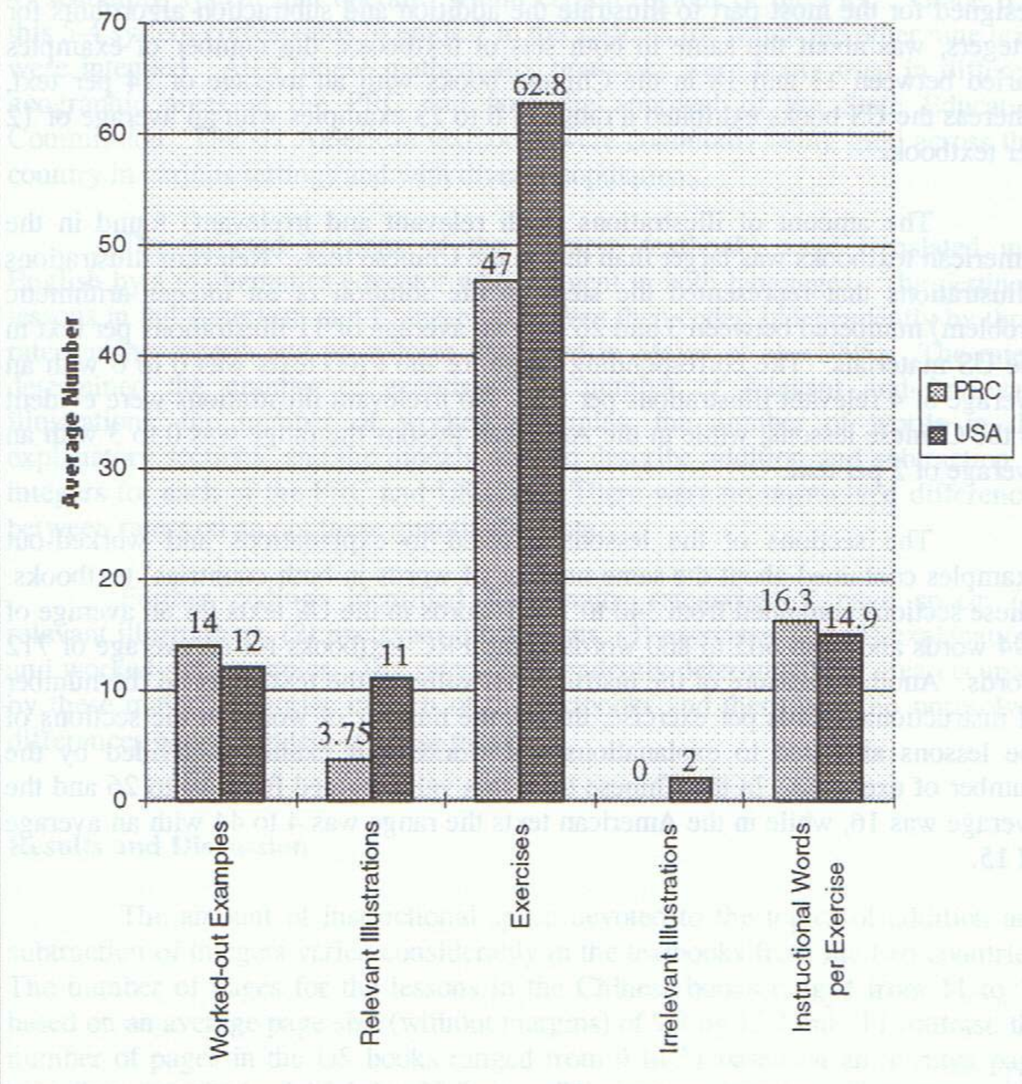

Figure 1 shows the US texts contained an average of one third more exercises than did the Chinese texts. The average number of worked examples was roughly the same in the texts from the two countries, but the American textbooks contained an average of 2 irrelevant illustrations per lesson and nearly three times as many relevant illustrations as did the PRC texts. None of the Chinese lessons was found to contain an irrelevant illustration.

The allocation of page space in the textbooks was determined by computing the area of those sections in each lesson devoted to explanations and worked-out examples, relevant illustrations, exercises, and irrelevant illustrations. The raters' consensus with respect to these area measures is shown in Figure 2.

Figure 2: Percentage of Page Space Devoted to Explanation, Relevant Illustrations, Exercises and Irrelevant Illustrations

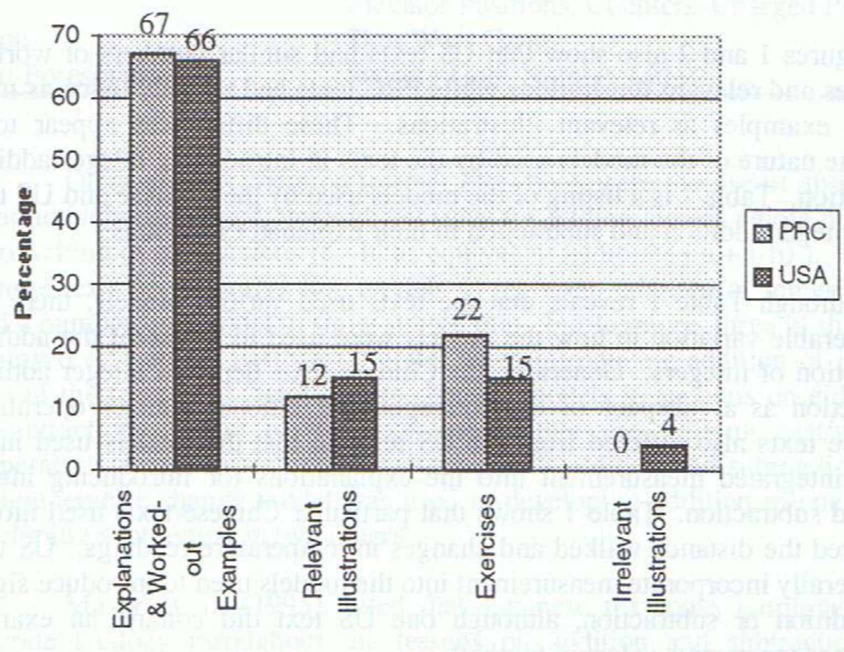

Figure 2 shows that US books devoted 81% of their space to explaining solution procedures and worked-out examples compared to 79% in PRC books. This contrasts with the comparable values of 36% and 81% for US and Japanese texts, respectively, found by Mayer et al. (1995).

An examination of Figures 1 and 2 also shows that the percentage of page space devoted to exercises is less in the US texts, but the US average number of exercises is greater. This finding reinforced the observations by raters that the exercise sets in the PRC texts were less crowded than those in the US texts. One or sometimes two integer addition or subtraction exercises were typically presented per line in the Chinese texts, while there were often three or more columns of such exercises in the American texts. Figures 1 and 2 also indicate that the average number of relevant illustrations in the US texts is twice the average in the PRC texts, but the percentages of page space devoted to relevant illustrations in the PRC and US texts are about the same. This finding also confirmed the raters' observations that, although Chinese books were smaller than the American books, the presentation of information in Chinese appeared to be less compressed or crowded than that presented in the English language texts.

Figures 1 and 2 also show that US texts had similar numbers of worked-out examples and relevant illustrations while PRC texts had roughly twice as many worked-out examples as relevant illustrations. These differences appear to be related to the nature of the models used by the texts in introducing integer addition and subtraction. Table 1 is a listing of the models used by the Chinese and US texts to present integer addition and subtraction in their explanatory sections.

Although Table 1 reveals that the texts used various models, there was also considerable variation in how the models were used to represent the addition and subtraction of integers. Generally, the Chinese texts depicted integer addition and subtraction as a subspace of the corresponding rational number operations. The Chinese texts also differed from the US texts in that the models used in the PRC texts integrated measurement into the explanations for introducing integer addition and subtraction. Table 1 shows that particular Chinese texts used models that measured the distance walked and changes in temperature readings. US texts did not generally incorporate measurement into the models used to introduce signed number addition or subtraction, although one US text did contain an example involving yardage gained or lost in football.

Table 1 : Models used to represent integer addition and subtraction in the textbooks from the Peoples' Republic of China and the United States

Publisher	Models
Beijing Normal	Walking East and West, Number Lines, Absolute Value, Temperature Changes
Guangdong Normal	Walking East and West, Number Lines, Absolute Value
People's Education Press	Walking East and West, Number Lines, Absolute Value, Thermometers
Southwest Normal	Income/Expenses, Thermometers, Absolute Value, Equation Solving
Addison-Wesley	Counters
Harcourt Brace	Counters, Number Lines, Water Pumping, Absolute Value
Heath	Counters, Number Lines
Houghton Miffin	Number Lines, Football Yardage, Absolute Value, Elevator Positions, Counters, Charged Particles
Saxon	Sign Wars Game
Scott Foresman	Scoring Golf, Number Lines

Other aspects common to each PRC book were the use of absolute values in inductively-developed, generalized rules for adding signed whole numbers, and the rewriting of subtractions (a - b) as equivalent additions (a + (-b)). Three of the Chinese texts used number line models to represent addition of integers, but none used a number line model to show subtraction. Furthermore, three of the PRC texts employed a "walking east/west" model when introducing addition of integers and three of the texts used "temperature change" models in sections on either addition or subtraction. None of the texts used either the walking east/west or the temperature change models for developing both addition and subtraction, although the temperature change model was used in developing addition in one text and in developing subtraction in two others.

Mayer et al. (1995) noted that Japanese textbooks employed the same concrete analogy throughout the lessons on addition and subtraction, but this practice was not evident in the Chinese texts. Mayer et al. also suggested that two other positive attributes of Japanese texts were (1) their coordination of multiple representations in word problem examples involving addition of integers with different signs; and (2) their use of an inductive order in introducing general rules for finding the sum of differently signed addends (p. 449). None of the PRC texts completely achieved such a coordination in their presentations, but portions of the necessary representations were apparent for some of the addition problem-solving steps that were explained in all the PRC texts. Moreover, an inductive order was apparent in all the Chinese texts' presentations of integer addition since each text presented and solved a number (2 to 6) of specific examples before stating general rules for the addition of integers with the same and different signs.

The US texts each devoted separate sections to the addition and subtraction of integers. This contrasted with the treatments given in the PRC texts wherein the integer operations were considered within the context of rational number operations. Four of the US texts used number line models to introduce integer addition and two of the textbooks presented an example in which there was a partially coordinated rendering of the verbal, pictorial, and symbolic representations in the solution of a word problem involving addends with different signs.

Two of the US texts presented generalized rules for the addition of integers that involved absolute value models, and one of these texts used an inductive approach in developing the generalizations for addition. This contrasted with the inductive approach being used by all the Chinese texts in developing general addition rules based on absolute values.

A commonality in the models of the US and PRC texts concerned the representation of integer subtraction as addition. All the PRC and US texts employed this model in their explanations. Also, as was the case with the PRC texts, none of the US texts used the same concrete analogy to introduce both addition and subtraction. As previously noted, the use of such common concrete analogies for both arithmetic operations was cited as a commendable aspect of Japanese texts by Mayer et al. (1995).

Another distinction between the use of models in the PRC and US texts concerned the use of concrete problem situations as introductory examples. All the PRC texts used commonly-applied situations represented in verbal form (walking east and west, temperature changes, or income/expenses) to introduce their sections on the integer addition, while four of the six US texts presented symbolic representations of additions and corresponding pictorial representations of counters or number lines at the beginning of their explanations. Stated in another way, all the PRC books introduced the notion of integer addition with a word problem involving a concrete analogy of adding integers. The solutions given for these introductory problems in the Chinese texts involved coordinating verbal, pictorial, and symbolic representations, (or the more difficult coordination of only verbal and symbolic representations) while the US books were more likely to introduce a less demanding symbolic addition example that was solved using a pictorial representation.

Differing cultural beliefs about the nature of mathematical understanding and the role of concrete experiences in the learning process are apt to contribute to some of the different expectations evident in the PRC and US texts. Chandler and Brosnan (1994) noted that US textbooks often tend to limit children's learning in various ways due to educational beliefs about what is feasible to teach at different age levels. For example, it has been documented that US elementary school textbooks introduce the magnitude of numbers at a much slower pace and present the introduction of regrouping of whole numbers in addition and subtraction at a later date than do Japanese and Chinese textbooks (Fuson et al., 1988). Such distinctions were also evident in the occasional presence of a calculator icon next to exercise sets in the US texts. This icon indicated that students might want to use a calculator to complete the exercises. Not only was such a suggestion omitted from each of the PRC exercise sets, but the PRC exercises commonly involved larger numerical values.

A cross-national distinction was also reflected in the more mathematically mature presentation of the topics in the PRC as opposed to the US texts. Integer addition and subtraction were accorded their own sections in the US texts and the explanatory models appear directed at students who are evidently less mathematically refined than their Chinese counterparts. The PRC texts presented models of the integer operations that were directly applicable not only to concrete physical situations, but also to more sophisticated number systems and to more advanced study in mathematics.

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