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Secondary School Students' Generalisations Of Patterns

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Abstract

This paper reports an investigation of 127 secondary school students' ability to recognise patterns and make generalised descriptions of them. Two spatial patterning tasks were presented in either concrete or diagrammatic formats and the students were able to use concrete, diagrammatic or verbal formats to represent them. The SOLO Taxonomy (Biggs & Collis, 1982, 1991) was used to classify the students' generalisations. Analysis identified a progression in the students' ability to recognise generalisations from their representations of spatial patterns, which fits the SOLO model. These were predominantly consistent with Biggs' and Collis' (1982, 1991) description of the ikonic mode, with some concrete symbolic support. Regardless of the SOLO level of their generalisations, the majority of the students chose to use blocks to model at least the fifth step of the pattern before they moved to working from an internal representation. Drawing diagrammatic representations was not a very frequent choice.

Introduction

The purpose of this paper is to report the outcomes of an exploration of types and levels of cognitive functioning underlying the conceptual development of spatial patterns. It focuses specifically on pattern formation and generalisation and links these to the SOLO Taxonomy (Biggs & Collis, 1982, 1991). The focus on spatial patterns was chosen because of the acknowledged importance of spatial thinking in its own right as well as its powerful contribution to mathematical thinking in general (Australian Education Council, 1991; Bishop, 1983; Lean & Clements, 1981; National Council of Teachers of Mathematics, 1989).

Expressing generality from patterns is a notion fundamental to the development of mathematical concepts such as algebra. It is an important component of the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) and is one of the three subheadings in the algebra section of *the National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). The *National Statement on Mathematics for Australian Schools* recommends that children "work with a variety of numerical and spatial patterns, and find ways of expressing the generality inherent in them....leading children to recognise that different descriptions can fit the same spatial arrangements" (p.191). It is important to explore the most effective ways of implementing these ideas in the classroom at all levels.

Several writers (Bishop, 1983; Presmeg, 1992; Thomas and Mulligan, 1994) have acknowledged the importance of encouraging students to use visual processing in order to succeed at mathematical tasks, and this is particularly true of spatial patterning. However, there is evidence that some children have difficulties with visual processing (Bishop, 1983) and that there is a need to understand more about how it can be developed. Kosslyn (1983) contributes to this understanding by defining four stages of image processing: generating an image, inspecting an image to answer questions about it; transforming and operating on an image; and maintaining an image in the service of other mental operations. This is reflected in the *National Statement on Mathematics for Australian Schools,* which claims that, to be able to represent a pattern internally, children first need to be able to see it, then find ways to express it verbally.

In this study, we are concerned with finding out more about the first of Kosslyn's stages, how students go about generating an image in order to be able to see the pattern. This is a critical step. In fact, Resnick and Ford (1981) suggest that "the important intellectual work is over once a representation has been developed" (p.220). In particular, this part of the study is concerned with the kinds of external representations students may need to create in order to be able to transer to a mental representation. First, however, it is important to understand more about what previous research has offered regarding learners' approaches to processing mathematical information, and whether some of these approaches lead to more successful outcomes than others.

There is evidence that successful mathematicians do not necessarily all use the same modes for processing information (Krutetskii, 1976; Shama & Dreyfus, 1994). The modes they use can include verbal-logical and visual-pictorial (Krutetskii, 1976), physical/kinaesthetic, ikonic or notational forms, or various combinations of these (Gardner, 1983; Thomas & Mulligan, 1994). While Mayer and Sims (1994) found that some students do not need visual prompts because they can generate their own representations, others have reported the manipulation of materials (Owens, 1994), drawing diagrams (Resnick & Ford, 1981), or a combination of these (Bishop, 1983) to be important in establishing internal representations and extracting meanings. Krutetskii (1976) claimed that students can be equally successful at mathematics with different correlations between visual-pictorial and verbal-logical components. Watson, Collis and Campbell (1994) comment on the need for all of these forms to be used to support instruction in the early high school years.

In spite of this knowledge, there is evidence of a mis-match between students' preferred methods of processing information and the way in which the information is presented to them (Resnick, 1992). Resnick suggests that this may be due to failure to encourage children to build on to their already established, intuitive ideas about mathematics. The contributions of Biggs and Collis (1991) and others (for example, Campbell, Watson & Collis, 1992; Collis, Watson & Campbell, 1993; Watson et al., 1994) explore this notion of multimodal functioning, particularly in relation to the ikonic and concrete symbolic modes of the SOLO Taxonomy (Biggs & Collis, 1982).

Earlier studies (Robertson & Taplin, 1994; Taplin & Robertson, 1995) with Year 7 students suggested a progression in the sophistication of students' expressions of generalisation which were consistent with those described by the ikonic mode of the SOLO Taxonomy (Biggs & Collis, 1982, 1991; Campbell et al., 1992). In particular, there was a suggestion of a progression through unistructural-multistructural-relational levels, with the relational level responses showing some hint of concrete-symbolic thinking. Furthermore, Robertson and Taplin (1994) found that when students were asked to describe the 5th, 10th, 100th and nth terms of patterns, the majority of students chose to make concrete or diagrammatic representations of at least the fifth and tenth terms before they began to work from an internal representation of the pattern. While the studies did not distinguish between the functional levels at which students *chose* to operate, and the optimum levels at which they were *capable* of operating, they posed the question of whether students eventually reached a stage in the development of their thinking where they transcended the need for concreteness (Collis et al., 1993, p.119). Consequently, two major questions arose from the earlier studies:

1. Is the suggestion of a unistructural-multistructural-relational cycle in the ikonic mode supported by similar styles in subsequent modes of the SOLO taxonomy?

2. At what stage in the development of expression of generalisations about patterns, are students able to internalise and generate the pattern without needing to make an external concrete or diagrammatic representation?

In attempting to answer these questions, data were collected from students in the first and last years of secondary schooling at three schools.

The Sample

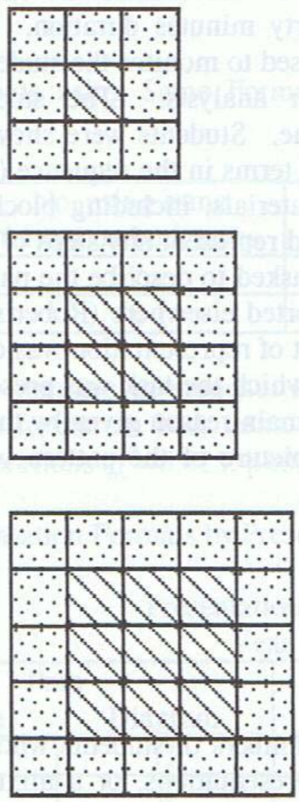
Three schools were used for data collection. One was located in a low-income urban fringe neighbourhood with high levels of public housing, unemployment and complex family structures. The second school was located in a middle income urban neighbourhood where employment and family structure appeared to be much more stable. The third was in a rural community. In each school, random samples of gender balanced groups were selected from each of Grades 7, aged 12-13 years (N=65), and 10, aged 15-16 (N=62), from populations described by their teachers as being of average ability.

Tasks

Two formal mathematical tasks were selected in which students were asked to express generalisations from patterns. These tasks were chosen for the following reasons: they are spatial in nature, they are suitable for representation in different formats, namely physical/kinaesthetic, visual-pictorial, and verbal-logical representation, and they are typical of patterning tasks recommended in documents such as the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics,1989) and *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991). The two patterns are represented, in pictorial form, in Figure 1.

In order to ensure that the format in which the Experimenter presented the task was not likely to influence the student's form of representing it, the tasks were presented in alternating formats: concrete modelling, in which a representation of the pattern was made from blocks or other materials, and diagram, in which a two-dimensional representation was given.

The Path Pattern



The Step Pattern

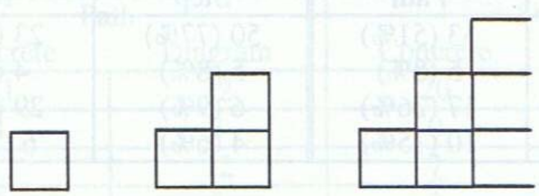


Figure 1. The Tasks

Procedure

Data were collected in individual clinical interviews, each of approximately twenty to thirty minutes duration. Observation and Teachback (Pask 1976) strategies were used to monitor the students' responses and interviews were tape-recorded for later analysis. The students were given the tasks individually and one at a time. Students were shown the patterns and asked to identify the 5^{th}, 10^{th} and 100^{th} terms in the sequence (Orton & Orton,1994). They were given a selection of materials, including blocks, squared paper and blank paper, and told that they could represent any steps of the patterns in whatever way they chose. They were then asked to describe the pattern and a generalisation for "any term". It has been reported elsewhere (Robertson & Taplin, 1994) that the most frequently chosen format of representation was concrete modelling. This was irrespective of the format in which the task was presented. Robertson and Taplin (1994) also reported that the main reason given by the students for this preference was that it gave a physical picture of the pattern which was quick and easy to construct.

Results

Format of Presentation

Table 1 shows the numbers of students who used, respectively, concrete, diagrammatic and verbal representations, or a mixture of these (that is, started with one and switched to another) for each of the two tasks.

Table 1 : Forms of Representation of the Tasks by Number of Students

	Year 7 (N=65)		Year 10 (N=62)	
	Path	Step	Path	Step
Concrete	33 (51%)	50 (77%)	23 (37%)	31 (50%)
Diagram	5 (8%)	5 (8%)	4 (6%)	3 (5%)
Verbal	17 (26%)	6 (9%)	29 (47%)	24 (39%)
Mixture	10 (15%)	4 (6%)	6 (10%)	4 (6%)

In the Year 7 group, there was a clear tendency for the majority of students to represent the pattern concretely. This was also the preferred format for the Year 10 students on the Step task although on the Path task there were more students who went straight to a verbal representation of the task. On both tasks, there were more Year 10s than Year 7s who represented the task verbally from the start. In both year groups, only small numbers of students chose to represent the problem in diagram format. This is particularly interesting, considering that most secondary teaching which focuses on giving visual representations is presented diagrammatically.

Table 2 : Numbers of Students using Same Format of Representation on Both Tasks

	No. using same format on both tasks	No. using different formats
Year 7	37	28
Year 10	48	14

Table 2 shows the numbers of students in each year group who were consistent in using the same format on both tasks. A chi-square test indicated a significant difference in proportions (χ^2=6.023, $p<0.05$).

Table 3 : Students' Representation Formats by Presentation Format of Task

Presentation Format
Year 7

	Path		Step	
	Concrete	Diagram	Concrete	Diagram
Concrete	18	15	28	22
Diagram	3	2	0	5
Verbal	5	9	2	4
Mixed	5	5	1	3

Presentation Format
Year 10

	Path		Step	
	Concrete	Diagram	Concrete	Diagram
Concrete	11	12	20	11
Diagram	1	3	0	3
Verbal	17	12	7	17
Mixed	3	3	3	1

As Table 3 indicates, for the Year 7 subjects this preference for representing in concrete form was independent of the format in which the task was presented. For the Year 10 group, on the Path task the chosen form of representation appeared to be independent of the presentation format. On the Step task, however, there was a tendency for the students to give concrete representations of the tasks which were presented in concrete form, and verbal represenations of those presented as diagrams. A small number of students was categorised as having given "mixed" representations because they began a task with, for example, a concrete representation, and switched to another form, such as diagram, before finally moving into verbal format.

SOLO Levels

In the preliminary study, Taplin and Robertson (1995) classified students' expressions of generalisations into four categories consistent with the SOLO Taxonomy. These categories were used to classify the responses of the Year 7 and Year 10 students in this study. It was interesting to note that there were no new categories added at all from the data of the Year 10 students. These categories are shown in Table 4.

Table 4 : Classification of Students' Responses by SOLO Taxonomy

SOLO Classification	Type of Response (Path Task)	Type of Response (Step Task)
pre-structural	no particular system: *unable to give explanation* *"probably have 50 more blocks - about 60-70"* *"huge, bigger than all the others"* *"in lower numbers, like 10, take away 1 from the step number; in higher numbers, like 60, take away 10"*	no particular system: *unable to give explanation* *"have to fill up the gaps to make a square or whatever shape it was"* *"putting on all the layers till it gets to the top"* *"as big as the wall of a room-half height of wall by about 1 metre"*

Table 4 (cont'd)

SOLO Classification	Type of Response (Path Task)	Type of Response (Step Task)
unistructural	visual description: *"all squares with a span in the middle of them (pretty big, huge, really big)"* counting on: *counting total number of squares - counting on in square numbers until required step reached* *1-1 matching (e.g. 100th has 100 along each side) - ignored squares in middle* *1-1 matching as above, but acknowledged 2 squares less in middle* *counting on in 4s*	visual description: *"like steps going up every time"* *"equal amount on each side then going uphill"* *"keep multiplying like a half-built wall"* counting on: *counting blocks along bottom and end, ignoring those in middle (5th has 5, 10th has 10 etc.)* *counting from representation* *counting back (start with step number and subtract one for each row)*

Table 4 (cont'd)

SOLO Classification	Type of Response (Path Task)	Type of Response (Step Task)
multistructural	looking for patterns which are logical to Ss, possibly based on previous experiences: *multiplied step number by 4 & subtracted 1 for middle* *Nx3 in middle (i.e. 10^{th} is 10x3) and 1 around edge* *recognised N+2 along each side but ignored number of squares in middle* *recognised NxN in middle but ignored number of squares on outside*	looking for patterns which are logical to Ss, possibly based on previous experiences: *multiples of previous terms (10^{th}=5thx2, 100^{th}=10thx10)* *2^{nd} adds on 2, 3^{rd} adds on 3, 100^{th} adds on 100 to 99^{th} etc., but adding from start each time (1+2+3...)*
relational	recognition of a generalisation (not expressed algebraically): *recognised NxN in middle <u>and</u> 2(N+2)+2N around outside*	recognition of a generalisation (not expressed algebraically): *numerical pattern - 3 6 10 15 (but unable to generalise a formula)* *times middle one by length along bottom (recognised that it would not work for even numbers)*

The summary of responses to the Path Task, shown in Table 4, indicates a clear progression in the sophistication of responses, even within categories. The least sophisticated of the unistructural responses were consistent with the ikonic mode of the SOLO taxonomy, and were based on visual descriptions. The counting on strategies started with the simple incorrect notion of one-to-one matching between the step number and the number of numbers along each side of the square. A more sophisticated response in this category was to count on in fours, each successive four representing the four new corner squares added for the subsequent term. The responses to this task also reflect the trend for most students to offer strategies employing counting on or looking for patterns. On the Step task there were, again, four distinct types of response: those which reflected no particular system, those which involved counting on, looking for patterns and correct recognition of a generalisation. The counting on and counting back strategies progressed from partial counting of the steps, omitting some, to directly counting all steps, and to counting back from the given step number. As with the previous task, several pupils applied the strategy of working from multiples of the fifth term. A more sophisticated approach, again similar to that used on the Path Task, was to recognise the arithmetic progression, but to need to start adding on from the first term. As with the previous task, the majority of students gave responses which involved counting on or looking for patterns - and the majority of these strategies did not lead to success.

Figure 2 shows the numbers of students whose responses were consistent with each of these categories.

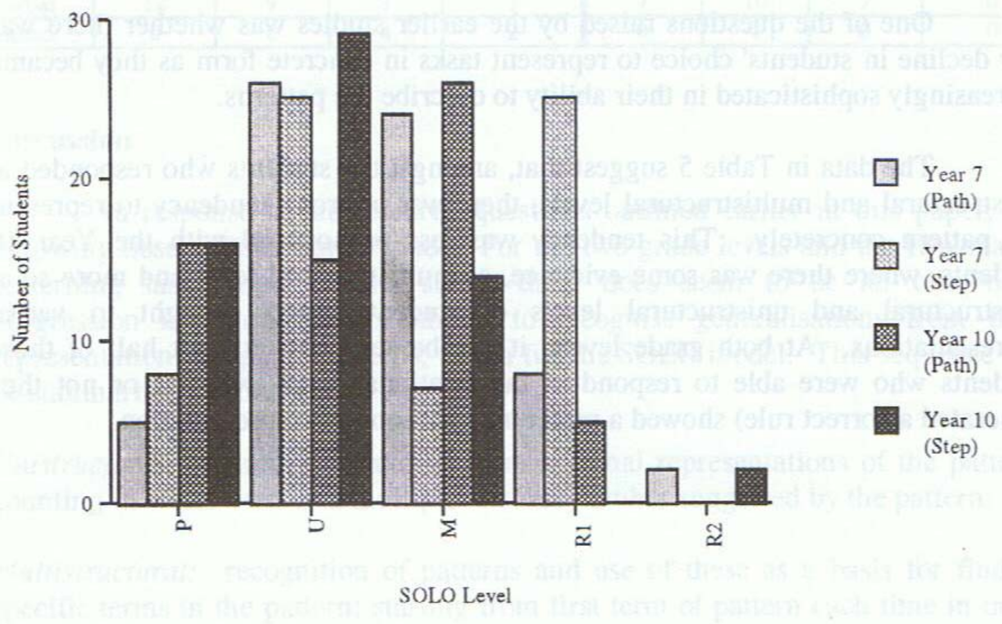


Figure 2. Frequency of Responses by SOLO Level and Grade

From Figure 2, it can be seen that the majority of Year 7 students had unistructural (U) and multistructural (M) responses to the Path task. The Year 10 students gave predominantly multistructural responses, although there were quite high numbers of prestructural (P) and unistructural responses from this group as well. On the Step task, 25 Year 7 students gave unistructural responses. A further 25 gave responses which reflected relational level thinking, although they were not able to give correct solutions (R1). There were very small numbers of students who could give correct, relational level responses which were hinting at the concrete symbolic mode (R2): 2 Year 7 students on the Path task and 2 Year 10s on the Step task. The Year 7 responses seem to be more sophisticated than the Year 10 students, a large number of whom responded at the prestructural, unistructural and multistructural levels. These responses were nearly all consistent with the ikonic mode of the SOLO Taxonomy (Biggs & Collis, 1982). While the students made some use of concrete symbolic counting and patterning strategies, they relied primarily on intuitive strategies to describe the patterns. It was interesting to note that this reliance on intuitive strategies was as evident with the older, Year 10, pupils as it was with the younger ones.

<u>Interaction Between SOLO Level of Representation and Preferred Format of Representation.</u>

One of the questions raised by the earlier studies was whether there was any decline in students' choice to represent tasks in concrete form as they became increasingly sophisticated in their ability to describe the patterns.

The data in Table 5 suggest that, amongst the students who responded at unistructural and multistructural levels, there was a strong tendency to represent the pattern concretely. This tendency was less pronounced with the Year 10 students, where there was some evidence, at multistructural level and more so at prestructural and unistructural levels of students going straight to verbal representations. At both grade levels, it can be seen that at least half of those students who were able to respond at the relational level (whether or not they generated a correct rule) showed a preference for concrete representation.

Table 5 : SOLO Taxonomy Level of Responses by Format of Representation of Task

Year 7

	Path SOLO LEVEL				Step SOLO LEVEL			
	Pre-structural	Uni-structural	Multi-structural	Relational	Pre-structural	Uni-structural	Multi-structural	Relational
Concrete	1	16	11	5	7	19	6	18
Diagram	0	2	3	0	1	2	0	2
Verbal	3	6	6	2	0	3	1	2
Mixed	1	2	4	3	0	1	0	3

Year 10

	Path SOLO LEVEL				Step SOLO LEVEL			
	Pre-structural	Uni-structural	Multi-structural	Relational	Pre-structural	Uni-structural	Multi-structural	Relational
Concrete	4	3	14	2	8	14	6	2
Diagram	0	3	1	0	1	1	1	0
Verbal	12	9	7	1	7	10	7	0
Mixed	0	0	4	2	0	4	0	0

Discussion

In response to the research questions outlined earlier in this paper, the following observations can be made. For the two grade levels and the two spatial patterning tasks used in this study, there does seem to be an observable progression in the students' ability to recognise generalisations from their representations of spatial patterns, which fits the SOLO model. This sequence can be summarised as follows:

Unistructural: counting on, mostly from external representations of the pattern; counting in either ones or a multiple of some number suggested by the pattern.

Multistructural: recognition of patterns and use of these as a basis for finding specific terms in the pattern; starting from first term of pattern each time in order to calculate a given term.

Relational: recognition of patterns and use of these to predict any given term directly, without needing to start from first term; articulation of generalisation, but not in algebraic terms.

The unistructural type of response was efficient for calculating the fifth and tenth terms of the patterns, but students either lost patience or made arithmetical errors when trying to calculate bigger terms, such as the hundredth. It was not possible for the students operating at this level to make generalisations about the patterns. Responses at the multistructural level offered more efficient systems for calculating bigger terms, but most students were still unable to generalise using these approaches. The relational level responses allowed a more efficient system for generalising. However, several of the students responding at this level gave incorrect formulae and did not seem to have systems for checking the validity of these formulae.

The above suggested a unistructural-multistructural-relational cycle (Campbell et al., 1992). Responses at the unistructural and multistructural levels were consistent with Biggs' and Collis' (1982, 1991) description of the ikonic mode, with some concrete symbolic support. The students drew on some concrete symbolic experiences with counting and patterning, but used intuitive strategies to try to make this previous knowledge fit the patterning tasks they were given. At the relational level there was some suggestion of transition to concrete symbolic mode. It is somewhat alarming that there was no suggestion of further development beyond the Year 7 level with this particular type of task - and perhaps even that some Year 10 students performed at lower levels than the younger group.

Regardless of the unistructural-multistructural-relational cycle, the majority of the students chose to use blocks to model at least the fifth step of the pattern before they moved to working from an internal representation. Drawing diagrammatic representations was not a very frequent choice. Furthermore, with the exception of the Year 10 group on the Step task, this generally occured regardless of the format in which the task was presented to them. This could support the idea proposed by Campbell et al. (1992), that teachers should be encouraging modelling and drawing as an important step towards efficient mental processing of the information, and in fact indicate that modelling should be encouraged to a greater extent than drawing. This question warrants further investigation. It also suggests that older students do not necessarily "transcend the need for concreteness" (Collis et al., 1993, p.119), as we might expect them to do.

Implications for Teaching and Further Research

Several teaching implications and further research questions have arisen from the findings of this study. One of these is why there were so many students, even at Year 10 level, who gave prestructural and unistructural responses. We need to find out more about why the students did not seem to have progressed in their ability to form generalisations from patterns, so that we can find strategies to move from ikonic to concrete-symbolic modes of thinking. One explanation could be that the design of this study did not allow for distinguishing between the functional level at which they *chose* to represent the task and the optimum level at which they *were capable of* representing it (Lamborn and Fischer, 1988; Watson, et al., 1994). However, another possible explanation that warrants further investigation could be that students receive some exposure to this type of activity in the lower secondary school, but it is assumed that they do not need it any more in the later years. If this is the case, then the results of this study suggest there may be a need to expose pupils to this type of task throughout their secondary schooling. A further research question would be to explore the extent to which the ability to generalise from patterns at a relational level contributes to later understanding of other algebraic principles.

Another implication for teachers arises from the need, indicated by this study, for students to use concrete materials to model tasks, even when they are older or capable of higher (relational) level thinking. This suggests that teachers should be encouraged to make greater use of concrete modelling throughout secondary school, not just in the lower grades. The infrequency of use of diagrams also has implications, since this is probably the form of representation used more commonly by teachers in the upper secondary school.

In addition, the findings of this study suggest some further implications for future research. One of these is the need to investigate the link between students' representations of spatial patterning in these structured algebraic tasks and problem solving in "real" space, such as the interpretation of maps, graphs and charts (Bishop, 1983). Another question which arises is the need to consider "non-mathematical variables, such as student motivation, work habits, teaching, and language performance which could contribute significantly to mathematical performance" (Lean & Clements, 1981, p.296).

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