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Children's Productive Thinking In Solving A Ratio And Proportion Problem

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Abstract

Based on the production system theory, an information processing taxonomy (IPT) model was postulated to extend the production system by including a second level of production of information. The aim of this paper is to report on evidence from children's thinking to verify the features of the model particularly the extension of the production system. The results of investigation show that both the written and verbal statements of children working on a ratio and proportion problem support the mathematical thinking features spelt out in the IPT Model.

Introduction

The Information Processing Taxonomy (IPT) Model (Fong, 1992, 1994) hypothesised five hierarchical levels of mathematical thinking to explain problem-solving processes. Each level has at least one of the following features: retrieving information from external sources, operation of information in Short Term Memory (STM) and evoking actions from the Long Term Memory (LTM) at primary level and/or secondary level. The following paragraphs describe briefly the essential features of the model.

The **primary level** refers to the level at which a problem solver is able to use external-source data to evoke further information from the LTM. At the **secondary level**, the production of information refers to the activation of the production of information from the secondary source (i.e. a source other than the external source). This requires the problem solver to relate the actions of the earlier production with the processes of the next stage of production. Thus at this later level, the "action (or elicited) information" from the primary level becomes the "condition information" of the secondary level productions of information.

The aim of this paper is to determine the extent of the model with special reference to those features mentioned above which could be used to explain the cognitive processes of pupils in solving a ratio and proportion problem. The following paragraphs describe the methods and procedures used to achieve this objective.

Methods and Procedures

The method of study was to tape-record some pupils' verbal statements of solving a ratio and proportion problem. As a result, a set of data was obtained from the tape-recorded statements of 12 pupils from various schools. They comprised 4 high, 4 average and 4 low ability pupils. The Model assumes the two levels of production of information which operate on the various types of information stored in different parts of children's memories (STM and LTM). The recorded statements from the interviewed pupils are evidence meant for supporting the assumptions concerning the two levels of production of information.

Children's mental processes were first represented using diagrams to show the links between various types of information and their operated information. From the diagrammatic representations of pupils' cognitive processes and their recorded verbal data, three cognitive processes are investigated upon: retrieval of information, primary production of information and secondary production of information. These are specially selected because they are the processes which can be explained in the pupils' verbal statements.

Within the realm of each process (retrieval of information, primary and secondary productions), interpreted cognitive processes from different strategies of the 12 pupils interviewed are identified in diagrammatic form. This was carried out by cross-referencing different pupils' verbal statements. Verbal statements are only reported wherever they are available.

Analysis of Pupils' Responses to a Ratio and Proportion Problem

The problem which was administered to the 12 pupils is as follows:

Circles P and Q overlap at R. The overlapping part represents 36% of circle P and 18% of circle Q. If the area of R is 54 cm^2, what is the total area of the non-overlapping part?

Seven strategies were used by the pupils to solve the problem. However, because of limitation of space, they are not discussed in this paper.

The ratio and proportion problem was pitched at the highest level according to the IPT Model. At this level, pupils are expected to apply the secondary production of information besides retrieving information from external sources and the primary production of information. The following paragraphs describe an example which illustrates how the features of the model can be used to explain the problem solvers' mental operation. They are examined in the following sequence: the retrieval of information from the external source, the use of primary and secondary productions of information. In each of them, pupils' verbal statements are extracted to suggest the written statements.

(i) Retrieving Information from External Source

The pupils' written solutions and verbal statements to the question were examined. Evidence shows that some statements were written by retrieving information from an external source. The following is an example of statements which indicate the retrieval of information from an external source. They are supported by evidence from verbal statements.

External source: The overlapping part represents 36% of circle of P and 18% of circle Q. (abbreviated by R=36%, R=18%)

Evidence from Written Solution:

% of overlapped part for P=36%
% of overlapped part for Q=18%

Evidence from Verbal Statement:

Pete: The overlapping part represents 36% of circle P....this shaded part is 36%.....18% of circle Q is the overlapping part. But the overlapping parts, the shaded parts represents 36%.

David: You know that this shaped part is 36% of circle P....and this same area here is 18% of circle Q....just match these two this R is 36% of P.

The example above illustrates how the retrieval of information feature of the Model could be used to explain the statements made by the problem solvers. Notice that the external source statement, the overlapping part of P=36% or Q=18% was directly decoded verbally by Pete and David. In Pete's case, the decoding of information was extended to associate the % of each circle with the overlapping part. In David's statement, productions of information led him to equate the two percentages (18% and 36%) representing the same area. The results here show that there are sufficient reasons to confirm that retrieving of external information occurs in the problem-solving processes.

(ii) Primary Production of Information

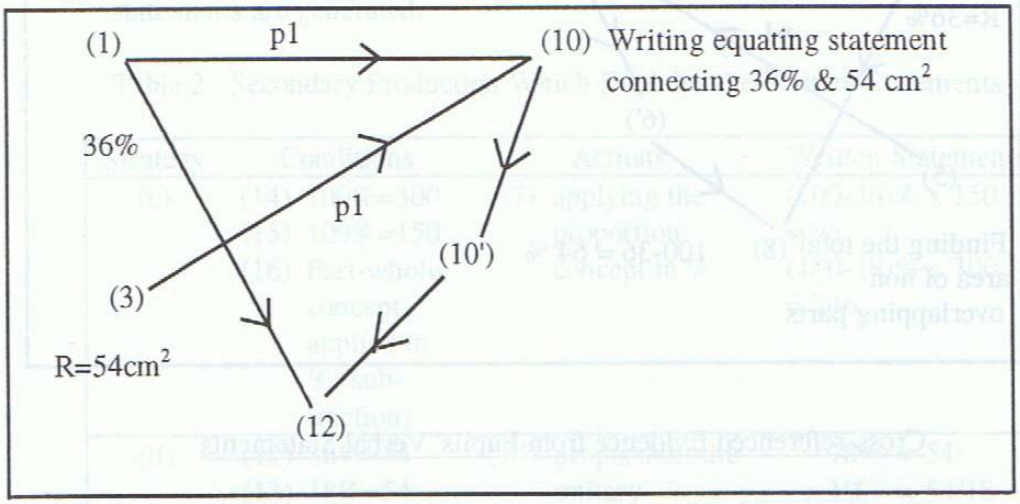
Primary production of information is one of the features of the IPT Model. An analysis of the seven strategies used for solving the problem shows that this feature could explain some of the written and verbal statements of the problem solvers' solutions. Table 1 provides an example which explains the use of primary-production rules leading to the written statement.

Table 1 : Primary Production Which Explains the Written Statements

Strategy	Conditions	Actions	Written Statements
(a)	(1) R=36% (3) R=54 cm^2	(6) Writing equating statement connecting 2 quantities 36% and 54 cm^2	36% = 54 cm^2
(b)	(5) Finding the total area of the non-overlapping parts	(19) Part whole concept to find the sum of two parts	96 + 246 = 342

Observations of statements from pupils' solutions show that there are two instances in which primary production of information could be explained. The first production pair of information is given by "R=36%, R=54cm^2" — p1→Writing equating statements connecting 2 quantities 36% and its area 54cm^2 (note: p1 means evoking information at the primary level). This explains the written statement 36%=54cm^2. In the second instance, the written statement 96+246=342 is explained by: "Finding the total area of the non-overlapping parts — p1→ part-whole concept to find the sum of parts". The above production pairs which explain the derivation of statements are supported by the verbal statements as shown in Figure 1 and 2 as follows.

Figure 1. Interpreted Cognitive Thinking Processes from Pupil's Strategy (a)



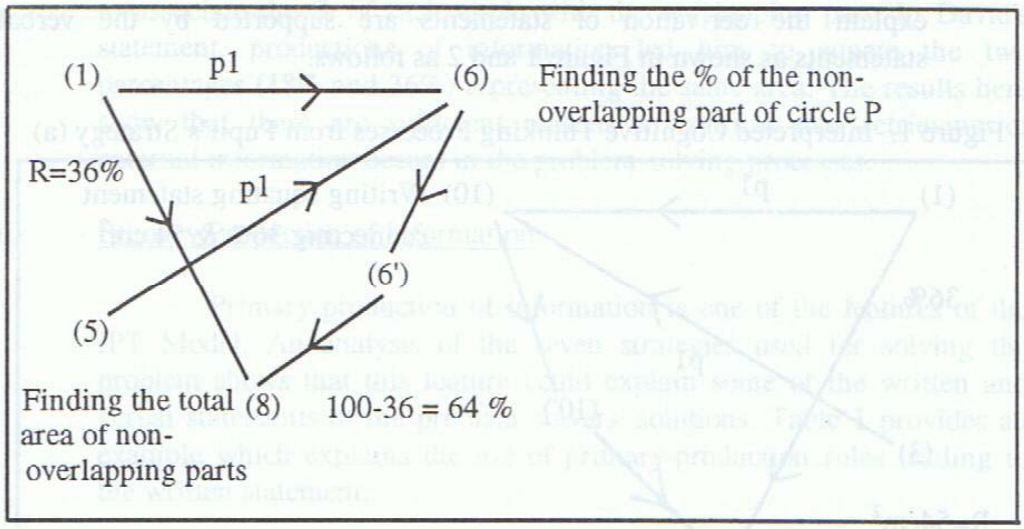
Cross-referenced Evidence from Pupils' Verbal Statements

Pete: (a) Because 36% here is 36%, the shaded part is 36% and the area is 54cm^2. And so I put this.... The question said if the area of R is 54cm^2. I think this should be 54.

(b) But the overlapping parts, the shaded parts represent 36%. If the area of R is 54cm^2, then here they didn't say R is what so I look at the diagram. So I found that R is the overlapping parts . So I presume 36% is 54 cm^2.

In Figure 1 above, statement (10) "writing an equating statement connecting 36% and 54 cm^2" evoked from statement (1) "R=36%" and statement (3) "R=54 cm^2". This primary production condition-action pair explains the written statement 36%=54cm^2. It is supported by the verbal statements of Pete. Specifically, in Pete's statements "...the shaded part is 36% and the area is 54 cm^2... So I found that R is the overlapping parts so I presume 36% is 54 cm^2". This is an indication of the production pair leading to the equation 36%=54cm^2.

Figure 2. Interpreted Cognitive Thinking Processes from Pupil's Strategy (b)



Cross-referenced Evidence from Pupils' Verbal Statements

David: (a) You know that the shaded part is 36% of circle P. Which means that non-shaded part of circle P is 64%.

(b) 'Cos anything, the whole thing is 100%, the whole circle is 100%,....so 36% of it. So this will be 64%.

David's statements "'you know the shaded part is 36%'" and "'cos' the question asked the non-overlapping parts" match the two condition statements, (1) and (5), which evoke the action statement to find the area of the non-overlapping part of P. This requirement to find the non-overlapping part leads to the statements 100-36=64%.

(iii) Secondary Production of Information

The preceding section describes the extent of the primary production of information which could explain the cognitive processes of problem solvers. The results show that only certain written statements could be explained using the primary production of the information features. The secondary production of information is required to explain other written statements. This is because, in some cases, the statement could not be written directly from merely external-source information. At the secondary production level, information is generated from processed information as well as external-source information. This is a feature which may account for other statements which could not be explained by the primary production feature. The following paragraphs provide some evidence of secondary production of information by which some written statements are generated.

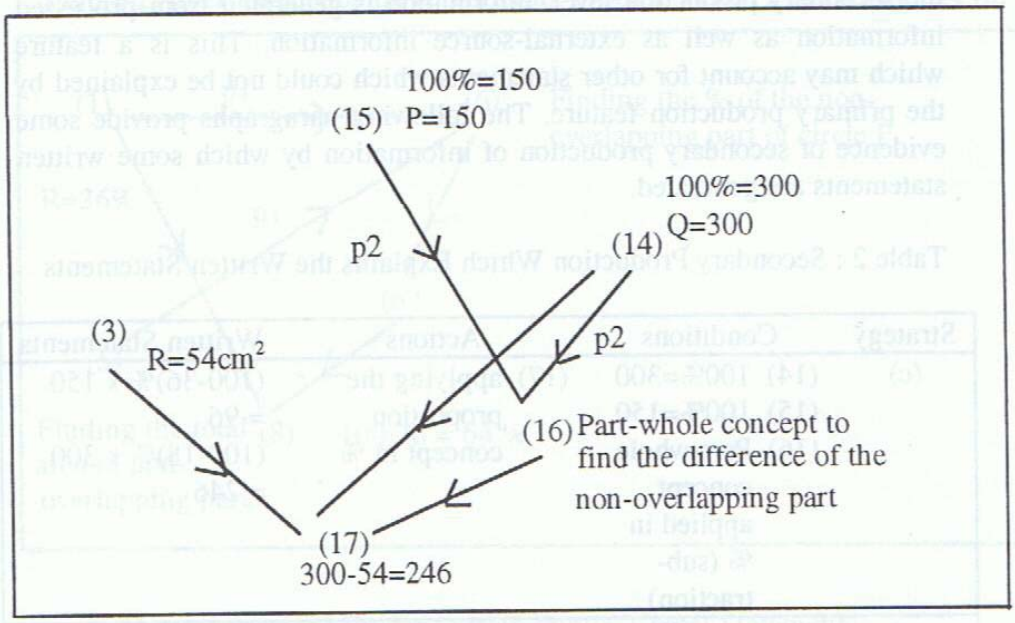
Table 2 : Secondary Production Which Explains the Written Statements

Strategy	Conditions	Actions	Written Statements
(c)	(14) 100%=300 (15) 100%=150 (16) Part-whole concept applied in % (sub-traction)	(17) applying the proportion concept in %	(100-36)% x 150 = 96 (100-18)% x 300 = 246
(d)	(12) 36%=54 (13) 18%=54	(14) proportion: the unitary method	18% = 54 → 1% = 54/18 82% = 246 36% = 54 → 1% = 54/36 64% = 96

Table 2 above shows the activation of the secondary production pairs of information which explain the written statements adjacent to them. These are some of those written statements from problem solvers' solutions which could not be explained by the primary production of

information. For example, in strategy (d) above, the condition statements (12) i.e.36%=54 cm^2 and (13) i.e. 18%=54 cm^2 evoke an action statement (14), using the unitary method of the proportion concept to find a certain value equivalent to 64%. This action-condition pair leads to the written statement 64%=96. Written statements of strategies (c) and (d) from Table 2 are supported by verbal statements of pupils as shown in Figure 3 and 4 as follows. (Note: p2 refers to secondary production of information)

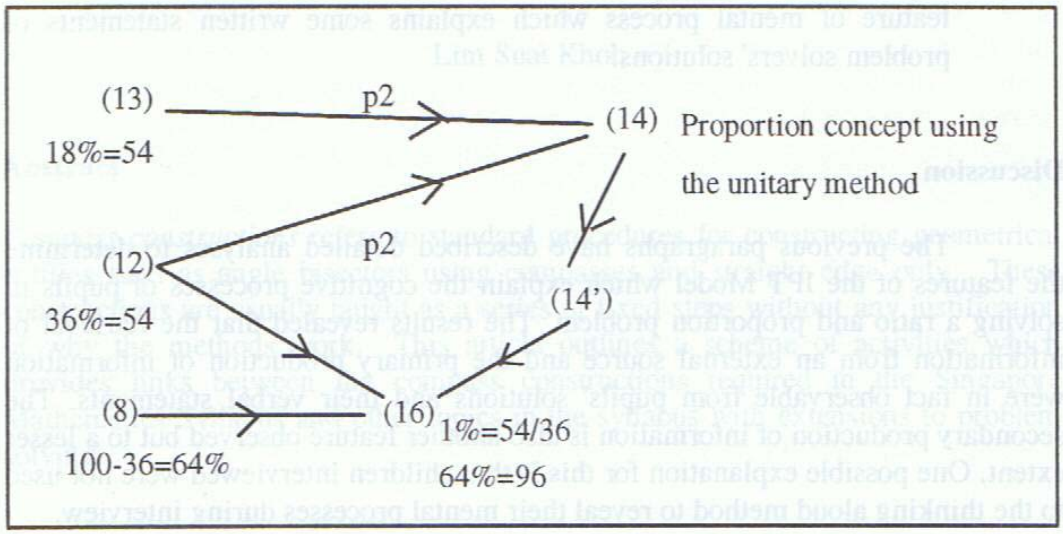
Figure 3. Interpreted Cognitive Thinking Processes from Pupil's Strategy (c)



Cross-referenced Evidence from Pupils' Verbal Statements

Pete: 96 is this non-overlapping part in circle Q. In circle Q, I'm sorry. This part is also the same. I use 300-54, get 246. This is the area of circle Q.

Figure 4. Interpreted Cognitive Thinking Processes from Pupil's Strategy (d)



Cross-referenced Evidence from Pupils' Verbal Statements

Pete: Because the first statement here is 54 and here is 36. I wrote 54/36.

Although quite a number of secondary production pairs were identified from the written statements as indicated in Table 2 above, only two incidents were observed to provide evidence to support the production pairs. In Figure 3, the part-whole concept to find the difference (the non-overlapping part) (16) is evoked from (15) 100%=150 or P=150 and (14) 100%=300 or Q=300. This production pair together with other information (i.e.(3) operates on (14) and operates on (16) → (17)) explains the written statement (17) 300-54=246. The production pair: (14) operates on (15) → (16) is supported by Pete's verbal statement: "96 is this non-overlapping parts in circle Q. In circle P, I'm sorry. This part is also the same. I use 300-54, get 246. This is the area of circle Q". Apparently, Pete is referring to the non-overlapping part of Q. It could be inferred from his statement that the part-whole concept (16) was evoked in his thinking processes. This leads to finding the difference between the area of the whole circle and the area of the overlapping part.

The results of the analysis of the secondary production of information indicate that the secondary production of information is a feature of mental process which explains some written statements of problem solvers' solutions.

Discussion

The previous paragraphs have described detailed analyses to determine the features of the IPT Model which explain the cognitive processes of pupils in solving a ratio and proportion problem. The results revealed that the retrieval of information from an external source and the primary production of information were in fact observable from pupils' solutions and their verbal statements. The secondary production of information is also another feature observed but to a lesser extent. One possible explanation for this is that children interviewed were not used to the thinking aloud method to reveal their mental processes during interview.

While analysing children's productive thinking in solving problems, insights were developed in relation to mathematical thinking. It was observed that some pupils tend to operate at a higher level of thinking (i.e. using the second level of production of information) to solve some intermediate steps which may be classified at lower level of operation. The other feature which was observed is that some children may 'self evoke' information which could not be explained using the production system model. The third feature derived from interview is that it is possible to evoke the same set of information using different sets of production pairs. It seems to imply that further research could be carried out more rigorously to look into these features in mathematical thinking.

References

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