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# Using Worksheets for Secondary School Mathematics

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## Introduction

Most teachers of primary mathematics are very familiar with worksheets which are used extensively to supplement the workbooks published by the Curriculum Development Institute of Singapore (CDIS). At primary levels, these worksheets are usually problem sheets where pupils do their working on the worksheets themselves, very much like pages of the workbooks. For mathematics at secondary level where the working has to be shown, problems are usually solved on separate sheets of paper and it is therefore unnecessary to provide space on problem sheets for students to do the problems.

However, it is proposed that worksheets can be used effectively for various purposes at the secondary level and it is the objective of this article to discuss how secondary mathematics teachers can design and use such worksheets in their classes. One should be clear that a worksheet for secondary mathematics should not be merely a list of problems (similar to tutorial problem sheets at junior college level) but, by its very name, should be for pupils to do the working on the worksheets themselves.

## Reasons for Using Worksheets

There are several reasons why teachers should, at appropriate times, use worksheets.

*When the working has to be done on the worksheet itself:*

If the progression of steps has to follow a clear sequence and it is therefore necessary to provide guiding steps (rather than allow total open-endedness) or where the provision of step-by-step procedures is required, it would be desirable for such a systematic sequence of instructions and the working for each step to be recorded on the same sheet of paper. Another instance where working should be done on the worksheet is when, for clearer pattern observation, tables might be provided for pupils to fill in the data.

Although pupils can design their own tables, a given format may facilitate the observation of the wanted pattern, especially for the weaker pupils.

*Time savings and better focusing:*

Worksheets can save time for the pupils as the tedious time-consuming parts of activities may be provided, leaving them to fill in the observations, the calculations and/or the working. The advantage is that pupils are then able to channel more energy and time to focus on the essential aspects of the problem.

For example, the teacher may wish to use a variety of examples of graphs to lead to a certain concept. Rather than simply provide the examples, pupils are required to draw the various graphs, thus actively engaging them in "discovering" the concept. However, pupils are usually rather slow in graph plotting and yet too few graphs plotted would not bring out the concept. Here, a worksheet with axes provided for each graph and the appropriate positioning of the various graphs would facilitate the efficient working out of the activity. An example will be given below.

*Pupil Motivation:*

The removal of tedious time-consuming chores as explained in the section above may be a motivating factor for students and the provision of help in terms of appropriately structured steps or tables will also give motivation to the weaker pupils. Moreover, worksheets can be designed as puzzles and can look different from normal textbook type pages of problems and, being used occasionally, can have a novelty effect on the pupils.

*Recording Purpose and Individual Progress:*

Teachers can use worksheets effectively to enhance learning, to supplement textbook materials and to promote investigations. At times, teachers may use various activities for pupils to discover a result. The "guided discovery" approach is particularly appropriate for the introduction of concepts, derivation of results or observation of properties and even for investigations. However, pupils merely doing the activity will soon forget the details of the activity whereas worksheets used with such activities will provide records of the instructions, the data collected and the observations made, thus recording how the results were "derived".

With the availability of computers or calculators, the discovery approach to learning can be enhanced if the simulations, graph plotting activities, complicated calculations, etc. can be done by the computer or calculator. However, with the use of such approaches, individual students necessarily proceed at different speeds and may be using different examples or performing different tasks. Furthermore, once the computer is switched off, the actual examples are gone. Hence, the worksheets serve two purposes: firstly, as guides providing step-by-step instructions for independent work and secondly, as records for work performed, data collected, observations from the screen, etc.

Worksheets can also be designed to cater to individual or group pace and they can have challenging extensions at the end for the more able pupils while weaker pupils can follow the instructions at home without the teacher's presence if they are unable to complete the work in the time provided.

**When To Use Worksheets**

In general, in any activity where a desired progression is necessary, worksheets can be used to accompany the activity and in fact forms an integral part of the activity for recording data, for observing patterns and for consolidating results. Needless to say, these completed worksheets should be filed for later retrieval and for revision purposes.

In learning mathematics, there are various stages such as (i) the introduction of concepts (ii) the establishing of properties and derivation of results (iii) the consolidation of a procedure through practice and (iv) open ended investigation involving mathematical concepts. In fact, worksheets can be used at these various stages and in the following, examples will be given to illustrate their use.

*Introduction of Concepts*

As mentioned above, activities can usually be designed to provide examples through which a concept may be abstracted. For example, to introduce trigonometric ratios, pupils may be required to observe that the ratios of relevant sides of a right-angled triangle are constant irregardless of the triangle and are dependent only on the size of the angle concerned. An example of such a worksheet is given in Appendix 1.

Another example is an activity to establish the link between the sign of the gradient and the types of straight lines they represent. Such an activity with one graph per type was planned by a trainee teacher for part of a one-period lesson. However, the time was insufficient for pupils to plot the graphs and as a consequence, the discussion of the conclusions was rushed and less effective than it might have been. I felt that at least two graphs per type was necessary and, to save time and yet achieve the proper focus on the concept, I subsequently designed the worksheet as given in Appendix 2 for such a lesson.

*Establishing of properties and derivation of results*

It is effective and appropriate in many cases for a result to be derived through a sequence of steps. At secondary levels, teachers may not use rigorous algebraic proofs but may rely on geometric analogies or experimental verification. The geometric "proof" for the formula $a^2 + b^2$ is commonly found in texts but could be taught through an activity with a worksheet to record the procedure. In Appendix 3, I have included a worksheet on the less common geometric analogy for the formula for $a^2 - b^2$ to accompany a paper arrangement activity.

For experimental verification, again the use of worksheets can set out the procedures in a systematic way and the "fill in the blanks" format allows pupils to focus on the data, on observations and on the inference of the result. For example, teachers could design worksheets for pupils to "discover" geometrical results through measurements or to observe certain properties of transformations through performing the transformations on the computer, on graph paper, or even on the worksheets themselves.

*Consolidation of procedures/remembering terminology*

I feel that the use of worksheets for this aspect of learning is less significant than for the above two stages but this is not to say that they cannot be used. Pupils sometimes have difficulty remembering a sequence of steps for a given procedure which has just been taught. Worksheets can be used with guiding steps outlining the procedure on one column, with the appropriate computation to be carried out in a parallel column to be filled in by the pupil. However, this is only for newly taught procedures and such a crutch should not be used on a long term basis. Once the pupil has mastered the steps, sufficient practice would serve to consolidate the process.

As for remembering terminology, CDIS Secondary Mathematics Project has designed some puzzles in the form of worksheets to help the normal stream students remember the terms through "play". For example, a crossword puzzle can be designed to promote recall of the special properties of a certain class of mathematical objects such as different types of polygons or quadrilaterals.

*Investigations*

Worksheets can be used to help investigations through the provision of properly designed tables, a guided sequence of observations to be made, etc. Some worksheets can be likened to observation sheets used in experiments for data collection. Activities could include data collection for statistical inferences or investigative activities to encourage pattern observation, etc. Such investigations can also be enrichment activities for topics outside the syllabus.

## Designing Worksheets

The design of a worksheet depends on the type of activity it is being used for. Teachers should remember that worksheets like any other teaching tool or media should only be used where it is appropriate. Worksheets should be seen as a material used in an activity just as geometrical instruments, the calculator and other materials are necessary ingredients contributing towards the activity. A learning activity could be designed based on text materials or other resource materials such as historical methods and a worksheet could then be designed to accompany the activity.

There are a few general steps which may be helpful to teachers setting out to design worksheets:

(a) First, decide whether the use of a worksheet would suit your purpose, bearing in mind the objectives of the activity concerned.

(b) Plan the activity in outline, thinking broadly of the steps involved, checking for the systematic progression of ideas. Also decide if the work is to be done individually, in groups or as a whole class activity. Also decide if the work is to be done independently or with teacher guidance.

(d) Check through the worksheet: Is it clear? Is the progression systematic? Does it have sufficient examples from which to draw inferences? Can the recording format be improved? Is there sufficient space for the tasks? Are the diagrams accurate? (The last is especially important if inferences are to be made from measurements.)

Teachers may of course use worksheets from other sources and modify as necessary, taking into consideration the lesson time, previous knowledge of students, cultural background, etc. Some good sources of worksheets are publications of the National Council of Teachers of Mathematics (NCTM) of the USA such as the Arithmetic Teacher, the Mathematics Teacher, Mathematics Teaching in the Middle Grades and the monthly bulletin of the NCTM. However, some of the worksheets need to be adapted as local students may not be familiar with the American context used.

**Conclusion**

The use of worksheets for secondary level mathematics is not very common and I believe that this is a medium which can be used more frequently than is done now in Singapore. It should however be used to enhance learning and not for its own sake.

I certainly hope teachers will find this article helpful in producing their own worksheets. Readers are at liberty to reproduce or modify those worksheets I have included in the appendices provided they acknowledge the source. I would also be pleased to receive feedback from readers on their use of these and other worksheets in the secondary mathematics classroom.

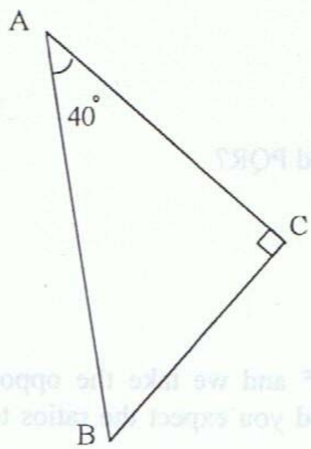
Appendix 1

**Worksheet: Introducing Trigonometric Ratios**

What you will need: Geometrical instruments, calculator.

Here is a right-angled triangle ABC where ∠C = 90° and ∠A = 40°. Check the accuracy of the angles with your protractor.

Now draw next to it a different sized triangle such that ∠P = 90° and ∠R = 40°.




1. Mark on each of the two triangles the hypotenuse and the sides which are opposite and adjacent to the angle of 40°.

2. Measure the lengths of the sides and fill in the first three rows of the table below.

3. Use your calculator to calculate the ratios of the sides and fill in the last three rows of the table.

| Sides/Ratio | ΔABC, ∠A = 40°. | ΔPQR, ∠R = 40°. |
|---|---|---|
| Hypotenuse | AB = | |
| Opposite | BC = | |
| Adjacent | | |
| Opposite/Hypotenuse | | |
| Adjacent/Hypotenuse | | |
| Opposite/Adjacent | | |

4. What do you notice about each of the ratios although they are obtained from different triangles?

5. Check with a classmate sitting next to you. Does he have a different sized triangle PQR? Are his ratios of the last three rows the same values as yours?

6. What is the same about triangles ABC and PQR?

7. If in $\Delta PQR$, $\angle R = 50°$ instead of $40°$ and we take the opposite and adjacent sides with respect to $\angle R$, would you expect the ratios to be the same as for triangle ABC?

<u>Conclusion</u>: (to be filled in after discussion with your teacher)

For a right-angled triangle, the ratios of sides with respect to a fixed angle are

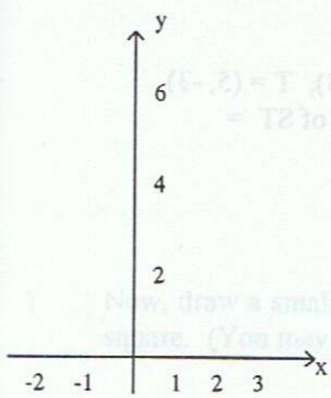
These ratios are called:

Appendix 2

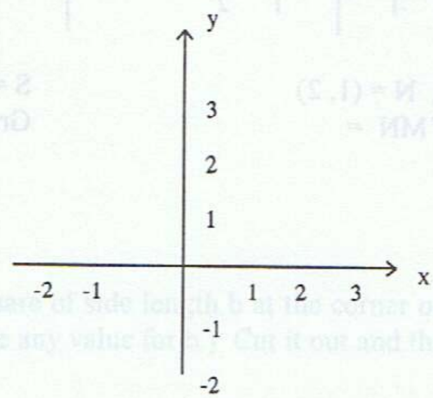
**Worksheet: Types of Gradients**

You have learnt how to calculate the gradient of a line passing through two given points. Recall that the formula for gradient of a line passing through $(x_1, y_1)$ and $(x_2, y_2)$ is

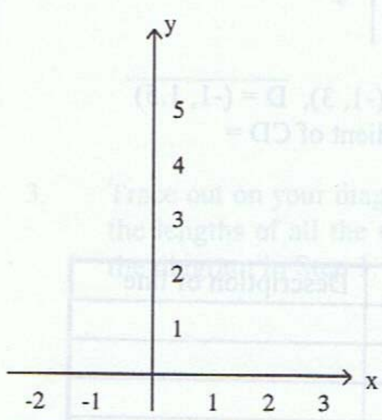
On each of the graphs given below, mark in the two given points, draw in the line passing through them and work out the gradient of the line.



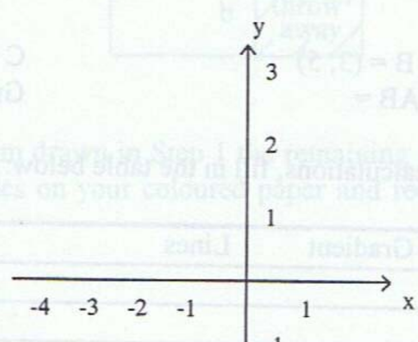

E = (-2, 0), F = (2, 6)
Gradient of EF =



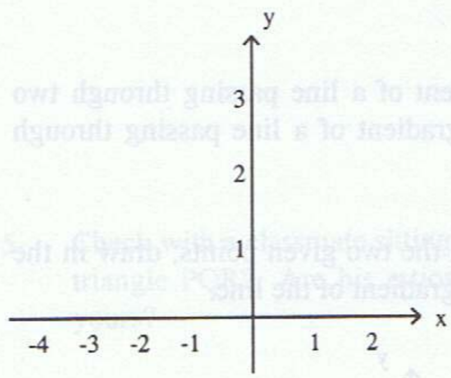

U = (-1, -1), V = (3, 2)
Gradient of UV =

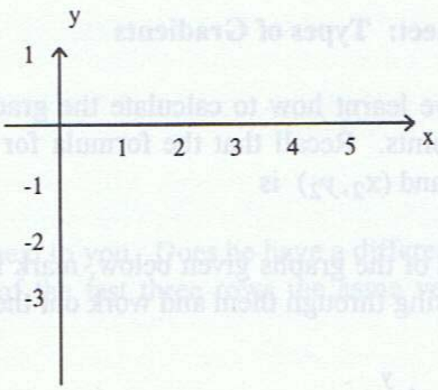



K = (1, 3), L = (-2, 5)
Gradient of KL =



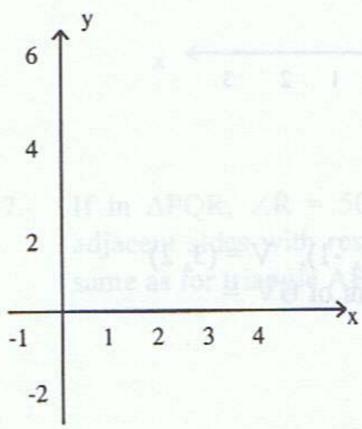

P = (-1, -1), Q = (-3, 3)
Gradient of PQ =

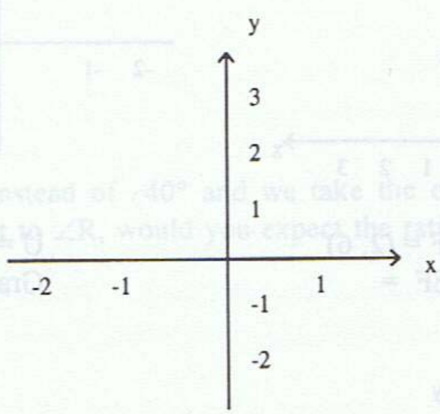



M = (-4, 2), N = (1, 2)
Gradient of MN =

S = (2, -3), T = (5, -3)
Gradient of ST =




A = (3, -2), B = (3, 5)
Gradient of AB =

C = (-1, 3), D = (-1, 1.5)
Gradient of CD =

From your calculations, fill in the table below:

| Value of Gradient | Lines | Description of line |
|---|---|---|
| Zero | | |
| Positive | | |
| Negative | | |
| | | |

Appendix 3

**Worksheet:** **What is $a^2 - b^2$ ?**
(A paper arrangement activity)

1. Cut out a square of any size from your coloured paper. Trace the outline here and label the length of the side a.

2. Now, draw a smaller square of side length b at the corner of your paper square. (You may choose any value for b.) Cut it out and throw it away.

What is the area of the remaining piece of paper ?

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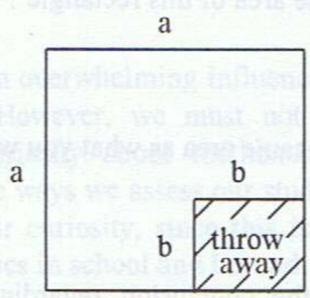
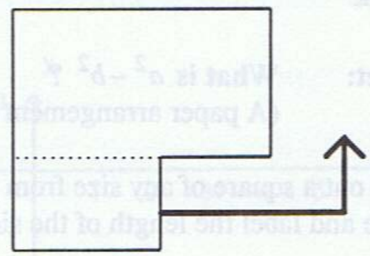



3. Trace out on your diagram drawn in Step 1 the remaining shape. Label the lengths of all the sides on your coloured paper and record them on the diagram in Step 1.

4. Cut your shape along the dotted line shown to form two rectangles. Then arrange them to form one rectangle (see arrow).



5. Trace your resulting rectangle here below and label the length and breadth in terms of a and b.

6. What is the area of this rectangle ?

7. Is this the same area as what you wrote for Step 2 ?

8. What is the conclusion regarding the relationship between the two expressions?