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Children Constructing Their Mathematics: Implications For Teachers

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Abstract

The ways in which mathematics educators think about the learning of mathematics have changed over the last ten years. The image of children waiting, open-mouthed (or was it open-minded?) to accept whatever knowledge the teacher was going to transmit is anachronistic. We know that children do not simply 'soak up' the knowledge around them but that they must actively construct this knowledge. How they do this and what effect the social milieu in which this action takes place has on the construction is much more problematic. Beyond this, the implications for teachers of acceptance of a theory of learning which relies on the children's own constructions are only just being investigated.

The purpose of this paper is to provide some background for teachers on the theory of knowing which has been recognised as 'constructivism', particularly 'social constructivism', and to investigate possible implications for teachers of mathematics in schools. The paper concludes with a description of an approach to teaching based on social constructivist principles which has proved successful for the author in both schools and tertiary environments.

Introduction

Mathematics learners and educators often have assumed that mathematics knowledge was something which was well-formed and immutable. In order to learn mathematics what one had to do was decode messages coming from people who already knew the mathematics (the teachers) in such a way that the knowledge was reproduced in the learner. The image was almost one of 'pouring' the finished knowledge 'into' the learner. There could be no changes made to the knowledge by the learner because there were no changes to be made in the 'perfect' knowledge being transmitted. Of course, while the knowledge may have been considered 'perfect', there were a number of 'imperfections' in the process of getting it from the teacher to the learner. Even if we assume that the teacher has a 'perfect' understanding of the particular piece of mathematics being taught (and this is unlikely to be the case), the coding and decoding of the knowledge can be so complicated that it is often a wonder that the learner ends up understanding anything at all, let alone gains a 'perfect' understanding which matches that of the teacher. The experience of teachers is such that they feel that this level of perfection is reached only very rarely. Cries of "Why don't (can't) they understand?", "I have told them a dozen times!", "What is wrong with them?" emanate from staff common rooms as the teachers become more and more frustrated with the children who seem to be failing as learners. But is it the learners who are failing or is it that the methods used to encourage them to learn are failing?

The purpose of this paper is to investigate an approach to learning and teaching which seems to provide some hope to both teachers and learners of mathematics who are caught in this dilemma. The discussion will consider the theory of knowing which has been recognised as 'constructivism', particularly 'social constructivism' and investigate possible implications of this theory for mathematics teachers. By way of example, an approach to teaching mathematics based on social constructivist principles, and which has proven successful for the author in both schools and tertiary environments, will be described.

Constructivism defined

The (radical) constructivist theory of knowing is based on two major premises:

> 1. *Knowledge is actively constructed by the cognising subject, not passively received from the environment.*
> 2. *Coming to know is an adaptive process that organises one's experiential world; it does not discover an independent, pre-existing world outside the mind of the learner.*
>
> *(Lerman, 1989, p. 211)*

The first of these does not specifically deny a place in mathematics teaching and learning for many of the 'traditional' approaches used by teachers. However, what it does indicate is that the learner will act on whatever is presented by the teacher in an active way. The second premise is much more problematic in that what it leads to is the notion of each learner constructing a unique body of mathematics knowledge based on the unique set of experiences through which the learner has lived. This idea is contrary to the traditional notion of mathematics as one body of finished knowledge. It says, for example, that two learners in the same class may have different notions of the same concept and both of them may be 'correct' in terms of their performance in the mathematics of the class. The notion that there is a body of mathematics knowledge which exists somewhere outside the learner and which must be discovered by the learner also runs counter to this second premise.

The thought that what is happening in a mathematics class is that all the learners are constructing their own unique mathematics knowledge is attractive from a theoretical point of view, but not very useful in the real context. Most people would agree that there is some mathematics knowledge which needs to be accepted as being 'true'. What this means is that we would all agree to it happily. For example, almost all adults would agree with the statement '2+2=4', but how do we move towards such agreement? Part of the answer to this question may lie in what has become known as 'social constructivism'. While adhering to the two major premises of radical constructivism, protagonists of social constructivism, such as Cobb (1990, p.209), argue that 'learning is an interactive as well as a constructive activity'. 'Viable knowledge is established by the learners through their individual constructions of their own meanings and their sharing and discussion in the social context of the classroom' (Perry & Conroy, 1994, p.54). An agreed 'class meaning' is reached which has been described as 'taken-as-shared' mathematical knowledge (Cobb, Perlwitz & Underwood, 1992, p.37).

> *"This term is preferred to 'shared' mathematical knowledge because all that can be said is that the classroom community, learners and teacher, has agreed on using certain meanings. It is not possible to tell whether everyone shares these same meanings. Nor is there any question about whether or not these meanings are 'true'. The question to be asked is one of viability. That is, will the knowledge which is currently taken-as-shared help children do what they are being asked to do? If a new experience introduces to one of the community an idea which questions the viability of the taken-as-shared knowledge, then further negotiation will be needed."*
>
> *(Perry & Conroy, 1994, p.54)*

A direct consequence of accepting an approach to learning mathematics which adheres to a social constructivist theory of knowing is that teachers and learners continually must be willing to refine their own knowledge so as to assimilate new experiences. In order not to undervalue the learner's own knowledge, teachers must be careful not to anoint particular pieces of knowledge as absolutely correct.

> *"Whatever one intends to teach must never be presented as the only possible knowledge - even if the discipline happens to be mathematics. Indeed it should be carefully explained that a fact such as '2+2=4' may be considered certain, not because it was so ordained by God or any extra-human authority, but because we come to construct units in a particular way and have agreed on how they are to be counted.*
>
> *(von Glasersfeld, 1992, p.9)*

Implications for teachers

What does all of this mean for teachers? Is there an approach to the teaching of mathematics which can be developed to take into consideration the social constructivist theory of knowing? Even if there is such an approach, of what benefit is it to learners of mathematics? In an attempt to answer these questions, an approach first introduced by the Purdue Mathematics Project team (see, for example, Cobb, Perlwitz & Underwood, 1992) and since adapted for use, with both primary school children and adult learners in Australia, is reported in the following discussion.

The key features around which the approach has been developed are an experiential learning cycle, principles of cooperative learning and the problem-centred approach of the Purdue Mathematics Project (Perry & Conroy, 1994). The approach is also firmly based on the following consequences of accepting a social constructivist perspective on learning:

> * *Teachers should provide instructional activities that will give rise to problematic situations for children.*
> * *Children's actions are rational to them, and teachers should attempt to view students' solutions from their perspective.*
> * *Teachers should recognise that what seem like errors and confusions from an adult point of view are children's expressions of their current understanding.*
> * *Teachers should realise that substantive learning occurs in periods of conflict, confusion, surprise, over long periods of time, and during social interaction in which negotiation of taken-as-shared meaning is essential.*
>
> *(Wood, Cobb & Yackel, 1992, p. 182)*

An approach which is designed to place the learners in conflict situations and to encourage their interaction as they move towards taken-as-shared meaning has the potential to create disruptive learning environments. Obviously, this is unacceptable. Consequently, an important pre-cursor to mathematics learning using the approach was the establishment of social norms under which the class would work. Attempts were made to create these norms in as interactive a way as possible, but it cannot be claimed that this always occurred. Interestingly, it seemed easier to establish norms interactively with school children than with adults! Most sets of norms included the following:

1. Students will work in pairs.
2. Activities will consist of problems. That is, it can be assumed that, when first meeting an activity, students may not be able to solve the problem or even know where to start on it.
3. Pairs of students are expected to develop solutions to the activities cooperatively and to reach consensus on the solutions. The teacher is expected to encourage problem solving and cooperation.
4. Pairs of students are expected to explain and defend their solutions or attempted solutions to the whole class. Other students are expected to indicate agreement or disagreement and encourage alternative solutions in a manner which does not denigrate anyone's work or effort.
5. The class is to see itself as a community of validators and not to rely on the teacher for validation of solutions. The aim is to reach a solution or set of solutions which can be taken-as-shared by the whole class.

This approach relies heavily on the development of problematic situations aimed at the individual construction of particular mathematical concepts. Consequently, the choice of activities is critical to the success of the approach. Control over the class curriculum is maintained by the teacher through these activities. We have found that suitably developed materials usually result in the students at least constructing their ideas in the mathematical idea desired by the teacher. Of course, the details of this construction belong to the individual and the teacher can be aware only of the learning which is taken-as-shared. However, the teacher is able to gather a great deal of information about individual learning among the students in the class.

1. Each lesson is broken into two parts of roughly equal times. The first part is devoted to small group problem solving where the students work in pairs on a problem or set of problems. All pairs in the class work on the same problems although some may get further than others. The second part of each lesson is devoted to the students discussing, first through pair presentations and later in the whole class, how they attempted to solve the problem(s).
2. Any introduction to the problems given by the teacher is restricted to an explanation of the symbols and words used. There is no input from the teacher on how to solve the problem or even approach a solution.
3. While the students are working on the problems, the teacher circulates, observing and discussing the students' attempts, encouraging continued work but not validating approaches or solutions.
4. In pairs, the students explain their (attempted) solutions. The teacher helps clarify the explanations, assists in the verbalisation of these and encourages alternative solutions from both the presenters and the rest of the class.
5. The teacher does not indicate whether answers are correct or incorrect but encourages reflection on a variety of solutions.
6. The class works as a whole to gain consensus on a possible solution to the problem. That is, the class as a whole develops a taken-as-shared solution.

Conclusion

The results of our work indicate that not only can students construct their own mathematics using the approach advocated, but also that they are able to utilise the social norms developed by the class to establish taken-as-shared knowledge about particular mathematics ideas. That is, the students are able to test the viability of their own mathematics constructions against those of their colleagues and assimilate new information which may arise from these discussions. These results have reinforced the work of the Purdue team and colleagues in Australia (for example, Wright, 1992) and have shown the applicability of the approach to learners of all ages.

It is instructive to note the similarities of our approach and results to those of Lo, Wheatley & Smith (1994, pp.45-46) who were working with Grade 3 children.

1. *Students can profit from explaining their group's solution to classmates in a whole-class setting.*
2. *Students can profit from opportunities to make sense of other students' explanations.*
3. *Inferring the expectations of the other members of the group is an essential component of communication.*
4. *Students can profit from being challenged when explaining their methods.*
5. *The negotiation of certain social norms and beliefs plays an important role in fostering mathematics learning.*
6. *The need to communicate mathematical ideas can promote meaningful learning.*
7. *Opportunities to communicate mathematics may foster positive attitudes about mathematics learning.*
8. *Class discussion can provide opportunities for individual students to connect and integrate their mathematical knowledge.*
9. *Students can increase their learning opportunities by interpreting the social norms being negotiated in the class.*
10. *Providing many opportunities for students to work out the social norms and to negotiate mathematics meaning may facilitate learning.*
11. *Teachers need to provide many opportunities for students to work out the social norms and to negotiate mathematics meaning.*

It is clear that the approach to mathematics teaching which has been described here has wide applicability across a range of educational settings. The strong problem-based approaches evident in recent mathematics curricula in many countries can connect well with the social constructivist theory of knowing. What has been described here is just one way in which this connection can be made.

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