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The Impact of Supplemental Self-Paced Instruction on Conceptual Understanding and Procedural Skill of Adolescents

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Abstract: We investigated causal relationships between conceptual and procedural knowledge in mathematics using an East Asian perspective. In particular, we studied if supplemental self-paced instruction that focuses on the mastery of either concepts or procedures through repetition with variation helps adolescents improve their performance in proportional reasoning tasks. We used a pretest-posttest control group design with 34 junior high school students and 44 undergraduates randomly assigned to three groups (conceptual, procedural, and control). The conceptual group did nonnumeric tasks and the procedural group did numeric tasks, answering one worksheet per day for fifteen consecutive days. Using non-parametric two-tailed tests of hypothesis, we found evidence that supplemental procedural instruction significantly differed from supplemental conceptual instruction and from the absence of supplemental instruction in affecting the degree of procedural knowledge. We found no evidence that the type of supplemental instruction affected the degree of conceptual knowledge.

Keywords: conceptual knowledge, procedural knowledge, proportional reasoning, repetition with variation

Introduction

Western educators and East Asian educators seem to have different views on conceptual and procedural instruction. While some studies done in Western countries imply that conceptual instruction is more effective and efficient than procedural instruction, students in East Asian countries perform better in international studies of mathematics achievement than their Western counterparts despite receiving instruction that is characterized as being mostly procedural.

A study by Noche (2013) used an East Asian perspective and found that, in the mathematical domain of proportional reasoning, procedural instruction (and not conceptual instruction) led to gains in procedural knowledge and neither procedural instruction nor conceptual instruction led to gains in conceptual knowledge. We replicate this study using a larger sample size, a longer treatment period, a broader class of participants, and improved instructional materials to strengthen the findings of Noche (2013).

Literature Review

Students in East Asian countries outperform their counterparts in other countries in large-scale international studies of mathematics achievement (Leung, 2006; 2017). Leung (2006) asserted that this is because their teachers have good conceptual understanding and procedural skills, their instruction is teacher-dominated, and their culture strongly emphasizes the importance of education, which creates high expectations for them and their teachers. Leung, Park, Shimizu, and Xu (2015) state that in the Confucian Heritage Culture that these countries share, success and failure are attributed more to effort rather than to innate ability, and examinations, memorization, repeated practice, and reflection are emphasized. Leung (2017) states that "East Asian languages seem to influence mathematics learning and assessment in ways different from Western languages," an observation that Leung et al. (2015) support. It also seems that East Asian teachers and students acquire their mathematics competence from instruction that focuses on procedures and repeated practice with variation (Leung, 2006).

Much research has been done on conceptual and procedural knowledge in mathematics and their pedagogical implications, mostly on pre-school, primary school, and secondary school students (Rittle-Johnson & Siegler, 1998). Some suggested that conceptual knowledge should be taught before, and be emphasized more than, procedural knowledge (Grouws & Cebulla, 2000, p. 15). This is in sharp contrast with the practices in most East Asian schools where procedural skills are often taught before and given more emphasis than conceptual understanding (Leung, 2006).

Some studies showed that procedural instruction can interfere with conceptual understanding (Pesek & Kirshner, 2000; Philipp & Vincent, 2003). Students who received procedural instruction and then conceptual instruction learned less (based on the results of individual written tests and oral interviews) than those who received only the conceptual instruction, whether the students usually received procedural instruction (Pesek & Kirshner, 2000; Simoneaux & Kirshner, 1994) or conceptual instruction (Philipp & Vincent, 2003). Other studies showed that procedural instruction can improve conceptual understanding (McNeil & Alibali, 2000). In either case, it seems that conceptual instruction is more effective and efficient in improving both conceptual and procedural knowledge (Matthews & Rittle-Johnson, 2007; Rittle-Johnson & Alibali, 1999). Nevertheless, calls have been made for more focus on procedural knowledge in mathematics (Rittle-Johnson & Star, 2004; Star, 2005).

Conceptual and procedural knowledge may develop iteratively (Rittle-Johnson, Siegler, & Alibali, 2001), rather than one type generally developing before the other. Instruction that uses this iterative model seems to yield greater improvements in procedural knowledge (and comparable improvements in conceptual knowledge) than instruction where concepts are taught before procedures (Rittle-Johnson & Koedinger, 2002, 2009). Schneider and Stern (2005) initially found evidence that supported the concepts-first view rather than the iterative model. They later found out that the inconsistent results might be explained by difficulties in validly measuring conceptual and procedural knowledge (Schneider & Stern, 2010). Schneider, Rittle-Johnson, and Star (2011) used the lessons learned in Schneider and Stern's (2010) study and found evidence that strongly supports the iterative model. Rittle-Johnson, Schneider, and Star's (2015) review further supports the iterative model.

Studies on conceptual and procedural knowledge in mathematics have usually focused on one mathematical domain such as counting, single-digit addition, multi-digit addition and subtraction, fractional arithmetic, or proportional reasoning (Rittle-Johnson & Siegler, 1998). There have also been studies on computational estimation (LeFevre, Greenham, & Waheed, 1993), decimal fractions (Rittle-Johnson et al., 2001; Schneider & Stern, 2005; Schneider & Stern, 2010), decimal arithmetic (Rittle-Johnson & Koedinger, 2002, 2009), and mathematical equivalence (Matthews & Rittle-Johnson, 2007; McNeil & Alibali, 2000; Rittle-Johnson & Alibali, 1999; Schneider et al., 2011). Since many adults have difficulty with problems involving proportions (Heller, Post, Behr, & Lesh, 1990, pp. 388–389), proportional reasoning seems to be a domain well-suited for studies involving adolescents.

Relationships Between Conceptual and Procedural Knowledge

There is not just one relationship between conceptual and procedural knowledge; there are many, and they depend on several factors such as the domain of mathematics involved, the characteristics of the human subjects, and the types of instruction and assessment used. "Relationships between conceptual and procedural knowledge change over time and are influenced by many forces, both internal and external to the learner" (Hiebert & Lefevre, 1986, p. 22).

One way to describe a relationship between conceptual and procedural knowledge is by looking at how they develop in time. Rittle-Johnson and Siegler (1998) saw four possibilities:

1. Conceptual knowledge develops after procedural knowledge.
2. Conceptual knowledge develops before procedural knowledge.
3. Conceptual and procedural knowledge develop concurrently.
4. Conceptual and procedural knowledge develop iteratively, with small increases in one leading to small increases in the other, which trigger new increases in the first.

Rittle-Johnson and Siegler (1998) presented the following principles for predicting the order of acquisition of conceptual and procedural knowledge:

1. Children will understand key concepts before they use the target procedure if (a) the target procedure is not demonstrated in the everyday environment or taught in school; or (b) children have frequent experience with relevant concepts, either in their everyday environment or in the classroom, before the target procedure is taught.
2. Children will use the target procedure before they understand the relevant concepts if (a) the target procedure is demonstrated frequently, either in the everyday environment or in formal instruction, before children understand key concepts in the domains; or (b) the target procedure is closely analogous to a procedure in a related domain and can be induced from that procedure before children understand key concepts in the domain.

Another way to describe a relationship between conceptual and procedural knowledge is by the type of evidence that could be obtained regarding the relationship (Rittle-Johnson & Siegler, 1998):

1. Use of routine procedures that do not violate concepts of the domain
2. Possession of both conceptual and procedural knowledge as indicated by performance on independent tasks
3. Positive correlation between individual children's conceptual and procedural knowledge
4. Consistent order of acquisition of conceptual and procedural knowledge
5. Possession of one type of knowledge predicts acquisition of the other
6. Effective interventions increase both conceptual and procedural knowledge
7. Possession of one type of knowledge is causally related to the acquisition of the other

This last type of evidence (causal relation) is of special interest to us.

> In [causal relation] studies, children are randomly assigned to intervention and control groups, children in the intervention group receive instruction in one type of knowledge, and children in both groups are immediately or subsequently tested for knowledge of the other type. For example, randomly chosen children might receive instruction in concepts within a domain. If this instruction resulted in increased procedural ability relative to the control group, this would indicate a causal role of understanding concepts for gaining procedural knowledge. (Rittle-Johnson & Siegler, 1998, p. 80)

Methodology

Statement of the Problem

We seek to answer the following:

In the domain of proportional reasoning, how does supplemental conceptual instruction and how does supplemental procedural instruction affect the conceptual understanding and the procedural skill of adolescents? In particular, can supplemental self-paced instruction that focuses on the mastery of either concepts or procedures through repetition with variation help adolescents improve their performance in tasks designed to assess their proportional reasoning understanding and skills?

Definition of terms

A *rate* is a quotient of two quantities in different measure spaces, a *ratio* is a quotient of two quantities within a single measure space, and a *product of measure* is a product of two quantities in different measure spaces that is a quantity in a third measure space, where Vergnaud's definition of *measure space* (as clarified by Lesh, Post, & Behr, 1988) is used. A *proportion* is an equality of two rates, or of two ratios, or of two products of measure.

Proportional reasoning is "reasoning in a system of two variables between which there exists a linear functional relationship." This "leads to conclusions about a situation or phenomenon that can be characterized by a constant ratio. (Karplus, Pulos, & Stage, 1983, p. 219).

Conceptual knowledge in the domain of proportional reasoning is knowledge of the multiplicative and additive mathematical principles of Harel, Behr, Post, and Lesh (1992).

Procedural knowledge in proportional reasoning is knowledge of correct arithmetic or algebraic procedures either to solve for an unknown in an equation describing a proportion or to find the order relation between two rates, ratios, or products of measure.

A task or problem is *numeric* if arithmetic computations are required in getting its solution; otherwise, it is *nonnumeric* and can be either number-free or quantified. Noche and Vistro-Yu (2015, p. 283) provide an example: "the task 'Which is greater, 3/8 or 2/9?' is a quantified nonnumeric task; it can be solved without numerical computation: because $3 > 2$ and $8 < 9$, it follows that $3/8 > 2/9$. The task 'Which is greater, 3/5 or 7/8?' is a numeric task."

A person's *amount of conceptual knowledge* in the domain of proportional reasoning is his/her score in a test consisting of nonnumeric proportion-related problems testing for understanding of Harel et al.'s (1992) mathematical principles. A person's *amount of procedural knowledge* in proportional reasoning is his/her score in a test for skills in solving numeric proportion-related problems. The *change in amount of knowledge* is the associated posttest score minus the associated pretest score.

The *adolescents* in this study are junior high school and undergraduate students officially enrolled during the time of the study in a private, Catholic, coeducational university.

Worksheets that use *repetition with variation* are those that follow a style similar to that of Kumon Math using Fuller (1991), Ukai (1994), What Works Clearinghouse (2009), and Kumon Math worksheets as references.

Supplemental instruction is self-paced, extracurricular instruction occurring daily outside class hours. It consists of worksheets that use repetition with variation to be done individually without books or calculators.

Conceptual instruction is instruction primarily aimed at increasing an individual's amount of conceptual knowledge in a certain domain (in this

case, proportional reasoning) while *procedural instruction* is aimed at increasing an individual's amount of procedural knowledge.

Research Design

Rittle-Johnson and Siegler (1998) made the following observation:

> A major gap in the literature is the lack of direct causal evidence regarding how acquisition of concepts and procedures influence each other. To draw causal inferences, we need experiments in which children are randomly assigned to groups, in which one type of knowledge is inculcated, and in which changes in the other type of knowledge are then examined. (p. 104)

This experimental research used a randomized pretest-posttest control group design with three groups (conceptual, procedural, and control). Random assignment minimized the threat of extraneous variables that might affect the outcomes of the study. The use of a no-treatment control group was necessary to determine if a treatment was indeed the cause of any differences in the outcomes. (Fraenkel & Wallen, 2000, ch. 13)

The independent (treatment) variable in this study is categorical: the type of supplemental instruction (conceptual, procedural, none). The dependent (outcome) variables are quantitative: the amount of conceptual knowledge and the amount of procedural knowledge in proportional reasoning. Figure 1 shows a block diagram of the research framework.

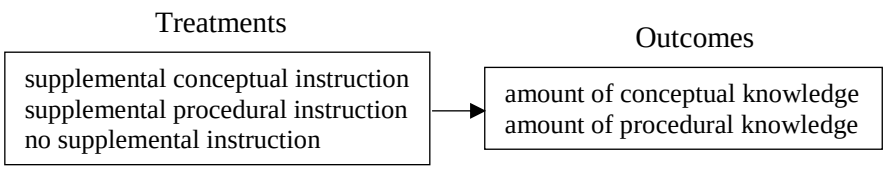


Figure 1. Research framework

The research hypothesis is that the type of supplemental instruction (conceptual, procedural, or none) affects the amount of any type of knowledge (conceptual or procedural).

Participants

A total of 267 students (92 from two junior high school sections, 175 from five undergraduate sections) were invited to participate in the study. The 181 participants who took the pretest were randomly assigned to one of three groups; 67 students were assigned to the conceptual group, 68 to the procedural group, and 46 to the control group.

Participants assigned to the conceptual or the procedural groups who did less than 10 of the 15 worksheets were removed from the study. Participants that did not take the posttest were also removed from the study.

The final sample consisted of 78 participants (35 male, 43 female; 34 junior high school, 44 undergraduate): 19 in the conceptual group, 19 in the procedural group, and 40 in the control group. Their ages at the start of the study ranged from 13 years and 1 month to 24 years and 3 months (with an average of 16 years and 4 months and a median of 16 years and 8 months).

Data Collection

The experiment lasted 17 consecutive days. On the first day, the participants were given a sample test (discussed below) and the pretest. On days when the participants were in the campus, the participants in the two supplemental instruction groups answered their worksheets at pre-scheduled times in rooms reserved for the study (not the students' regular classroom) supervised by the experimenter. On days when the participants were not in the campus, the participants in the two supplemental instruction groups did their worksheets outside the campus unsupervised. (The intervention was modelled after the Kumon method (Ukai, 1994) where worksheets are "practiced every day of the year" (p. 91).) On the last day, the participants took the post-test.

As mentioned above, before the pretest was given, a sample test was presented to the participants to ensure that they understood the terms used in the pretest (e.g., liquid level height) and were prepared for questions where the answer cannot be determined from the given information (e.g., those that test for understanding of mathematical principles, Harel et al., 1992). The participants were guided as a group in finding the correct answers to the questions in the sample test.

On days when the participants were in the campus, the participants in the supplemental instruction groups spent around 15 minutes a day answering the worksheets (cf. Ukai, 1994, p. 91, "One packet is to be completed each day, requiring 15-30 minutes' study."). Participants signed up for the schedule most convenient for them. Each participant recorded the time taken to answer each worksheet. The supplemental instruction was self-paced, with an emphasis on mastery. The participants were told to prioritize performance over speed when answering the worksheets. The worksheets were to be answered in order (question 1 before question 2, page 1 before page 2, worksheet A before worksheet B, and so on) (cf. Ukai, 1994, p. 90, "highly sequential presentation"). Participants were allowed to approach the experimenter for short clarifications regarding the worksheets (cf. Ukai, 1994, p. 95, "The job of instructors [...] is not to 'interfere' with the child's learning, which will come through internalizing calculation skills, but to offer praise, hints, and individualized feedback on work.").

At the start of each daily session, each participant individually consulted with the experimenter who showed him or her how he or she performed in the previous worksheet. If there were few errors (10% or less), then the participant corrected the errors with the help of the experimenter, then answered the next worksheet alone. If there were many errors (more than 10%), then the experimenter provided some brief feedback and the participant repeated the whole worksheet alone. If the participant took much longer to finish a worksheet than the time allotted for it (twice the time or more), then the participant repeated the worksheet (cf. Ukai, 1994, p. 91, "They progress to a higher level of work only after they show the ability to complete sheets accurately within prescribed time and mistake limits. If either of the limits is exceeded, additional drilling is assigned." Compare also p. 104, "The concept of a Standard Completion Time [...] provides specific time limits for each page. Mr. Kumon and others observed that the highest achievers worked quickly and that when children had unlimited time to finish, their attention drifted. He also came to believe that a perfect score achieved too slowly did not constitute mastery."). The time allotted for each worksheet was based on the average time taken by students during pilot tests.

If the previous worksheet was done outside the campus, then the participant submitted it at the start of the session and was given the next worksheet even though the submitted worksheet had not yet been checked (cf. Ukai, 1994, p.

91, "The completed homework is turned in and that day's packet is done at the classroom."). The submitted worksheets were checked by the project staff before the next session to determine what worksheets to assign during the next session (cf. Ukai, 1994, p. 91, "The instructor charts the child's progress in a detailed record book and, according to the most recent results, assigns more difficult work or repetition of previous pages." Compare also p. 107, "If immediate grading and record-keeping is not carried out, the wrong level of worksheet may be assigned, destroying the seamless progression of work that successful application of Kumon requires.").

Some participants repeated worksheets many times and were thus not able to finish all fifteen worksheets in the fifteen days. This was expected; an essential characteristic of the supplemental instruction used in this study is that students learn at their own pace. However, due to the time limitations of the study, no additional sessions were given to participants who did not complete the fifteen worksheets during the fifteen days. If a participant would not be in the campus the next day, then at the end of the session he or she was given the next one, two, or three worksheets to be answered outside the campus and submitted on the day he or she returned to the campus.

Data Analysis

Non-parametric (two-tailed) statistical tests of hypothesis were used. Dataplot software (Heckert, 2010) was used to implement Kruskal and Wallis's (1952) *H*-test, a one-way analysis of variance by ranks (corrected for ties), with pairwise multiple comparisons done using Conover's (1999, p. 290) procedure (A. Heckert, personal communication, June 6, 2011).

Instruments

All the research instruments used in the study were created by the first author with many of the questions based on the work of others (duly cited). The terms "conceptual," "procedural," "rate," "ratio," and "proportion" were not explicitly used in the research instruments or by the experimenter during the study. All the research instruments were to be answered individually without using books or calculators.

Pretest and Posttest

Our main data gathering tools were the pretest and the posttest. Since they were exactly the same (except for their labels), they will henceforth be called

the test. The test and the sample test (discussed in the Data Collection section above) we used are exactly the same as those Noche (2013) used.

In Noche's (2013) dissertation, the test was pilot tested on undergraduate students who were not part of the experiment, and was revised three times. To provide content-related evidence of validity, the test was reviewed by a colleague with expert judgment in the domains of the study (the dissertation adviser).

To measure the test's test-retest reliability (stability), the students who pilot tested the third version were asked to retake the same test two to three weeks later. For the four students who were able to comply, the Pearson product-moment correlation coefficient for their total test scores was 0.9245 (a value greater than 0.70 is acceptable, Fraenkel & Wallen, 2000, p. 179) and the Spearman rank-order correlation coefficient was 0.8 with $p > .05$ (one-tailed and two-tailed) because of the small sample size (Spearman rank-order correlation coefficient, Lowry, n.d.).

The test was an answers-only, written-response test composed of 69 multiple-choice questions in a booklet 24 half-letter-sized pages long. It was designed to be completed in around one hour.

The test contained questions involving four tasks—the balance task (based on Siegler's (1976) physical comparison task), the fullness task (based on Siegler and Vago's (1978) version of Bruner and Kenney's water jar problem), the mixture task (based on Noelting's (1980) orange juice experiment), and the blocks task of Harel et al. (1992)—having different physical principles: conservation of angular moment, the fact that the pressure is the same at all points at the same level within a liquid at rest, uniform diffusion of liquids, and homogeneous distribution of weight (homogeneous density of matter), respectively.

To avoid bias due to subject fatigue, the four tasks were arranged in different orders, resulting in the test having 24 different sets. (For the pretest, the sets were randomly assigned to the participants. For the posttest, each participant given the same set that he or she was assigned in the pretest.)

Each task had a set of numeric questions that tested for quantitative skills and a set of nonnumeric questions that tested for understanding of Harel et al.'s (1992) mathematical principles. The test had 45 nonnumeric questions and 24 numeric questions.

Worksheets

The supplemental instruction consisted of two sets of fifteen worksheets: one set had nonnumeric (conceptual) tasks and the other involved numeric (procedural) tasks. The tasks differed from those in the pretest and the posttest.

Each worksheet was a booklet eight half-letter-sized pages long, included a short discussion of the concepts or procedures involved, with examples, and was designed to be completed in around 15 minutes. The worksheets used repetition with variation, in a style similar to that of Kumon Math (Ukai, 1994) where performance, speed, and continuous practice are emphasized and tasks are to be done in a strictly sequential order and repeated until mastery is attained.

Revision, creation, and pilot-testing. Noche and Vistro-Yu (2015), in describing the 22 worksheets used in Noche's (2013) dissertation, noted that the procedural worksheets took significantly longer to finish than the conceptual ones. It was thus not clear if the difference in performance between the conceptual and the procedural groups was because of the type of instruction or because of the time taken in the instruction. In addition, some of the worksheet completion times in Noche's (2013) dissertation were way beyond the targeted range of 15 to 30 minutes.

We revised the worksheets of Noche (2013) to shorten their completion times and to improve the chances that the two sets of worksheets have the same completion times. Also, we created eight additional worksheets to take into account the longer treatment period. We pilot tested the worksheets on senior high school students and undergraduate students who were not part of the experiment.

Aiding the tasks in the test. The 15 worksheets were labeled A to O. Worksheets A to F indirectly aided all four tasks in the test because they involved understanding and skills that may help in improving proportional

reasoning. The National Mathematics Advisory Panel (2008, pp. xix, 29) of the United States mentions the importance of fractions, which are covered by worksheets A (*Fractions: equality, addition, subtraction*) and B (*Fractions: multiplication, division*), and of number lines, which are covered by worksheets C (*Locating numbers on a number line*) and D (*Identifying points on a number line*).

Worksheet E (*Ratio and proportion using a linear scale*) used Adjiage and Pluvinage's (2007) linear scale register (in particular, the double scale), which they consider as "a privileged tool for interpreting and processing ratio problems" (p. 157). Worksheet F was a review of worksheets C to E.

The balance task involved conservation of angular moment (the product of a ring's weight and distance from a fulcrum). It was directly aided by worksheet I (*Interlocking toothed gears*) which involved conservation of linear speed (the product of a gear's number of teeth and angular speed) and by worksheet M (*Volumes of liquids in different containers*) which involved conservation of volume (the product of a liquid's height and area in a container).

The fullness task involved comparing the fullness of different containers of liquid. It was directly aided by worksheet G (*Ratio comparison problems*) which consisted of context-free tasks and by worksheet K (*Water rectangle*) which involved comparing the volumes of liquid in identical containers but with different orientations.

The mixture task involved mixing two liquids to get a solution whose volume is the sum of their volumes. It was directly aided by worksheet G and by worksheet J (*Sugar and water*) which involved dissolving grains in a liquid to get a solution with the same volume as the liquid.

The blocks task involved the decomposition and composition of a solid with uniform density. It was directly aided by worksheet H (*Mass of a liquid*) which involved finding the mass of a liquid given its density and its volume and by worksheet L (*Masses of chocolate bar pieces*) which involved the decomposition and composition of a solid with uniform density.

Worksheets N and O were reviews of worksheets G to M and directly aided all four tasks. (The Appendix shows some examples of the worksheets.)

Results

Worksheets

The conceptual group completed a total of 272 worksheets; the procedural group completed the same number of worksheets. For the conceptual group, the times to complete each worksheet ranged from 3 to 46 minutes (average of 13.9 minutes, median of 12 minutes). For the procedural group, the times ranged from 3 to 48 minutes (average of 14.3 minutes, median of 13 minutes). The times for the two groups did not differ significantly (Kruskal & Wallis $H = 2.5292$, $p = .1118$).

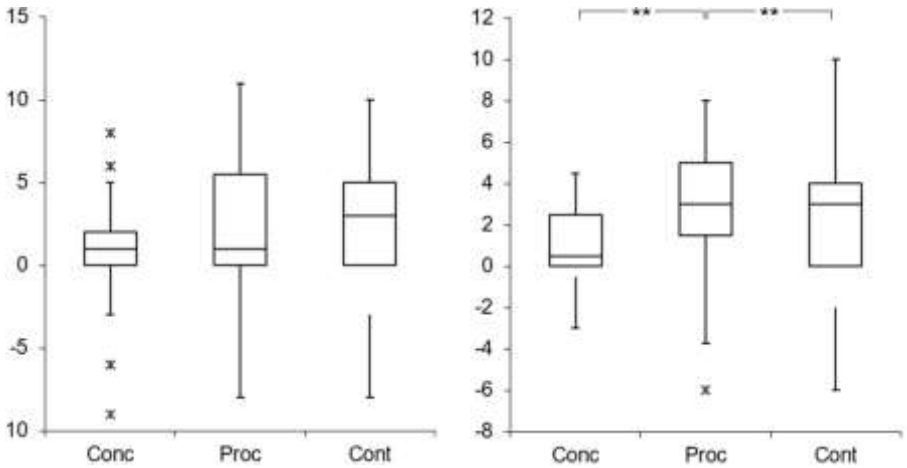
The conceptual group did 67.3% (183 out of 272) of their worksheets under the supervision of the experimenter; the corresponding figure for the procedural group was 65.4% (178 out of 272). A Fisher exact test for 2×2 tables shows that the differences were not significant ($p = .7167$).

Pretest and Posttest

The test had 69 problems, of which 45 were nonnumeric and 24 were numeric. The conceptual, the procedural, and the control group had 19, 19, and 40 participants, respectively. At the start of the experiment, the three groups of participants had no significant differences in their amount of conceptual knowledge ($H = 5.4706$, $p = .0649$) and in their amount of procedural knowledge ($H = 0.7006$, $p = .7045$).

Figure 2(a) shows the nonnumeric problems posttest scores minus pretest scores of each group. The changes in amount of conceptual knowledge of the three groups had no significant differences ($H = 4.1769$, $p = .1239$).

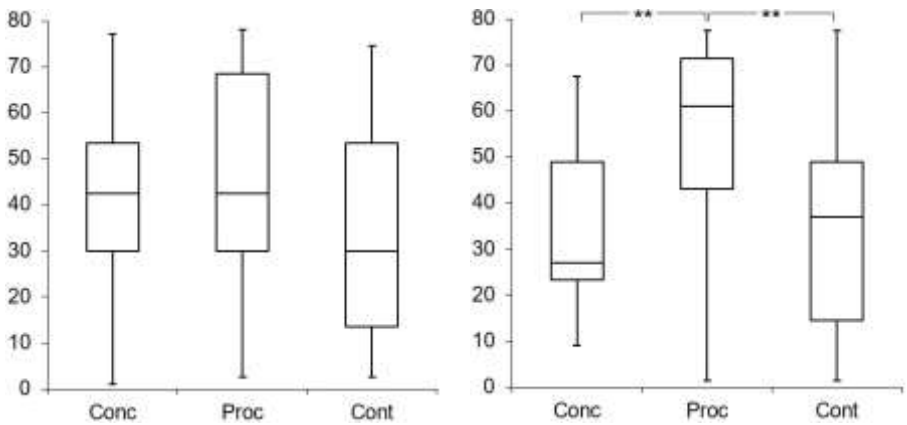
Figure 2(b) shows the numeric problems posttest scores minus pretest scores of each group. The changes in amount of procedural knowledge of the three groups had significant differences ($H = 14.2670$, $p = .0008$). The procedural group results significantly differed from the conceptual group results ($p < .01$) and the control group results ($p < .01$).



(a) (b)

Figure 2. (a) Changes in amount of conceptual knowledge (raw scores); (b) Changes in amount of procedural knowledge (**$p < .01$) (raw scores)

Because the *H*-test uses ranks, the differences among the groups are more apparent if the box plots show the ranks of the raw scores as they do in Figure 3.



(a) (b)

Figure 3. (a) Changes in amount of conceptual knowledge (ranks); (b) Changes in amount of procedural knowledge (**$p < .01$) (ranks)

Discussion

Comparison with Earlier Work

In an earlier study involving undergraduates only, Noche (2013) used a conceptual, a procedural, and a control group having 10, 12, and 24 participants, respectively, and a treatment lasting eleven days. He found no significant differences in the changes in amount of conceptual knowledge of the three groups (H = 0.06, p = .968). He did, however, find a significant difference in the changes in amount of procedural knowledge (H = 12.54, p = .002), with the procedural group results significantly differing from the conceptual group results (p < .05) and the control group results (p < .01). Figure 4 shows the main results of his study.

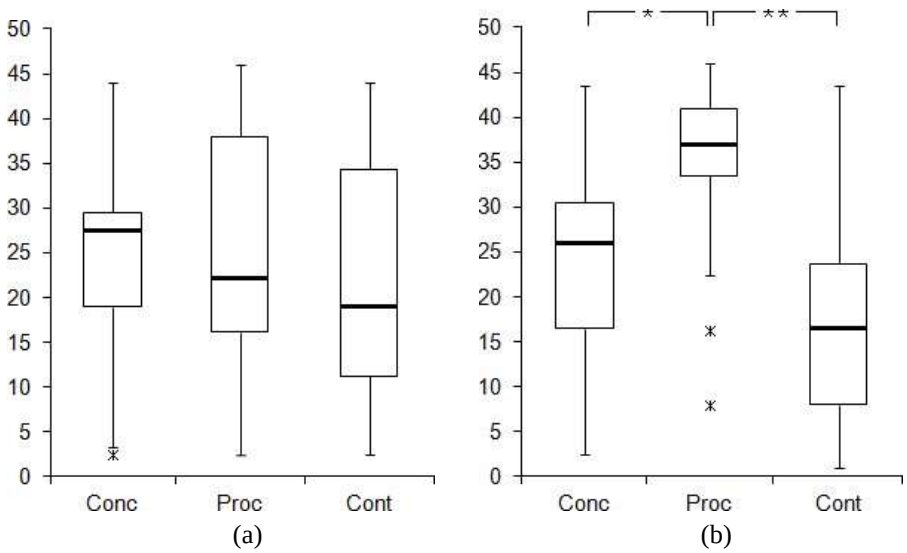


Figure 4. (Noche, 2013) (a) Changes in amount of conceptual knowledge (ranks); (b) Changes in amount of procedural knowledge (*p < .05, **p < .01) (ranks)

One major concern with Noche's (2013) study was that the conceptual and the procedural groups significantly differed in the times to complete the worksheets (H = 13.69, p = .000). It was thus possible that the differences in

performance between the two groups were due to the different times taken in the instruction rather than the different types of instruction.

It was also possible that the lack of significant differences in the changes in amount of conceptual knowledge was due to the small sample size or the short treatment period.

We aimed to address these concerns by using supplemental instruction that had no significant differences in completion times (p = .1118 vs. p = .000), by using a larger sample size (78 students vs. 46), and by using a longer treatment period (15 days vs. 11). We also attempted to extend the generalizability of the results by including junior high school participants and not just undergraduates.

From Figures 3 and 4, it can be seen that our results support Noche's (2013) results.

However, the causal relations we found differ from those found in other studies. Rittle-Johnson et al. (2015) describe the findings of Rittle-Johnson and Alibali's (1999) study on mathematical equivalence done on elementary-school children as follows: "Children who received the procedure lesson gained a better understanding of the concept than the control group [who were given no lesson], and children who received the concept lesson gained greater procedural knowledge than the control group" (p. 590).

Furthermore, Rittle-Johnson et al. (2015) describe the findings of Canobi's study on arithmetic done on elementary-school children as follows: "Solving conceptually sequenced practice problems supported gains in procedural as well as conceptual knowledge relative to the control condition [solving nonmathematical problems]. Solving practice problems in random order supported only modest gains in procedural knowledge and did not support gains in conceptual knowledge" (p. 591).

Conclusion

Our research hypothesis is that the type of supplemental instruction (conceptual, procedural, or none) affects the amount of a type of knowledge

(conceptual or procedural). It can be rewritten as two hypotheses: (1) the type of supplemental instruction affects the amount of conceptual knowledge, and (2) the type of supplemental instruction affects the amount of procedural knowledge.

Our results did not support hypothesis 1. We found no evidence that the type of supplemental instruction affected the amount of conceptual knowledge.

Our results supported hypothesis 2. In particular, we found that supplemental procedural instruction significantly differed from the other types of supplemental instruction in affecting the amount of procedural knowledge. This difference does not seem to be significantly due to differences in treatment times (number of worksheets done, worksheet completion times) or treatment conditions (percent of worksheets done under the experimenter's supervision).

We used a no-treatment control group and random assignment of participants to treatment groups to be able to draw causal inferences. We found a causal relationship between procedural instruction and procedural knowledge. We did not find any other causal relationship, not even between conceptual instruction and conceptual knowledge.

Our results support the view that procedural instruction is more effective and efficient than conceptual instruction. Perhaps our results will provide additional insight on why students in East Asian countries perform well in international studies of mathematics achievement despite receiving instruction that is characterized as being mostly procedural.

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Appendix

Shown here are some "water rectangle" tasks used in the study. Page Na5 is from a worksheet to be done by the conceptual group; page Nb5 is from a worksheet to be done by the procedural group.

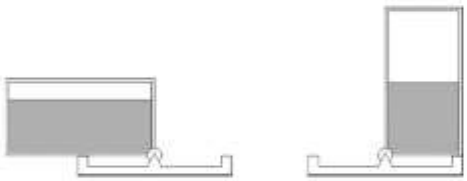
Na5

Colored liquid is put into each of a pair of identical rectangular containers, then the containers are sealed. The amount of liquid put into the containers may differ or may be the same.

1. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

2. What can be said about the two containers shown below?



(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

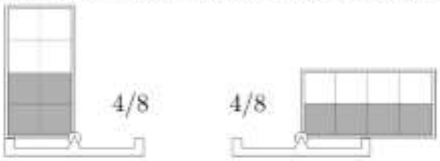
3. What can be said about the two containers shown below?

(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Both containers have the same amount of liquid.

Nb5

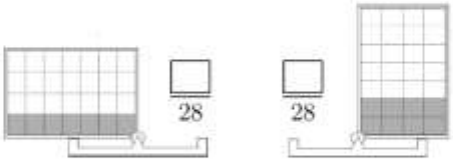
Colored liquid is put into each of a pair of identical rectangular containers, then the containers are sealed. The amount of liquid put into the containers may differ or may be the same.

1. Which of the two containers shown below has more liquid?



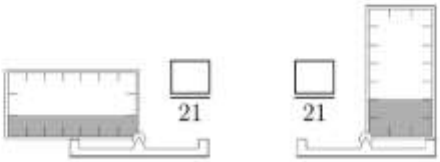
(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Neither. Both have the same amount of liquid.

2. Indicate the fraction of the grid covered by the liquid in each container shown below. Which container has more liquid?



(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Neither. Both have the same amount of liquid.

3. Indicate the fraction of the grid covered by the liquid in each container shown below. Which container has more liquid?



(a) The container on the left has more liquid.
(b) The container on the right has more liquid.
(c) Neither. Both have the same amount of liquid.