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Metacognitive Regulatory Phases during Problem Posing of Grade 9 Students

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Abstract: This study investigated the problem-posing phases of regulation of cognition of 466 grade 9 students from three Secondary schools in Singapore. Schoenfeld's episode-based framework for examining regulation of cognition during problem solving was used as a basis for hypothesizing the different phases in problem posing. To validate these phases, students' responses to a questionnaire after they had completed two problem-posing tasks were investigated using exploratory factor analysis. Three distinct emerging regulatory factors of *Planning*, *Checking* and *Looking Back* during problem posing were found and that the phases on *Property Noticing* and *Problem Construction* formed part of the factor on *Planning*. The study contributed to addressing the paucity in the knowledge about regulation of cognition during problem posing and pointed to some directions for future work. These findings could also help shape better instructions for promoting problem-posing activities in the classroom.

Keywords: problem posing, regulation of cognition, problem solving

Introduction

In this study, mathematical problem posing is defined as the generation of new problems or the re-formulation of existing ones (Silver, 1994). A problem involves a context with an initial state and an unknown (goal) state that has to be resolved. Implicitly an answer is needed. Although the field of problem posing in mathematics education research has received active research inquiry only in the "last two decades or so" (Silver & Yankson, 2017), several authors have pointed to the importance of students' mathematical problem posing. Much of the work is linked to students' exploration in mathematics (Cai, Hwang, Jiang & Silber, 2015; Cifarelli & Sevim, 2015; Cai, 2003) and to the teaching and learning of mathematics

(Crespo, 2003; English, 1997; NCTM, 2000). Bransford et al. (1996) opined that to develop mathematical thinking needed to solve complex real world problems, it is important for students to be able to generate and to formulate their own problems. Researchers like Brown and Walter (1993) highlighted that problem-posing activities within mathematics lessons can help in reducing mathematics anxiety, in surfacing misconceptions and in promoting group learning and that "we learn mathematics when we were actively engaged in creating not only the solution strategies but the problem that demand them" (p. 187). The emerging significance of problem posing as a mathematical activity that could promote engaged learning provides the main impetus to the present study.

Theoretical Framework

Problem-Posing Actions

The interpretative framework shown in Figure 1 synthesizes most of the ideas espoused in previous studies and includes a tentative classification of the underlying processes (Chua, 2011). The context of the framework is on students posing their own problems to a given stimulus. Consistent with Kontorovich and Koichu's (2009) four-facet framework to describe problem posing, the interpretative framework draws on Schoenfeld's (1985a) model comprising categories for understanding problem solving. Various research also suggests the close relationship between the processes of problem solving and problem posing (Mamona-Downs & Downs, 2005; Polya, 1971; Silver, 1995). The framework also draws on the notion of recursion in the Pirie-Kieren model of how students develop mathematical understanding (Pirie & Kieren, 1994). The interpretative framework characterizes much of the actions and behaviours during problem posing. However, it is not intended to depict any sequencing of problem-posing actions.

Strategizing the problem formulation involves drawing topics (from the poser's resources) to be used for posing the problem. Christou et al. (2005b) suggested that the posing may involve processes like *association*, *analogy* (Kilpatrick, 1987), *editing*, *selecting*, *comprehending* and *translating* of quantitative information. The poser starts by sieving the key components of a given mathematical stimulus, and explores how the inter-related components can be linked to the objective of formulating the problem. For example, given the stem “$2x^2 + x - 1$” to pose a problem, the poser may draw on the related quadratic concepts for inclusion into the problem formulation.

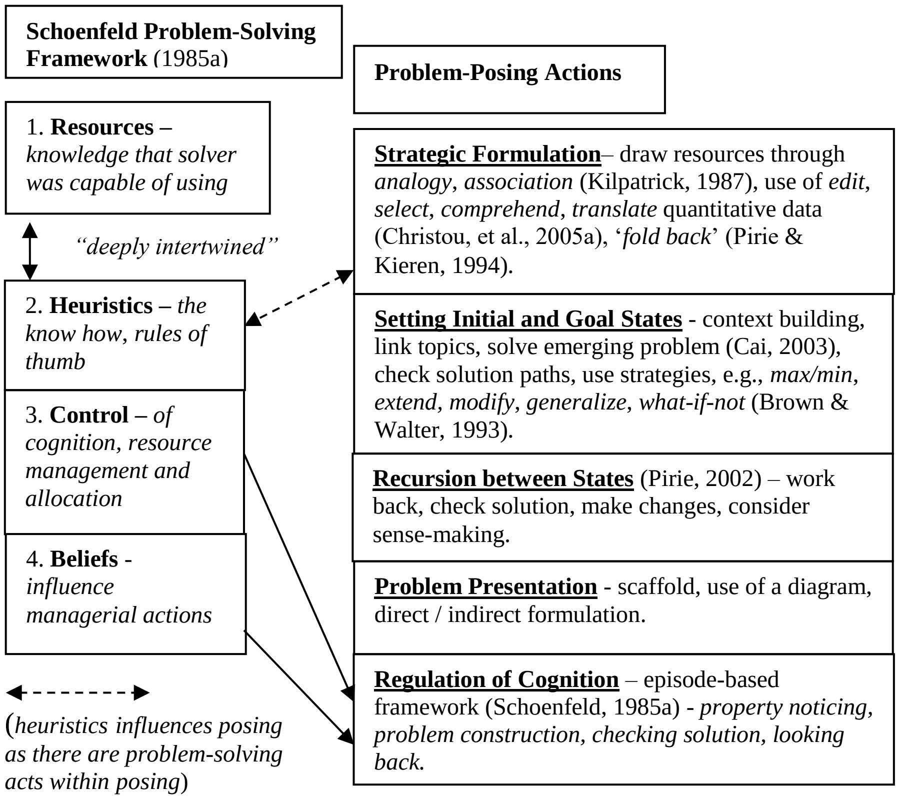

Figure 1. Towards a Conceptual Framework of Problem Posing (Chua, 2011)

In problem posing, students have to decide which of the related concepts should be included in the building of the initial state and the goal state for the emerging problem. This strategic action affects the formulation of the problem.

Setting of initial and goal states involves setting the context for the emerging problem. Students’ knowledge of problem-solving heuristics may influence the setting of these states because they may think about the solutions as they pose their problems (Cai, 2003; Lowrie, 2002). Students could use problem-posing strategies such as "*what-if-not*" (Brown & Walter, 1993), or formulate problems that involve maximum or minimum conditions. The problem-posing actions could be seen in the example:

> "*Mary has 20 coins that she had to put into two boxes with one of them having to contain at least six coins*. [initial state] *What is the maximum number of coins that the other box can contain?* [goal state arrived at by using maximum/ minimum strategy] (Chua, 2011)

Recursion between states is about the switching between the initial state and the goal state to validate the emerging problem and to check whether the states are consistent with the solution path. Through planning and analysis, successive refinements are made to the emerging problem, for example, by checking if the posed problem makes sense. Proulx and Maheux (2017) argued that the posing of a problem by itself is not static because the "posing triggers a solving process that in turn transforms the initial posing" (p.163). Chua (2011) noted that *recursion* in problem posing involves "a complex process of *folding back* between prior knowledge in the poser's resources and the checking on the emerging problem." For example, with the initial state as "*given that* $2x^2 + x - 1 = 0$*"* and a tentative goal state as "*find x"*, the poser may *fold back* on his or her prior knowledge about the properties of quadratic graphs and the quadratic discriminant to modify the goal state which could appear as "*by sketching an appropriate graph, find the number of real roots.*"

Problem presentation is about creating the final form of the posed problem. The final form of the posed problem can be a direct or an indirect problem, or a problem containing various degrees of scaffolding. This can be done, for example, by the inclusion of a diagram in the original problem to provide contextual support. The problem

> "*find the number of real roots in* $2x^2 + x - 1 = 0$"

is a direct problem since the *discriminant* is positive. Although the problem

> "*if the quadratic* $px^2 + x - 1 = 0$ *has real roots, what is the minimum value of p?*"

could be considered indirect since p is an unknown to be computed and is constrained by having to be a minimum (Chua, 2011).

Studying problem posing purely as a cognitive process would not be sufficient given the complexities involved in its processes. One needs to study how the cognitive process is being regulated, as argued by Garofalo and Lester (1985): "successful cognitive performance depended on having not only adequate knowledge but also sufficient awareness and control of that knowledge" (p. 163).

Regulation of Cognition

In problem posing, the initial and goal states have to be created and the accompanying solution paths have to be considered. Far from being a straight-forward linear process, it involves *recursion* between the two states, as the poser checks for coherence as the solution path is being worked on. The knowledge of the mathematical ability of the intended solver is also another consideration in the process of problem posing. Because the problem poser needs to keep track of what he or she is doing and thinking, the poser requires a control of the cognitive processes.

Described simply, metacognition is "thinking about thinking" (Livingston, 1997). In mathematics education research, the use of the term metacognition does not have a common definition. In the context of acquisition and of application of learning skills, researchers like Sperling, Howard and Staley (2004) and Schraw and Moshman (1995) pointed to two of the metacognitive components that are consistent with Flavell's (1981) original notion of metacognition. The component on *knowledge about cognition* points to one's level of understanding of one's own memories, of the cognitive system and of how one learns. The other component on the *regulation of cognition* is about how one could regulate one's own approach to learning, including goal setting, choosing and applying strategies and monitoring of one's actions.

Georghiades (2004) however noted that different researchers provided different definitions, portraying different focuses on the processes and the mechanisms associated with metacognition. This led to the development of different instruments to study the construct. Some instruments that had been developed served a more general purpose, like studying metacognition in reading comprehension which are not suitable for use in this study.

Schoenfeld (1985b) specifically pointed out that "the techniques of the psychological community for exploring metacognition, while useful, would prove far too limited for the purposes of mathematics education" (p. 379). He suggested the need for a variety of techniques for the analysis of problem-solving protocols such as clinical interviews.

Problem posing studies can draw from Schoenfeld's (1985a) notion of "control" which is about resource allocation during problem-solving performance and which he pointed as being a "major determinant of the problem-solving outcome" (p. 143). Just like in problem solving, where "it is not only what you know but how you use it that matters" (Schoenfeld, 1987, p. 192), decision making in problem posing comes about from drawing from the poser's prior knowledge and understanding the context of the proper use of the knowledge to formulate the problem.

Schoenfeld's (1985a) "*control*" of cognition in problem solving refers to "global decisions" regarding the selection and implementation of resources and strategies. Processes include *planning*, *monitoring* and *assessment*. The present study follows Schoenfeld's notion of regulation of cognition as it is used in his episode-based framework for analysis of problem-solving protocols. His framework which focuses on decision-making behaviour during problem solving involves the parsing of verbal protocols into episodes which he described as "periods of time during which the problem solver was engaged in either one large task or in closely related body of tasks in the service of the same goal" (p. 316).

Schoenfeld (1985a) pointed to five episodes during problem solving (*read, analyse, explore, plan / implement and verify*). The solver's decisions at the transition points between episodes could have implications on the outcomes of the solution attempts. According to Schoenfeld (1985a), *reading* the problem could be overt or silent. In *analysis*, the solver attempts to understand the problem by identifying task-specific knowledge, including familiar problems and being cognizant about the initial conditions and goal state. In *exploration*, as the need dictates, the solver makes a decision on the progress of the solution path, including working on less structured ways of solving. *Planning / implementation* involves making selection of solution steps and strategies and checking of the follow through in the planned course of action. *Verification* involves checking the solution for sense making and meeting the goal state. Given the complexity of problem solving, the five phases may not occur in a linear manner. The framework specifies the characteristics of each episode which could be compared with students' observed problem-solving behaviour.

Phases in Problem Posing

A summary of the descriptions about the phases in problem solving and in problem posing is shown in Table 1. Yimer and Ellerton (2006) noted that numerous metacognitive frameworks are "minor variations" of Polya's (1971) four-stage model. The proposed problem-posing phase framework also takes reference from earlier work from Polya's (1971) framework on problem solving and also pulls descriptions from Schoenfeld's episode-based model to describe the problem-posing phases. The resulting framework is useful in putting the problem-posing phases into the contexts in which they occurred.

In the *Property Noticing* phase, a problem poser has to decide on what to use from his or her knowledge to set the initial and goal states. Under the *Problem Construction* phase, the problem poser has to build context and use posing strategies to link the two states. During *Checking Solution*, a problem poser after finding that the solution path is not compatible with the initial and goal states, may have to revert back to the *Property Noticing* phase. The problem poser then retrieves other suitable knowledge or creates new connections to continue with the posing. The problem poser may have to make recursions between the states as he or she checks and makes changes to the emerging problem. The problem poser may reflect on the validity of the problem and again may recursively make changes during the phase on *Looking Back* (Chua, 2011).

A series of related problem-posing actions in the conceptual framework for problem posing may be linked to the problem-posing phases. For example, during the *Property Noticing* and *Problem Construction* phases, the problem poser may have to draw upon his or her prior knowledge through *analogy* and *association* to start the strategic formulation of the problem and to set the initial and goal states.

Table 1
Regulation of Cognition in Problem Posing and in Problem Solving (Chua, 2011)

Problem-Posing Phase Framework	Brief Description (with Schoenfeld's terms)	Polya's Problem-Solving Framework (1971)	Brief Description
Property Noticing	Decide on what to use from resources to set initial and goal states. [Schoenfeld's *reading, exploration* episodes]	Understand the Problem	Include understanding initial and goal states, representing the problem. [Lester's (1985) 'orientation' category]
Problem Construction	Create context and use posing strategies to link states. [Schoenfeld's *planning-implementation* episode]	Devise a Plan	Use problem-solving strategies and processes. [Lester 's (1985) 'organization' category]
Checking Solution	Check on solution path of emerging problem, modifying states if necessary. [Schoenfeld's episodes of *exploration, implementation, analysis*]	Carry out the Plan	Work on the plan - check if it works, otherwise go back to first step. [Lester's (1985) 'execution' phase]
Looking Back	Reflect on how posed problem can be done differently, reflect on quality of posed problem and confidence	Look Back	Reflect on solution, any alternatives. [Lester's (1985) 'verification' category]

Method

Subjects

A total of 466 grade 9 students from three Secondary schools in Singapore took part in this study. Students in this study were novice problem posers, and they would not be expected to exhibit many of the problem-posing characteristics.

Instruments

Students worked on two geometric problem-posing tasks which were contextualized in a form that required them to pose a problem for their friends to answer (see Appendix 1). They also solved their own posed problems. Students were then asked to complete a questionnaire on the regulation of cognition immediately after the completion of the two tasks. This retrospectively captures aspects of regulation of their posing actions.

The aim of the questionnaire is to test empirically the problem-posing phases (*Property Noticing*, *Problem Construction*, *Checking Solution*, *Looking Back*) described in the Problem-Posing Phase Framework. The primary purpose is to build an emerging inventory specific to the regulation of cognition so as to further understand the metacognitive regulatory phases during problem posing.

A check on research literature in mathematics education showed there was no known instrument that was designed to measure specifically metacognitive regulatory processes in problem posing. Since problem solving and problem posing are closely related, the questionnaire design therefore had to be adapted from a suitable source within research on metacognitive processes during problem solving.

Schoenfeld's (1985a) episode-based problem-solving framework was adopted as a basis for drawing out items of relevance for problem posing. In particular, a version of the Schoenfeld's 21-item metacognitive statements used by Goos, Galbraith and Renshaw (2000) in their study of the metacognitive aspects of grade 9 students solving combinatorics problems was adapted for the present study (Chua, 2011). Changes were made on the statements and new items were added to serve the purpose of checking on the actions within the problem-posing phases.

The first section on "*before you started posing*" contains five possible strategies related to the *Property Noticing* phase. The second section on "*during posing your problem*" refers to the *Problem Construction* phase involving eight possible actions. The third section on "*after you had finished posing*" points to six actions regarding *checking* the products of the posing. The seven items in the fourth section on *"some other thoughts as I was posing the problem"* ask students to *look back* at the problem-posing task after they had completed their work.

To check for face validity, ten grade 9 students who were not part of the main study, were asked to take the resulting 26-item questionnaire survey. Two mathematics teacher-educators and four teachers with knowledge of problem posing were also asked to read the 26 items to evaluate the appropriateness and relevance of the items. Feedback from the students and the teachers on some of the item wordings were used to improve item clarity and relevance. A 6-point Likert scale was used, with 1: *strongly disagree,* 2: *disagree,* 3: *slightly disagree,* 4: *slightly agree,* 5: *agree* and 6: *strongly agree*. The final version of the instrument shown in Table 2 comprises 26 items distributed across the four phases with at least five items per phase.

Table 2
Metacognitive Questionnaire

Before you started posing the problem, what do you do?	
M1	I read what was required in the task at least twice.
M2	I understood what the task was asking me to do.
M3	I describe how I was going to pose the problem.
M4	I tried to remember if I had seen a problem that could be used for this task.
M5	I thought about what topics could be put together to form the problem.
During posing your problem, what did you do?	
M6	I thought about the solution steps involved as I was posing the problem.
M7	I read my posed problem at least twice.

M8	I thought of different ways of posing the problem.
M9	I kept looking back at the posed problem at each solution step.
M10	I made changes to the posed problem while solving it.
M11	I had to stop and rethink each step if I was getting to the problem that I wanted.
M12	I checked the solution to my posed problem step by step.
M13	I made a mistake and had to redo my step(s) as I was solving my posed problem.
After you had finished posing the problem – what did you do?	
M14	I looked back at the completed solution.
M15	I checked to see if I my calculations were correct.
M16	I asked if I could have posed a different problem after I had finished.
M17	I thought my friends had seen my posed problem before.
M18	I looked back at the posed problem to see if it made sense.
M19	I thought about different methods that I could have used to solve my posed problem.
Here were some other thoughts as I was posing the problem.	
M20	I used methods of asking questions that I had seen before while posing the problem.
M21	I liked the problem that I posed.
M22	I felt that posing a problem was easy.
M23	I knew how well I had done once I finished the task.
M24	The problem was the one I had planned for.
M25	I was sure I could solve the problem I had posed.
M26	I wrote down important points when I was posing the problem.

Results

Factor analysis was used to analyse the data set comprising students' responses to this survey. The purpose of the analysis was to surface the emerging problem-posing phases hypothesized. To explicate the potential factors in the 26-item instrument, exploratory factor analysis using principal component analysis with direct oblimin oblique rotation was used. The Kaiser-Meyer-Olin measure verified the sampling adequacy of the analysis, with KMO = .898, and with Bartlett's test of sphericity χ^2 (325, N = 466) = 4533.09, $p < .001$. These indicated that the correlation structure was adequate for factor analysis. By setting the maximum likelihood factor analysis with a cut-off point of .30 and the Kaiser's criterion of eigenvalues greater than 1, a four-factor solution as the best fit for the data was found. For all the items, almost all of the communalities were greater than .40. The results of this factor analysis are presented in Appendix 2.

An initial labelling of the emerging factors was attempted. Factor 1 contained several variables which involved some forms of '*checking*' on the work done. Factor 2 appeared to indicate the phase of '*looking back*' while Factor 4 pointed to some form of '*planning.*' Factor 3 with its two variables was not a strong interpretable factor.

Factor 1: Checking of Work

The high communalities together with the Cronbach's *Alpha* of .869 across the ten variables in Factor 1, suggested that the variables collectively gave a good fit to the factor. M18 (*checking if posed problem makes sense after posing*) and M11 (*re-thinking at each step during posing*) were also loaded into other factors. Because they offered a better interpretability to Factor 1 which was on *checking* than to the other factors, they were included in in this factor. The summary of the variables and the loadings for Factor 1 is shown in Table 3.

Table 3
Factor 1 - *Checking*

Description	Var	Load
I checked the solution to my posed problem step by step. [during posing]	M12	.794
I made changes to the posed problem while solving it. [during posing]	M10	.758
I made a mistake and had to redo my step(s) as I was solving my posed problem. [during posing]	M13	.736
I kept looking back at the posed problem at each solution step. [during posing]	M9	.721
I wrote down important points when I was posing the problem. [others]	M26	.523
I looked back at the completed solution. [after posing]	M14	.506
I looked back at the posed problem to see if it made sense. [after posing]	M18	.456
I checked to see if I my calculations were correct. [after posing]	M15	.438
I had to stop and rethink each step if I was getting to the problem that I wanted. [during posing]	M11	.397
I thought about different methods that I could have used to solve my posed problem. [after posing]	M19	.395

The phase of "*during posing*" featured strongly in the first four highest loading, with M12 (*checking step-by-step*) and M13 (*re-doing steps*) indicating some form of checking of the solution to the emerging problem. The checking of the final solution was indicative in M14 (*looking back at completed solution*) and in M15 (*checking calculations*). M18 (*making sense*), M19 (*thinking about different methods*) and M26 (*writing important points*) suggested some checking of the final posed problem. M10 (*making changes*), M9 (*looking back*) and M11 (*rethinking each step*) also pointed to some of the types of actions during the checking of the posed problem while working on the task. The *checking solution* phase during problem posing characterized much of Factor 1, with students checking before and checking after posing the problem.

Factor 2: Looking Back

Factor 2 had a Cronbach's *Alpha* of .760. M4 (*trying to remember a similar problem that could be used*) was placed in Factor 4 (*planning*). A summary of the variables and the loadings for Factor 2 is shown in Table 4. The three largest loadings, namely, M22 (*feeling posing problem was easy*), M21 (*liking the posed problem*), and M23 (*knowing how well it was done*) were on reflections of the task. M6 (*thinking about the solution steps*) probably reflected the students' thoughts about the method of approach during posing, that is, students thinking in a recursive way as they switched between checking of the initial and goal states.

Table 4
Factor 2 – *Looking Back*

Description	Var	Load
I felt that posing a problem was easy. [others]	M22	.829
I liked the problem that I posed. [others]	M21	.670
I knew how well I had done once I finished the task. [others]	M23	.601
I thought about the solution steps involved as I was posing the problem. [during posing]	M6	.588
The problem was the one I had planned for. [others]	M24	.562
I asked if I could have posed a different problem after I had finished. [after posing]	M16	.457
I was sure I could solve the problem I had posed. [others]	M25	.465

Various forms of students' reflections in Factor 2 suggested that issues involving *affect* could be part of the *looking back* phase. For example, M25 (*sure could solve*) could be an indicative of the students' confidence in the task that they had done. Pondering if they could have *posed a different problem* (M16) and confirming that the posed problem was what they *had planned for* (M24) were the different types of reflections in the phase of *looking back*.

Factor 3*: *Indeterminate Solution

Negatively loaded to Factor 3, M5 (*thinking about what topics to be put together*) was a better fit under *planning*. Both M20 (*thinking about familiar questions*) and M17 (*thinking about questions friends had seen before*) suggested the notion of familiarity with prior experiences while posing the problems. As shown in Table 5, although both M20 and M17 were highly loaded in this factor, the low number of variables suggested that interpretation of this factor had to be done with great circumspection (Kline, 1994, p. 175). Factor 3 was not clearly determinate.

Table 5
Factor 3 – Indeterminate Factor

Description	Var	Load
I used methods of asking questions that I had seen before while posing the problem. [others]	M20	.771
I thought my friends had seen my posed problem before. [after posing]	M17	.747

Factor 4: Planning

As shown in Table 6, the three largest loadings in Factor 4 pointed to the work on *planning* that had to be done before posing a problem. The Cronbach's *Alpha* for this factor was .795.

Table 6
Factor 4 – Planning

Description	Var	Load
I read what was required in the task at least twice. [before posing]	M1	.811
I understand what the task was asking me to do. [before posing]	M2	.809
I describe how I was going to pose the problem. [before posing]	M3	.656
I thought of different ways of posing the problem. [during posing]	M8	.440
I tried to remember if I had seen a problem that could be used for this task. [before posing]	M4	.344
I read my posed problem at least twice. [during posing]	M7	.430
I thought about what topics could be put together to form the problem. [before posing]	M5	.332

Factor 4 lent much support to phases on *Property Noticing* and on *Problem Construction* in the hypothesized Problem-Posing Phase Framework. Students' initial thoughts and their planning were reflected in M1 (*reading task more than once*), M2 (*understanding what the tasks asked*) and M3 (*describing how to pose*). M1, M2 and M3 together described much of the *Property Noticing* phase where students had to make decisions on what to draw from their prior knowledge to start crafting the initial and goal states. The actions during the *Problem Construction* phase, including setting context for the posed problem, drawing upon prior knowledge including posing strategies, can be gleaned from M5 (*what topics could be put together before posing*), M4 (*recalling an earlier problem*), M8 (*thinking of different ways of posing*), and M7 (*reading the posed problem*). That these actions occurred *before* and *during* problem posing strongly suggested that the *Problem Construction* phase involved students going through a recursive process of posing, and then re-assessing their work before moving to make modifications to the posed problems.

To further examine the labelling of the factors from the factor loadings, two mathematics educators were asked separately to examine the sets of items without the labels, and then asked to suggest a label to best represent each set of items. Except for some minor variations in terms selected, for example, *"building up the problem*" and "*planning the problem*" for *planning*, there were agreements on the factor labels.

The factor solution in this study also had high number of matches with the factors uncovered earlier in a pilot study using the same instrument on a group of 152 students after they had completed a problem-posing task involving a geometric stimulus (Chua & Yeap, 2008). This lent weight to the viability of the factor solution.

Linkages between Factors

A summary of the scales of the regulatory factors is shown in Table 7.

Table 7
Regulatory Scales with Cronbach's Alpha

Scale	Variables	No. of variables	Cronbach's alpha
Checking	M9, 10-15, 18-19, 26	10	0.869
Looking Back	M6, 16, 21-25	7	0.760
Planning	M1-5, 7-8	7	0.795
Whole Scale		24	0.902

Table 8
Correlation Matrix of the Three Factors

Component	*Checking*	*Looking Back*	*Planning*
Checking	1	.518	.682
Looking Back		1	.498
Planning			1

The results shown in the correlation matrix in Table 8 pointed to the strong links between *planning* and *checking*. This can be explained by the close relations between the process of planning the formulation of a problem and checking on its solvability and viability. The process of reflection of the progress made during the posing and the adaption of new plans of action to proceed were shown to be correlated in Table 8, with *looking back* and *planning* correlated at .498.

Unlike the research work on mathematical problem solving and gender, the effect of gender on problem-posing performance has not been well researched (Chua, 2011). Table 9 shows that gender and the regulatory phases in problem posing are not significantly associated.

Table 9
Gender and Regulatory Phases

Scale	Male (n = 210) Mean (SD)	Female (n = 256) Mean (SD)	Difference in Means
Checking	4.37 (0.77)	4.35 (0.72)	0.02 (p = .765)
Looking Back	3.92 (0.81)	3.72 (0.81)	0.02 (p = .601)
Planning	4.09 (0.82)	4.01 (0.76)	0.08 (p = .286)

Discussion

Through the analysis from the questionnaire on the regulation of cognition, three emerging regulatory factors of *checking*, *looking back* and *planning* had been uncovered in this study. The various items contained in each of the factors afford researchers who are studying the different stages of problem posing, useful descriptors and a means for further analysis by looking at the sub-scales. The study also linked the hypothesized phases of *Property Noticing* and *Problem Construction* in the Problem-Posing Phase Framework to the factor on *planning*. In *planning*, it would be instrumental to study the extent which students' pre-existing domain knowledge (*Property Noticing*) shapes the posing of problems. Such findings could shed light on how students are able to draw from the specific domain knowledge in posing problems.

The study pointed to the strong correlations between the three clear factors of *planning*, *checking* and *looking back* during the problem-posing phases. This could suggest that students' problem-posing actions like *setting initial and goal states*, *recursion between states*, and *strategic formulation* may be at play during the problem-posing phases. Further studies may unravel the close interplay of these actions during these phases in problem posing. Because problem-posing tasks have to be situated within a context, future studies may also investigate to what extent are these problem-posing actions task-specific, and whether they may differ across different mathematical domains. Given the close relation between problem solving and problem posing, findings from these studies may shed light on the types of students' problem-posing strategies beyond the *what-if* strategy, and in that process, support instructions on problem-posing within mathematics lessons.

Mathematical problem posing can be learned just like problem solving. There are various problem-posing strategies and pedagogies advocated by researchers to improve students' learning in the mathematics classroom. For example, Manouchehri (2001) developed the "Four Point Instructional Model" to promote "genuine mathematical inquiry" in a grade 6 classroom through problem posing, while Lowrie (1999) developed a ten-week arithmetic problem-posing intervention programme to help grade 3 and 4 students produced more sophisticated problems. Such interventions can create a classroom environment amenable to problem-posing activities, and in that process also encourage students' explicit reflection on their own thinking processes. Just like being proficient in solving problems, posing good problems requires effective metacognition (A. H. Schoenfeld, personal communication, February 10, 2006). Explicit discussion with students on problem posing may bring about metacognitive experiences, just as learning problem solving would. For example, with the teacher role-modelling the problem-posing metacognitive processes, students can pick ideas about the components of a problem, then use that knowledge in formulating problems. In particular, the explicit classroom demonstrations of regulatory factors of *checking* and of *looking back* during problem posing could engender such good habits among the students. This may have a good 'translation' effect in improving students' problem solving.

The role of *beliefs* in shaping the "managerial actions" in Schoenfeld's Problem-Solving Framework (1985a) has also direct relevance in influencing how a student poses a problem. Specifically, the present study points to the presence of *affect* in the *looking back* phase during problem posing. For example, a student who believes in the inter-connected nature of mathematical inquiry would pose differently from another who believes that all mathematical problems must be solvable within a short period of time. Further studies may look into the effect of *beliefs*, and other aspects of the affective domain like *attitudes* and *self-efficacy* (McLeod, 1992), in influencing problem-posing actions. Such understanding can help in engendering problem-posing behaviour in the mathematics classroom.

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Appendix 1

<u>Task 1</u>

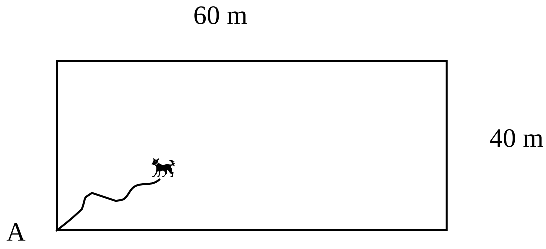
The first task involves the writing of a problem for their friends from a known answer:

"Write a problem so that the final answer is 60°"

<u>Task 2</u>

The focus of Task 2 is on building and developing a problem from a given context. The task requires the students to pose a problem for their friends based on this situation:

"A goat is inside a 60 m by 40 m rectangular fence in a farm. It is tied to a pole at A by a 30 m rope which could not be stretched "



Appendix 2

Factor Pattern Matrix for Four-Factor Solution

	Variables	Factor				Communality
		1	2	3	4	
M12	I checked the solution to my posed problem step by step. [during posing]	.794				.621
M10	I made changes to the posed problem while solving it. [during posing]	.758				.504
M13	I made a mistake and had to redo my step(s) as I was solving my posed problem. [during posing]	.736				.610
M9	I kept looking back at the posed problem at each solution step. [during posing]	.721				.556
M26	I wrote down important points when I was posing the problem. [others]	.523				.401
M14	I looked back at the completed solution. [after posing]	.506				.470
M18	I looked back at the posed problem to see if it made sense. [after posing]	.456		.378		.527
M15	I checked to see if I my calculations were correct. [after posing]	.438				.543
M19	I thought about different methods that I could have used to solve my posed problem. [after posing]	.395				.429
M22	I felt that posing a problem was easy. [others]		.829			.621
M21	I liked the problem that I posed. [others]		.670			.461

Variables		Factor 1	2	3	4	Communality
M23	I knew how well I had done once I finished the task. [others]		.601			.472
M6	I thought about the solution steps involved as I was posing the problem. [during posing]		.588			.430
M24	The problem was the one I had planned for. [others]		.562			.395
M25	I was sure I could solve the problem I had posed. [others]		.465			.414
M16	I asked if I could have posed a different problem after I had finished. [after posing]		.457			.373
M4	I tried to remember if I had seen a problem that could be used for this task. [before posing]		.400		.344	.470
M20	I used methods of asking questions that I had seen before while posing the problem. [others]			.771		.640
M17	I thought my friends had seen my posed problem before. [after posing]			.747		.561
M5	I thought about what topics could be put together to form the problem. [before posing]			-.352	.332	.485
M1	I read what was required in the task at least twice. [before posing]				.811	.627
M2	I understand what the task was asking me to do. [before posing]				.809	.672

Variables		Factor				Communality
		1	2	3	4	
M3	I describe how I was going to pose the problem. [before posing]				.656	.528
M8	I thought of different ways of posing the problem. [during posing]	.352			.440	.453
M7	I read my posed problem at least twice. [during posing]				.430	.342
M11	I had to stop and rethink each step if I was getting to the problem that I wanted. [during posing]	.397			.421	.493