What Constructed Stories for Qualitative Bar Graphs and Line Graphs Tell about Graphicacy of High Attaining Primary 5 Boys: A Case Study

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Abstract: Graphicacy is an important skill particularly when graphs have become such an essential means to convey information. This study investigated whether children were able to construct appropriate stories to match qualitative graphs. Twenty-five high attaining Primary 5 children (11+) from an intact class of an all-boys school in Singapore who have completed the standard Singapore primary mathematics curriculum where they were taught picture graphs, bar graphs and line graphs participated in this study. They were asked to construct stories to match two qualitative bar graphs, one line graph and one quantitative line graph. The findings show that these children did not have difficulties constructing stories to match bar graphs. However they had more difficulties constructing appropriate stories for line graphs, particularly for the quantitative line graph. The stories for the qualitative graphs were similar to presented in instructional materials. Stories for the remaining quantitative line graph showed some children had very good understanding of primary mathematics. Instead of focusing on graphs as a means to convey information, the findings suggest that these children would benefit from experiences which would challenge them to elaborate the underlying properties captured by the graphs.

Keywords: Graphicacy, high attaining, qualitative graphs, quantitative graphs, line graphs, bar graphs

Introduction

Graphicacy, the ability to read graphs, is an important skill particularly when graphs have become such an essential means to convey information. Graphs are often the representations of choice used by government, businesses, and the news media to provide statistical information to the public because graphs
display distilled information which would otherwise require long descriptive narratives.

In statistics the context is important to analysing and interpreting the data (Franklin & Garfield, 2006; Franklin et al., 2005). The shape of the graph, the spread of data and unusual data points such as outliers are explained based on the context. Graphs convey factual information (e.g. growth of a selected population over a certain period, or the number of families with n family members within a certain age group) and they also provide opportunities to make inferences that are not directly observable in the graph (e.g. what can the population be in the next two years if the existing trend of growth continues). A fundamental learning outcome of how to read graphs is to develop a sense of how the data are spread out or grouped, what characteristics about the data set as a whole can be described and what global information about the population can be discerned from this data set (Van de Walle, Karp & Bay-Williams, 2014). If presented with a qualitative line graph, i.e. a graph where no numerical values are provided for the horizontal and vertical axes, (as opposed to quantitative graphs, i.e. graphs with numeric labels) what story would such a graph suggest to the reader? For example, when presented with two line graphs, A and B, with very different intercepts on the vertical axes but line graph A shows very little fluctuations about an imaginary horizontal line parallel to the horizontal axes than B, the reader would hesitate a guess that the data set was collected from the tropics, perhaps from Singapore where the daily time for sunrise changes within a narrow band of about 5 minutes. However if the data were collected in the United Kingdom, there would be a reasonable amount of fluctuation as one moves from the winter season to spring. Hence how the points are clustered about a line helps the reader interpret the daily fluctuations of the sunrise time but the global distribution of the points over a period of time helps identify the location of the region, perhaps one is in the tropics and the other from a temperate country. Or both graphs reflect information from the same region but one in winter and the other in summer.

Curricular materials help children acquire a sense of data through the introduction of different forms of graphs. The same information presented using different graphs provide a different snapshot of the data as a whole. Information presented in pie charts show how the each part of the data compares against the whole whereas bar graphs provide an added dimension
of quantity but do not compare parts against the whole. Furthermore different types of graphs are used for different contexts. Bar graphs represent a category or event, whereas line graphs represent continuous data, such as temperature change over a period of time. Both axes on a smooth line graph involve a continuous scale.

In Singapore work with graphical representations is part of data analysis component of Statistics Strand and work with graphs starts in Primary 1 with the introduction of picture graphs presented vertically or horizontally (Ministry of Education Singapore (MOE) 2007) and continues throughout the primary years, culminating with the teaching of pie charts in Primary Six. Analysis of the questions posed in local curricular materials suggests that such materials prepare children to answer problem solving questions using information presented in quantitative graphs. Questions required children to construct bar graphs, count and tally objects in graphs. Problem solving questions require children to read information from the graphs and answer questions such (i) find the total given the tally, find the difference given the number in each category. Such materials are more likely to develop “algorithmic graphing” that focus more on answering factual or low-level interpretative questions (Pereira-Mendoza, 1995) instead of statistical thinking of the nature highlighted by Van de Walle, Karp & Bay-Williams (2014) which emphasizes looking at the data and deciding what story can be told and what reasonable conclusions can be drawn from the data (Rosenfeld, 2013).

Given the nature of the curricular materials, it is likely that Singapore primary children would succeed with questions involving quantitative graphs. How would these children respond to tasks which require them to create stories to match selected qualitative graphs? What stories would they construct? What would these stories inform educators of these children’s perceptions of the graphs? Would these stories be mere iconic or visual representations of the physical nature of the graph or would these children create stories that are statistically meaningful? Thus it is the objective of this paper to report on the stories created by 25 high attaining Primary 5 children from one intact class and their attaining to interpret qualitative graphs. Although the sample size of this study is small the findings are no less important as it is vital for mathematics educators and mathematics curriculum designers to prepare children to interpret graphs as this skill can be utilized in other subjects and
more importantly to respond in a considered and informed manner to the vast amount of data ever present in a fast changing world (You, 2009).

The next section reviews the relevant literature on graphicacy and this is followed a discussion of the study and the research question to be addressed in this paper. The penultimate section discusses the findings from the study and the conclusions drawn from this study closes this paper.

Review of Related Literature

Graphicacy, the ability to interpret spatial-visual information seems to come naturally to children. Compared to literacy, graphicacy is a more “basic skill” because young children seem to be able to read graphs with almost no instruction. A child is able to differentiate one-fourth from one-third of a pizza and that one-fourth is smaller than one-third before he or she learns to distinguish one fraction from another (Wainer, 1980). However children find some graphs more difficult to read than others (Padilla, Mckenzie & Shaw, Jr 1986; Wainer, 1980). Wainer’s work with a total of 360 children in a suburban school in United States, where equal number of participants from grades three, four and five were asked to interpret information presented in pie charts, tables, bar charts, and line graphs. Children found it most difficult to interpret line charts although performance increased with grade level but the differences between fourth and fifth grade children were modest. These findings were replicated by Padilla, Mckenzie & Shaw, Jr (1986).

Interpretation of graphs that represent situations can occur at different levels (Janvier, 1987). The questions posed can focus on local processing of information where the reader makes point-to-point interpretation. For example given a graph representing the sales of luxury cars in the last five years, the question in what year did the company make the most sale requires point-to-point interpretation. The question ‘From which year did the company show an increase in sale?’ requires trend detection or interval reading. Finally fully global interpretation requires reading of the whole graph such as in which year was the sales most rapid?

The use of iconic interpretation, in other words interpreting graphs as representations of physical phenomena rather than one representing an
abstraction was found to be a common error made by children. Some children may erroneously perceive an inverted V-shape graph for a hill displaying “pictorial distractors”. Others with “situational distractors” may confuse the experience of a situation with the abstract nature of that graph. For example some children confused a graph on the growth of two populations of microbe cultures in relation to their feeding times with “eating curves” and the rise and the fall of the curves to the consumption of food (Bell & Janvier, 1981, p37-38). Leinhardt, Zaslavsky & Stein, 1990) argued that children with iconic interpretation did not have symbolic understanding when asked to interpret and construct graphs. Symbolic interpretation of a graph focuses on understanding the mathematical relationship represented. Leinhardt, Zaslavsky, and Stein (1990) pointed out that readers of graphs should be able to consider the literal picture and reach a symbolic understanding of graphs. To test this point You (2009) conducted a study with 78 undergraduate students in a middle school teacher education programme and 170 seventh grade students with the specific aim to assess the solutions strategies used by each group when they were asked to choose four given graphs that represented the height of a flag above the ground as the flag was raised and to state their reasons. The responses found that 91% of the seven grade students committed iconic interpretation of the graphs. They chose the graph that represented the actual action of the flag going up the flag pole, i.e. a vertical line parallel to the vertical axis. It was not surprising that more undergraduate students provided symbolic understanding of graphs and these students were able to provide reasonable justifications for their choice. However You concluded that middle school students and undergraduate students encountered difficulty understanding the real-life situation portrayed in the flag raising task at a symbolic level.

McMillen and McMillen’s (2010) work with 18 children in a diverse second-grade class at a New York school is noteworthy. These children had prior knowledge of working with quantitative graphs. They could construct bar graphs, count and tally objects in graphs but they had not experience working with qualitative graphs. In their study the children were asked to create and identify "mystery" bar graphs. Their culminating activity was to match nursery rhymes, songs, and stories to the correct qualitative graph. The study showed that these children were able to match nursery rhymes, poems, songs and stories to almost all the selected qualitative bar graphs and create a numbered vertical axis as labels for the axis after the intervention. These children did not
focus on the visual features of the graph such as taller and shorter than to describe the bars but rather they used appropriate mathematical language such as less, more and fewer during the process when comparing the heights of the bars.

The nature of curricular materials could influence children’s graphicacy. If curricular materials merely require children to plug in information into a bar graph and answer questions that can be answered directly from the data, such as who is the tallest and by how much and what is the favourite fruits of the children in a hypothetical class and other similar factual and low-level interpretative questions, then graphicacy is rather limited. Pereira-Mendoza (1995) argued that although such “algorithmic graphing” is a necessary part of any child’s learning such tasks have serious limitations.

Pereira-Mendoza and his colleagues conducted two important studies. It is not sufficient that children be able to read the obvious information presented in a graph. Because graphs contain a wealth of information it is important that children are able to realise the full potential of the graph by interpreting and generalizing data from the graph (Kirk, Eggen, & Kauchak, 1980, cited in Curcio, 1987). It is important that readers of graphs are able to draw inferences and make predictions from the data captured in graphs (National Council of Teachers, 1980). Pereira-Mendoza and Mellor (1990) conducted a study with 121 grade four children and 127 grade 6 children in Canada. The purpose of their study was to find out children’s understanding of the information conveyed by bar graphs by examining the effects of various characteristics of graphical displays on these children’s ability to read, interpret and predict from such displays. These children took a paper and pencil test and a number of these children were selected for interview, the selection was made with the intent to provide as complete a spectrum of results as possible. The findings showed that these children had very few problems with the reading of the bar graphs with both groups of children achieving scores of 95% and above.

However these children found interpretation questions challenging with grade the four children achieving mean scores of 52% and grade six children with mean scores of 78%. Common errors in this category were computation errors, or reading or language errors, or scale errors.
These children’s performance with predict questions were even lower than for interpretation questions with the grade four children achieving a mean score of 16% and grade six children, 18%. These children’s ability to predict was affected by the arrangement of the data which was specifically included in the design of the instrument. There were three types of arrangements.

*Patterned data in order of magnitude*: When given graphs involving height, these children predicted, based on the pattern of the graph, that the height of a ten-year-old child was more than that of a nineteen-year-old child.

*Non-patterned data in order of magnitude and non-patterned data not in order of magnitude*: The need for a pattern was so strong that some of the children imposed a pattern or “any attempt to search for a pattern made no conceptual sense” (Pereira-Mendoza & Mellor, 1990, p. 155). During the interviews these children explained that the absence of a pattern prevented them from making appropriate predictions and a prediction was possible if there was a pattern in the data.

These children had difficulties answering questions when the information was not on the graph. For example, these children explained that they could not predict the allowance for 1990 since this was not presented on the graph although the allowances for the previous years were presented. The authors concluded that the findings suggest that these children seemed to think that a pattern must exist in a graph and the graph must be complete and hence were unable to predict beyond the given graphs.

*This Study*

Singapore spiral mathematics curriculum introduces graphical representations progressively, starting with the introduction of picture graphs in Primary 1, picture graphs with scales in Primary 2, bar graphs in Primary 3, line graphs in Primary 4, and pie charts in primary 6. This study builds upon work by Bell and Janvier (1981), Leinhardt, Zaslavsky, and Stein (1990), You (2009), and McMillen & McMillen (2010). Because the focus of the current curricular materials was on quantitative graphs, the purpose of this study was to explore how children engaged with the different data presented in qualitative graphs, something with which they were not familiar. Specifically if they were given
qualitative graphs what contexts would they provide to match these graphs. Although such open-ended tasks were non-existent in the primary curricular materials this does not preclude the possibility that these children may have internalised the fact that graphs are used to convey information. The stories could provide educators valuable information of how primary children perceive graphs. Did they perceive graphs as representing some form of statistical information, and have symbolic understanding of graphs or did they perceive graphs as visual representations of some physical phenomenon or a literal picture of some situations without symbolic understanding, misconceptions discussed by Leinhardt, Zaslavsky, and Stein (1990). The research question is

What do stories created by primary children tell of their interpretations of qualitative bar and line graphs?

Participants
The participants were a group of 25 high attaining Primary 5 boys from an intact class of an all-boys primary school in Singapore. They followed a common syllabus and they have been taught picture graphs, bar graphs and line graphs. These children were taught by the cooperating teacher who collected the data for this study which was part of her higher degree study but wishes to remain anonymous.

Instrument
The instrument comprised four graphs, two qualitative bar graphs, a vertical bar and a horizontal bar graph and one qualitative line graph and a quantitative symmetrical straight line graph (see Figure 1). Pie-charts were excluded from this study because its form and related concepts were introduced only in Primary 6.

Bar graphs represent a category or event. To assess whether children were affected by the orientation of the bars, two bar graphs, one vertical and one horizontal were included in the instrument. Would the stories focus on the visual features of the bars such as taller and shorter or would they use appropriate mathematical language such as ‘more than, ‘less than’ in their stories (McMillen and McMillen, 2010)?
Line graphs are more challenging than bar graphs (Padilla, Mckenzie & Shaw, Jr 1986; Wainer, 1980). Line graphs are effective for showing trends over time. In line graphs, points on a grid are used to represent continuous or uninterrupted data. When each axis is clearly labelled, the data shown can be interpreted properly. A wide variety of line graphs exist and are used, but two basic assumptions are inherent: (i) The data are continuous rather than discrete. (ii) Change is accurately represented with linear functions (that is, by lines) rather than some other curve. For example line graphs can be used to represent the length of a shadow of a flag pole as the sun moves across the sky. In the case of a broken line graph, one scale may not be numeric but is at least ordered, as in the case of the months of the year along the horizontal axis.

Two line graphs are included in this study. Line graph 3 shows five points joined by lines and these points fluctuate across the horizontal axis. Graph 3 may encourage children to construct stories that require point-to-point interpretation and perhaps local interval reading and possibly complete global reading.

Although Graphs 1, 2, and 3 are part of the primary mathematics curriculum Graph 4 is not. To provide some scaffolding, numerical values were provided for the vertical axis. The symmetrical structure of line Graph 4 suggests a steady increase over a certain period and a steady decline over a similar duration. Of the two graphs it is hypothesized that children in this study may find it more challenging to create stories to match line graph 4 because of the novelty of its symmetrical structure. What stories would these children create to match a line graph where there is a steady increase for a period of time followed by a steady decrease over a similar period? Would they create stories which require point-to-point interpretation or intermediate global reading of the graph which requires interval reading of the data and then complete global reading which requires interpretation of the graph as a whole?
Findings and Discussion

Table 1 provides a summary of the overall findings. Consistent with the literature (Padilla, McKenzie & Shaw, 1986; Wainer, 1980), the children were most successful with the two bar graphs. All but one of these children were able to create stories to match the two bar graphs with about half providing appropriate labels for the horizontal and vertical axes. The mathematics used was accurate and the context and nature of the questions were very similar to those in the curricular materials. There was only one error in the matching of bar graphs. Instead of line Graph 3 this child used the horizontal bar graph to tell his story.

Similar to findings by Padilla, McKenzie & Shaw (1986) and Wainer (1980) these children found constructing stories for line graphs more challenging than bar graphs with Graph 4 proving to be the most difficult. This was not unexpected for the reasons presented in the instrument section. It is difficult to think of stories where there is a constant increase in vertical value.
corresponding to a constant increase in time represented by the horizontal axis followed by a similar decrease over a similar period of time.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>Vertical Bar Graph (Graph 1)</th>
<th>Horizontal Bar Graph (Graph 2)</th>
<th>Line graph with fluctuating Gradient (Graph 3)</th>
<th>Symmetrical Inverted V Line Graph (Graph 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct Match</td>
<td>25</td>
<td>24</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>Correct Mathematics</td>
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<td>22</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>No mathematics</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Content of graphic stories similar to those found in local curricular materials</td>
<td>22</td>
<td>19</td>
<td>13</td>
<td>1</td>
</tr>
</tbody>
</table>

Most of the correct stories for the two bar graphs and line Graph 3 were similar to those found in local instructional materials. The example in Figure 2 is a case in point. When the stories were correct, the mathematics used was correct and the relationships between the numbers correctly expressed through the use of relative terms such as ‘less than’, ‘more than’, ‘fewer than’, not unlike the findings reported in the literature (McMillen & McMillen, 2010). None focused on the visual features of the bars such as ‘taller than’ or ‘shorter than’.

The examples in Figure 3 provide further examples of these children’s work. Because it is difficult to read the reproduction of the scanned images of these children’s handwriting their graphs are presented to the left and the corresponding word processed form of the created stories are presented, word for word, to the right. To give the reader a sense of children’s written work, scanned images of the Shot Putt and Sweet stories are provided in the Appendix.
Figure 2. Stories created bore a strong resemblance to instructional materials presented to the right of the figure, including the problem solving focus of the primary mathematics syllabus (Collars, Koay, Lee, & Tan, 2009).

The examples in Figure 3 suggest that these children were meticulous in their work and each story created used mathematics precisely and accurately and the mathematics content was represented accurately with the two axes of the bar graphs labelled accurately to reflect the content and the numbers used in the stories. The Shot Putt story showed that this child had good command of the mathematics. Not only was relational comparisons “more than” used to compare the groups, this child also incorporated the use of fractions, the number of people who threw 8m was one-half of the number of people who 6m. The amendments made to the numbers in the vertical axis of the vertical bar graph showed that he checked the numbers and when the difference between numbers was not two, he adjusted and he ensured that these numbers matched the context. Also the distances thrown were meaningful as these were humanly possible. He concluded the story with a question. There is very clear evidence that graph reading is for solving problems as most stories ended with a question that could be answered by referring to the graphs. The Cookies, Books and Sweets stories are further examples of these children proficiency with the intended Primary 5 mathematics syllabus.
One day there was a shot putt competition, 54 people competed. After the competition there were 4 groups 5m, 6m, 7m & 8m. There were 18 people who threw the ball 6m, the people who threw 8m was $\frac{3}{2}$ of the number of people who threw 6m. The number of people 6m was 3 more than the number of people who three 7m. How many people threw 5m?

Once upon a time there was an earthquake. People were surviving on cookies. What is the total of Alibaba and Mr Fish’s cookies. Rapunzel was walking then she tripped over a rock and lost 5 cookies but super man stole them. Now Rapuzel has ________ cookies and super man has ________ cookies.

Four children share some books. Alan has 30 books less than Betty but has 15 books more than Denny. Betty has 45 books more than Denny. Carl has 15 books more than Alan. Denny has 45 books. How many books does Alan have?

Sandy had 8 sweets which was $\frac{2}{3}$ the total number of sweets Celia had. Celia had 3 less sweets than Andy and 3 more than Bob. The total number sweets Sandy, Candy, Andy, Celia and Bob had was 55 sweets. How many sweets did Candy have?

Figure 3. These examples show that the mathematics coupled with the labels used to convey the story were precise and accurate.
Line graphs: Although the literature reported that children found line graphs difficult to interpret (Padilla, McKenzie & Shaw, 1986; Wainer, 1980), the current study showed that about half of these Primary 5 boys constructed credible stories to match line Graph 3 with increasing slope. About half of the stories included graphs with the axes labelled clearly. Again the stories to line Graph 3 bore strong resemblance to the instructional materials. The Water story and the Electricity story are evidence of the influence of National Education in the mathematics instructional materials. However the Beyblades story shows the influence of Manga stories on some of these boys. The stories demonstrate how these children worked with point-to-point interpretation of the points in Graph 3. The context directs the reader to the relationship between each value on the horizontal axis, and its corresponding value on the vertical axis. That the independent variable happens to be time is unsurprising as this reflects children’s experiences with the instructional materials.

Every day Singaporeans use water. John recorded the amount of water used in 5 days. On day 1, we used 40000, on day 2 we used 80000, on day 3 we used 100000. On day 4 70000 was used and on day 5 110000 was used.

If the Electric Bills for Monday is $400, and the Electric Bills for Tuesday is $800, what is the electric bills for Friday.

In May, the most candy floss was sold. In January, most people were on holiday so there was little business. In March most people were back so there was quite a lot of business.
In the month of February, Elijah had 40 of his beyblades sliced by long sword mantis. In March, he loss twice as much beyblade to long sword mantis than it February. He lost a hundred in April. Long sword Mantis trashed 30 more of Elijah’s beyblades than in February. He lost another 110 in June.

One day, there were 40 people in the town of Goomba. Over the next 2 years, the number of people doubled. Then as the country advanced, the population grew by 20, leaving the city with \(2 \frac{1}{2}\) times the number of people at first. Sadly, \(\frac{3}{10}\) of the people were killed in a riot. Luckily, as the country recovered, the population shot up to 110 people.

Some may argue that these children’s tendency to imitate the content of the curricular materials reflects the narrowness of the pedagogy. However a “teacher who wishes to develop his students’ ability to do problems must instil some interest for problems into their minds and give them plenty of opportunity for imitation and practice” Polya (1957/1973, p. 5). Hence the similarity of the created stories to the curricular materials need not be seen as a negative outcome of the teaching and learning process. That these children’s tendency to create stories which bears a striking resemblance to the curricular materials suggests that they could remember the content taught a year ago. What is needed now is for teachers to provide more challenging instructional materials which would stretch these children who were already proficient with the basic materials. Challenging materials could include questions which require children to discuss the characteristics of the data set as a whole and what global information about the population can be discerned from this data set (Van de Walle, Karp & Bay-Williams, 2014) and deciding what reasonable conclusions can be drawn from the data (Rosenfeld, 2013).

However these children found it difficult to construct stories for Graph 4. Only 7 children used the correct mathematics to create stories to match the
inverted V-graph line graph. The two examples of correct stories for Graph 4 were selected to show two possible interpretations of the graph. The stories were accepted as ‘correct’ because the mathematics used was correct but each child’s story suggests a different interpretation of the shape of the graph. The Justin Bieber story suggests the child looked at the graph as a whole or globally and then he created a story to match the structure of the graph. He had an interval interpretation of each point on the graph as he explained for the steady increase in number and for the positive slope of the line by stating that “every 1 hour 20000 more people went to the concert. To explain for the negative gradient, he explained that “for every hour more 20000 people left the area”. He had a pointwise reading too as he marked at each relevant point the different times the concert goers arrived and left the venue. It could be that because he had a good command of proportional reasoning he was able to construct a story to match the global picture presented by Graph 4 of increasing slope and the decreasing slope, although the concept of slope is taught only in secondary school mathematics. Only this child was able demonstrate this level of knowledge of proportional reasoning. Furthermore he was a meticulous child as he used a straightedge to construct each point of intersection.

Initial reading of the Burger story suggests that this child may have a pointwise interpretation of the graph and not a more global interval interpretation of the graph. But by studying the points used to illustrate the story suggests otherwise. The child may have seen the symmetrical structure of the line graph but may not have the mathematics language to explain the symmetrical structure of the graph as elegantly expressed in the Bieber story. This child drew the reader’s attention to the number of burgers sold at each point but on the graph he drew in equal intervals by distributing the four days equally over the horizontal axis. The choice of 4 days was also very strategic as this allowed for the first two days to account for the positive gradient and the next two days for the negative gradient. The same goes with the beyblade story. The incomplete sentence “He produced the same number of beyblades on day 6, same number of beyblades produced on day 7 and day 3” on day suggests that this child saw the symmetrical nature of line Graph 4. It is very important to read the narrative together with the structural presentation of the graph. Otherwise one comes away with a limited understanding of a child’s interpretation of graphs.
From 7 – 8 pm 20000 people had gathered to see the Justin Bieber Concert in California. The relationship was for every 1 hour, 20000 more people went to the concert. At the end of the concert which was 11.45 pm. Also for every hour more 20 000 left the area. At what time were there no more people in that area?

There are 4 days. The amount of burgers that are sold in the world. On day 1 40 000 burgers were sold. On day 2, 100 000 burgers were sold. On day 3, 40 000 burgers were sold. On the last day, all burgers were out of stock and 0 burgers were sold.

On the first day it open, Elijah produced 20000 beyblades. He produced two times the number of beyblades on the 2nd day. Elijah produced 60000 beys on day 3 20000 more on day 4 and a hundred thousand on day 5. He produced the same number of beyblades on day 6, same number of beyblades produced on day 7 and day 3 on day 10 he closed down.

Figure 5. Local point-to-point interpretation could also reflect a more global interval reading of the graph.

Graphs with no labels but the mathematics was correct: About half of the stories created matched the graphs but the horizontal or vertical axis or both the axes were not labelled. Examples of such cases are found in Figure 6. Initial analysis of such written work suggests that the children did not know how to label the axis or that they not place importance on the labelling of the axes. However deeper analysis of such works suggests otherwise. The mathematical relationships using the expressions of ‘more than’, and ‘less than’ were accurate and likewise where fractions were used to expression specific relationships between objects or events. Children’s created stories suggest that they knew what they were doing and they were posing a challenge to the reader of their stories. It was left to the reader to decipher the accuracy of the story and whether the graph was a good representation of the story. It was the responsibility of the reader to decide whether the story was correct,
and whether the mathematics was accurate. If the reader could do the sums, then the reader should have no problem identifying which bar is which. If the reader so wish, the reader could label the axis and also the numerical values. The creator of such stories could be said to have a good grasp of mathematics and were using them appropriately.

Sean has twice as much money as Joseph. Kurt has $\frac{2}{3}$ as much money as Sean. Jared has $15 \ more \ than \ Kurt. The total amount of money they have is $270.$

8 children in a classroom have 40 cents for pocket money, 5 children have 50 cents, 17 children have 60 cents, 14 children have 70 cents and 11 children have 80 cents.

Singapore has 55 tanks. Malaysia has 15 more tanks than Singapore. England has 85 tank. Vietnam has 25 tanks. Laos has 15 tanks less than Vietnam.

*Figure 6.* Which bar is which? It is the responsibility of the reader to read the story, make sense of the mathematics in the story and decide which bar matches the objects in the story.

**Line Graphs as Iconic Interpretation of a Physical Phenomenon:** Line graphs proved to be most challenging to many children who struggled to create stories to match the two line graphs. Those who did not create acceptable stores saw the two line graphs as iconic interpretations (Bell & Janvier, 1981; Leinhardt, Zaslavsky, and Stein, 1990) of some physical phenomenon or a literal picture of some situations without symbolic understanding. Because of the global interpretation of a trend, no mathematical content was provided to explain the
meaning of the graphs. No mathematics was used to support the Card Game story and no pointwise interpretation was provided for each of the 5 points. Instead Graph 3 was used to describe the success or otherwise of Tom progress in a game. The Cool Joe story used the inverted V graph to represent a mountain and the story describes Mr Cool Joe’s journey up and down that mountain. The Hikers story is mathematically correct but again the inverted V is an iconic representation of a mountain. The story provides a point to point description of the Hikers journey up and down the mountain but there was no attempt to explain for the unit rate of increase and unit rate of decrease up and down the mountain respectively.

In a card game, Tom was winning at first. Then when his opponent made a good move. He went to the losing streak. He used his best card and won the game.

Mr Cool Joe wanted to get to the other side of the mountain. He slowly walked up the mountain. Finally, after climbing the entire mountain, he started to travel down. Eventually he reached the other side and left the mountain.

On the first day, a group of hikers hiked up 20 000 ft. On the second day, they climbed up another 20 000 ft. On the third day, they climbed up to 60 000 ft. On the fourth day they climbed up 20 000 ft. On fifth day, they reached the peak at 95 000 ft. On the sixth day, they descended 15 000 ft. The next day, they descended 20 000 ft. On the eight day they descended 20 000 ft. On the next two days they descended 40 000 ft and reached ground level.

*Figure 7.* Line Graphs as Iconic Interpretation of a Physical Phenomenon

*Errors due to incomplete learning:* Globally Graph 4 was used to describe how hypothetical populations grew at a constant rate, reflected by line with
the positive gradient and how the same populations were destroyed at the same constant rate. The Tsunami story (see Figure 8) showed how the population grew by an exponential function of $10^n$ and reached its peak at 111111 and how this entire population was systematically destroyed by a tsunami at the same rate the population grew. He could not work with the values presented on the vertical axis and wrote over the original values. Why he chose to replace the given values with (1, 11, 111, … 111111) is a mystery as interview was not used to gather further information from the children. But this error could be due to incomplete learning as primary mathematics syllabus did not discuss exponential growth of population. The Rabbit example also used the disaster of the tsunami to explain for the complete destruction of a population. These stories showed how children were affected by events on the ground as this data set was collected at the time of the Fukushima earthquake and tsunami disaster. Themes of stories for Graph 4 were largely centred on growth followed by destruction, profit and loss, good health followed by ill health and death. But these stories should be read with a huge pinch of salt as they required the reader to suspend their sense of reality as the same amount of time was necessary for say a business to grow and then to suffer loss and become bankrupt.

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{figure8.png}
\caption{The Tsunami story and the Rabbit story show errors that could be due to incomplete learning.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{figure8.png}
\caption{From zero, the population grew till around 11111 then a tsunami came and destroyed everything. No one survived.}
\end{figure}

Wrong graphs were used to support the stories (see Figure 9). Instead of bar graphs, these children used line Graph 3 to represent the Eiffel Tower and the Soldiers story. But such errors are the exception and not the norm for line
Graph 3. These children may not be aware that it is appropriate to use bar graphs for categorical data, where there is no obvious connection between the categories and line graphs for bivariate, interval data. There could be different reasons for such errors. Bar graphs are taught at Primary 3 and line graphs in Primary 4. When the concept of line graphs was introduced and developed, it could be that there was no discussion for the need of another form of data representation. What were the limitations of the bar graph representation and how would line graphs improve data representation? After the unit on line graphs perhaps children were not asked to compare data sets and which representation would they use to represent different data sets. Overall stories involving war and destruction were also a favourite theme among the stories created.

The Eiffel tower is 200m taller than the Pyramids and the Singapore flyer is 400m shorter than the pyramids. The London eye is 300m shorter than the Eiffel tower and the Empire state building is 100m taller than the Eiffel tower.

At a camp, there are some soldiers. 10 of them have guns. There are 10 more soldiers who drive tanks. 30 soldiers drive submarines. The number of soldiers with laser guns is 30 more than the soldiers with grenade launchers. There are 15 soldiers with grenade launchers.

Figure 9. These examples show the wrong use of line graphs.

Conclusions

This seminal study demonstrates that high performance Primary 5 boys used graphs to display information and they have good command of the mathematics they were taught. The minority who used the wrong graph to represent their constructed story nevertheless used mathematics correctly. Although there was no requirement to use fractions in their stories those who did applied the concepts of fractions accurately by specifying clearly what was the whole. Majority of the stories constructed for the first three graphs resembled those provided in the instructional materials. This should not be
taken as a negative outcome of the teaching of statistical graphs. Rather these children’s stories suggest they remembered what they were taught. However these children are ready for more challenging work with graphs. The Primary Mathematics curriculum recommends that children not only learn to construct, read and interpret the various graphical representations but that children should be able to read and interpret data from tables/line graphs (MOE, 2012). The stories to the first three graphs are evidence that these children were very able to make point-to-point reading of the graph but they seemed to lack experiences making more global interval reading and they do not use graphs to predict possible outcomes, i.e. they have yet to interrogate the graphs to discuss the properties of the underlying situation.

The stories constructed for Graph 4 was insightful as one-third of the responses show that these children were able to perform point-to-point interpretation of a graph which they have not seen. Also they also show flashes interval reading of data. Their examples showed the beginning of interval reading of the global data but these have yet to take root in the children scheme of graph reading. The stories created for Graph 4 suggest that these children did make attempts to consider the properties of the underlying situation, the rise of the graph representing an increase and that part of the graph with negative gradient with a decrease of the variable represented by vertical axis. Such stories suggest that these children saw how the points were clustered to show uniform increase followed by a uniform decrease and the distribution of the points holistically showed an increase followed by a decrease over time (Van de Walle, Karp & Bay-Williams, 2014).

One limitation of this study is that the children were not interviewed to gain deeper insights how they interpret line Graph 4. Future studies could engage children with graphs with which they are not familiar and to discuss their interpretations of possible information distilled in such graphs.

**Implications for Teaching**

This study suggests that these high attaining children, who are well conversant with things graphical, are ready for more challenging mathematics tasks. Therefore it is important for policy makers, textbook writers and school teachers to introduce tasks which require children to interpret information
captured in graph and to stretch children’s knowledge of graphs and to improve their graphicacy. A good reading/interpretation activity is to give children a line graph with a title and the nature of the units for each axis and ask them to describe orally or in writing what is happening. A teaching experiment using “graph language” approach (Bell & Janvier, 1981, p. 41) where children used mathematical language to discuss graphs suggest using analysing complex graphs in graphical terms without reference to situations. This may help children overcome issues with situational distractors (Bell & Janvier, 1981).

References

Graphicacy of High Attaining Primary 5 Boys


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**Appendix**

Scanned images of two children’s written work, Shot Putt Story to the left and the Sweets story, to the right.