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Participants' Perceptions and Experiences with Worked Examples in Calculus

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Abstract: This study investigates students' perceptions and experiences in a calculus class with worked examples. The theory of worked examples is based on Sweller's cognitive load theory. In this study, participants attended voluntary discussion sessions that transitioned from showing worked examples to the participants to the participants working problems themselves. In particular, the experiences and perceptions of two weaker students and another average student are presented to give a picture of how the worked examples helped students in the class. These results are similar to what another study found when describing how the worked examples helped "good" and "poor" mechanics students change their attitude towards mathematics and self-efficacy in calculus, along with how it affected their learning in the course.

Keywords: calculus, worked examples, perceptions, experiences, qualitative research

Introduction

Calculus is one of the most challenging courses in college. One of the difficulties stems from the fact that during class students work diligently to copy all the notes, but have little time to digest the material. Also many times students' knowledge of algebra and trigonometry is inadequate. When the instructor is lecturing in class, many times the material seems understandable and easy. However, when students attempt the assigned work on their own, they find that the problems are more difficult and confusing than anticipated. Gunawardena states that "students who enter college are often under prepared and lack the background and motivation to succeed in college-level mathematics" (2002, p.108). Ainsworth et al. (1994) argues that students who come to college without an adequate background in math will likely withdraw from the course or quit

performing when a math class becomes difficult. Students who are under prepared --- and even some students who are adequately prepared -- fail to be successful because the class becomes difficult and they do not believe that they can succeed (Bandura, 1977; Wolters & Rosenthal, 2000). Half of the battle of helping students become successful in a course is to get them to believe that they can succeed and that they have the ability to learn and to do mathematics. This study discusses students' perceptions and experience with using worked examples to help them learn the course material in calculus and increase their confidence in their math ability.

Memory and Worked Examples Theory

There are three types of memory: sensory, long-term, and working (short-term). Our senses, sight, sound, smell, taste, and touch, serve as stimuli for our sensory memory. Long-term memory is where a person's immense body of knowledge and skills is located and working memory is where we think, solve problems, and are expressive. In general, everything that we "know" is stored in long-term memory and, through a query of working memory, is activated when needed. Miller (1956) says that working memory has a limited capacity, which can deal with no more than seven chunks of information simultaneously. Combining the senses to present information helps to expand the capacity of working memory. Some or all of the information will be lost during processing if the working memory's capacity is exceeded, unless information is recorded in a permanent form as it is being processed.

Generally, mathematics classes, as well as other Science, Technology, and Engineering, courses, are taught by lecturing on the new topic, presenting or demonstrating the concepts through a few examples, and assigning homework practice problems so students will learn the material that has just been discussed. Students are expected to practice the assigned problems shortly after the lecture. When students procrastinate or simply cannot focus on the covered material until a later time, they have more difficulty remembering what was said during lecture and/or details of the instructor's examples. Most, if not all, instructors use examples in class to illustrate the content's key principles to their students. However, students have little or no

time to absorb the examples when taking notes in class before another example or more theory is covered. Sweller and Owen (1989) state that "some views of mathematics and the way it should be taught owe more to tradition than to our current knowledge of cognitive processes" (pg. 322). The worked example theory would place emphasis on worked examples by coupling problems solved in class with active student participation by having students work similar problems. In fact, in several research studies (Sweller & Cooper, 1985; Ward & Sweller, 1990; Zhu & Simon, 1987; Carroll, 1994, Tarmizi & Sweller, 1988), instructors presented students with a worked example on paper and told them to study the example. Once the students were done studying the worked example, the instructor asked the student to solve a similar problem without any help from the worked example. It has been suggested that worked examples reduce the cognitive load on a student and might optimize schema acquisition (Sweller & Owen, 1989; Sweller & Cooper, 1985).

Worked examples are focused on skill acquisition in a subject. Trafton and Reiser found that "the most efficient way to present material to acquire a skill is to present an example, then a similar problem to solve immediately following" (1993, p. 1022). Worked examples have been used from middle school to college and in different areas (see next section for examples). The questions guiding this study were:

1. What are student's perceptions and experience with worked examples in Calculus?
2. What are the student's perceptions on how the worked examples contributed to their learning?
3. In what ways, if any, do worked-examples build self-efficacy in student's ability to learn material and instill confidence that they will be successful in Calculus?

The author used worked examples in voluntary discussion sessions for technical calculus students (see three step method in the methodology section) because it lends easy to a structured learning environment similar to other out of class help sessions (supplemental instruction, peer-led learning, learning assistant model, emerging scholars program, etc.). Through a pilot study, the author learned that worked examples helped students at all levels, especially students that struggled.

Brief Literature Review

Sweller and Cooper (1985) conducted one of the first studies on worked examples. Through five experiments they examined the use of worked examples as a substitute for problem solving. Zhu and Simon (1987) demonstrated the feasibility and effectiveness of teaching mathematical skills through chosen sequence of worked examples and problems in a Chinese-middle school's algebra and geometry curriculum – and without lectures or other direct instruction. In engineering, Chi et. al. (1989) showed that while students studied worked examples, "good" students generally monitored their own understanding and misunderstanding through self-explanations. Compare this to "poor" students who did not generate sufficient self-explanations or monitor their learning inaccurately. They found "poor" students relied heavily on examples. Ward and Sweller (1990) established that students who used worked examples (in physics) formatted to reduce the need for students to mentally integrate multiple sources of information achieved test performances superior to either those exposed to conventional problems or to those shown worked examples that required students to split their attention. Catrambone and Yuasa (2006) found that action elaborations in the Structured Query Language for databases improved procedural performance the most, in both the active and passive conditions, compared to both active and passive conditions with instructional elaborations. Finally, Crippen and Boyd (2007) found that the combination of worked examples with a self-explanation prompt in a chemistry web-based learning tool produced improvement in students' course performance, problem solving skills, and self-efficacy compared to students who were only provided with worked examples.

Perception is the way a person thinks about or understands something. Numerous studies have shown students' perceptions of their learning situation are important (Jackson & Prosser, 1989; Crawford et al., 1994; Lizzio, Wilson, & Simons, 2002; Trigwell, Hazel, & Prosser, 1996; Struyven, Dochy, & Janssen, 2003). The study conducted by the Mid-continent Research for Education and Learning (Marzona & Pickering, 1997) state that students' learning is affected by their perceptions of what they are learning. Centra and Gaubatz (2000) state that indicators of student learning "might include student perceptions of their increase in interest in

the subject, critical thinking skills, interpersonal outcomes (e.g. cooperative abilities), intrapersonal outcomes (e.g. self- understanding) and other broad course outcomes (Koon & Murray, 1995)" (p. 2). In addition, Campbell and Mislevy (2012) states that students' perceptions do matter and has an affect on institutions retention and attrition.

The studies on worked examples concentrated quantitatively on how worked examples helped students in science, technology, engineering, and mathematics courses, but did not examine students' perceptions and experiences with worked examples. It is true that Chi et. al. (1989) studied students' self-regulation of their solutions but they did not look at students' perceptions and experiences with worked examples.

Worked examples are important because it helps: 1) reduce the cognitive load during learning process, 2) assist in the transfer of information from working (short-term) memory to long-term memory, and 3) build schema. This study takes a look at students' perception and experience with worked examples in a technical calculus course to add to the literature on worked examples. The study will also look at the affect that the discussion sessions and three-step method (scaffold use of worked examples) had on student performance.

Methodology

Participants and setting

The participants were 20 students (out of 87 registered for the course) enrolled in a technical calculus class at a research university in the southwest. Students majoring in fire protection safety, mechanical and electrical engineering technology, and construction management technology comprise the majority of the students enrolled in the course. Students in both sections of technical calculus were given the opportunity at the beginning of the semester to attend the discussion and students voluntarily participated by signing an individual consent form. The demographics of the participants and the whole class are shown in tables 1 through 4.

Table 1
Demographics - Sex

	Male	Female
Participants	90%	10%
Non-Participants	88%	12%

Table 2
Demographics - Class

	Freshman	Sophomore	Junior	Senior
Participants	20%	40%	25%	15%
Non-Participants	22%	46%	23%	9%

Table 3
Demographics - Race

	Caucasian	Hispanic	Native American	African American	Asian	Other
Participants	90%	5%	5%	0%	0%	0%
Non-Participants	78%	4%	12%	4%	1%	1%

Table 4
Demographics - Major

	Fire Protection and Safety	Mechanical Engineering Technology	Electrical Engineering Technology	Construction Management Technology	Other
Participants	35%	20%	15%	25%	5%
Non-Participants	31%	19%	8%	25%	17%

The average age of the participants was 22.75 compared to 21.83 for the entire class. All of the participants and 89.9% of the entire class met the prerequisites of college algebra and trigonometry, and 10% of the participants (19.1% of the entire class) had taken technical calculus at least one time prior to the spring 2005 semester. Participants entered the course with pre-algebra assessment scores, on a basic algebra exam, that were not significantly (at the 0.05 level) different from the non-participants. In addition, to having similar demographics, participants and non-participants

did not significantly differ in past grade point average, but participants on average did attend class more than non-participants. Participation in this study was voluntary and participation was not factored into student's final course grade. The instructor of the course was a seasoned full professor (not the author) and the author lead the supplementary worked example sessions. In addition, the interviews were conducted by the author near the end of the semester of the course.

Three Step Method and Connection to the Worked Example

To assist students in learning the first semester of technical calculus, the author introduced and used a method called the "three-step method" in voluntary out-of-class discussion sessions to support student learning. The three-step method begins by looking at a sample problem (worked out example) on the Technical Calculus Learning Supplement. The learning supplement includes the following components: worked out examples, algebra review, TI calculators, applications of calculus, and a review of trigonometry to help prepare students for the subsequent course (Miller, 2010). During a session, the researcher leads the group in reviewing a worked example, and interacts with the students to make sure that they understand the steps used to solve the problem. Next, a similar problem is presented on the blackboard. In this stage, the researcher asks the students to assist in solving the problem by telling him/her how to work through the steps of the given problem as a group. The amount of interaction depends on how much understanding they have of the topic at this stage and on the difficulty of the topic. The final stage of the three-step method is to give the students another problem or problems and have the students work alone or in small groups. More independence comes during this stage, since students are solving a problem without the instructor's lead. Furthermore, during this stage, assistance is given to any student who is having difficulty with solving a given problem or having difficulty understanding the topic. This is where students will reinforce the skill acquisition being stored from short term to long-term memory. After adequate time has passed for most students to solve the problem, the final answer was given. A complete solution is given only if requested by a student. Through this type of collaboration, students will build confidence so that they can successfully work homework problems and the method will further help with skill and concept

acquisition. The three-step method is very different than what students experience in the traditional lecture course where the instructor presents the theory and gives examples. Finally the worked examples help students build understanding of calculus. For this study, we define *understanding* as procedural understanding (Rittle-Johnson & Alibali, 1999), action sequences for solving problems, and not so much conceptual understanding, explicit or implicit understanding of the principles that govern a domain and of the interrelations between pieces of knowledge in a domain.

Research Instruments and Data Analysis

The data came from structured, audio taped interviews (Patton, 2002) with each of the 20 participants who voluntarily attended the discussion sessions and used the worked examples. The following were the interview protocol questions on worked examples and the discussion sessions (where worked examples were used in the three-step method):

1) Describe to me what you perceive the purpose of the worked examples?
2) Tell me how you used the worked examples when working on technical calculus?
3) Tell me how the worked examples were helpful to you this semester?
4) What aspects of the discussion sessions are the most helpful to you?
5) What aspects of the discussion sessions are not helpful to you?
6) Tell me how the discussion sessions helped you in the course?
7) Tell me about some of your reasons that you attended the discussion sessions? and
8) In what ways, if any, have you experiences changed in this class versus a previous math class that you have had?

The interviews were transcribed for each participant and line numbers were added to each interview, along with a number for each student starting with 1 (i.e. first student interviewed was student 1, second interviewed was student 2, etc.…). In order to best capture and summarize the students' perceptions and experiences of worked examples, we employed a pattern and emerging themes analysis of the transcribed interviews (Patton, 2002). After the interviews were transcribed verbatim, the author asked each interviewee to check the accuracy of their interview. Any discrepancies

were corrected. This was followed by the author highlighting important phrases from each interview and documented those important phrases by labeling them with the student number and line numbers from the corresponding interview. The author identified an important phrase for this study as a statement, associated with the worked examples, discussion session, which stood out to the author when reading the transcription. For example, student 1, on lines 281-282, stated "Overall it (worked examples) has just made me more confident with how I step through problems. Like I say it has been kind of like my safety blanket throughout this whole semester", which the author considered an important phrase. Important phrases could be as small as several words to as large as a whole paragraph (see case study on Alex for a whole paragraph). At this point the researcher categorized all the important phrases into groups with a similar theme. The groups were checked to make sure all phrases were grouped correctly and themes emerged from the analysis. To ensure validity, experts, three full professors, all with over thirty years of experience in mathematics and mathematics education, reviewed the research and suggested improvements.

Data

Students' perspectives on worked examples

The stories of a few of the participants provide a snapshot into students' perceptions and experiences with worked examples and reveals some more details on their mathematical background. The following three students, Alex, Rachel, and Henry, are three of the twenty students who participated in the voluntary discussion sessions and used the worked example method. These three stories form the following viewpoints: (1) how two under prepared algebra students used the worked example method to change from a state where they were not confident in mathematics and thought that they would not be successful in the course to a state where they were very confident with calculus and were successful in the course, and (2) how a more prepared algebra student who had little interest in mathematics due to past experiences and low confidence in his ability to earn a grade above a C became very confident in his ability, loved mathematics again, and was successful in the course. These viewpoints are examples of how the worked example method helped three students. Although it is not implied that these results could be generalized to other students with similar beginning states,

these three examples were chosen to show the reader how worked examples helped students who had a variety of difficulties in past mathematics courses or who had experiences in past mathematics courses that created negative attitudes about mathematics. All students in this study stated that worked examples helped them in some shape or fashion and a majority of these students struggled with mathematics in the past. Therefore the author chose these three stories to convey students' perception of worked example and how they played a role in students learning calculus. The author could have easily told the stories of three other students and came to very similar results. The students in these three stories varied in major, age, pre-assessment on a basic algebra test, math background, and previous college G.P.A. Pseudonyms are used instead of the participants' real names to ensure confidentiality.

The case of Alex

Alex was a junior majoring in construction management technology. It had been five or six years since Alex graduated from high school and so he did not take the ACT to gain admittance to the university. He became frustrated with his previous college math courses including his many troubles with college algebra and trigonometry. As a result, he was quick to quit trying when he did not understand the material. Alex stated it this way, "there was no help (in college algebra) that I could get from anyone really. It was just all me and if I could not understand anything then I just could not understand it and I would throw my hands up."

In essence, Alex had the mindset that if he did not understand the mathematics right away, he would not be able to understand it. He did not understand that struggling to learn a concept is a very valuable process and this is where learning can occur. Alex's method of throwing up his hands when he did not comprehend the material was one of the reasons he struggled so much with college algebra and trigonometry. Alex failed at his first attempt at college algebra and decided to enroll into a general mathematics course. Although he successfully completed the general math course, Alex again withdrew the next time he took college algebra. This did not stop Alex from persevering: he enrolled in college algebra a third time only to result in additional failure. Alex's determination drove him to enroll for the forth time the following summer. This time, Alex passed with an A. The next semester he enrolled in trigonometry and completed the course

with an F. During the semester of this study, Alex was enrolled in both trigonometry and technical calculus. His cumulative G.P.A. was 2.143 and he scored an eight out of twenty-five on the pre-algebra assessment and did not take the post-algebra assessment.

Before enrolling in technical calculus, Alex knew that calculus was one of the harder courses on campus. Alex enrolled in the course knowing that he was going to struggle and that he may not be successful. The first day that Alex showed up to the voluntary sessions, he did not know "which way to look or go about fitting into this … But once you explained overall what we were doing as far as looking at an [worked] example, working through a [worked] example with you, and then working on our own, it became more and more easy to fit in and feel comfortable." Alex attended most of the voluntary sessions and would work to understand the material of the previously covered lecture material by working many problems through the worked example method. It took quite a bit of practice for Alex to retain things into long-term memory. He would use the confidence that he gained by solving problems and work on more problems in his individual study time. He stated that he immediately started seeing an impact on his performance on his homework by getting scores of 9's and 10's out of 10. This had an impact on his exam scores and Alex became very confident with his mathematics ability. Alex stated that without the worked examples, "I am not sure if I would even pass. I would either fail it or get a D. I would be really low." From Alex's past performances with college algebra and trigonometry and the heavy emphasis of these courses in calculus, Alex would have had a high probability of being unsuccessful in the course and continuing his past mathematical failures. The worked examples not only helped him understand and to work calculus problems, but through using the worked example method, Alex stated the worked examples made him more confident with "overall it (worked examples) has just made me more confident with how I step through problems."

Alex became so confident that he became cocky with other classmates before exams. He stated that his classmates would be jealous when they found out he did not need a formula sheet like they did and they would ask "I guess you know how to work them out, don't you?" Alex would look them in the eye and say, "I sure do. I know how to work every single problem and that feels good."

Alex had a very different disposition about mathematics at the end of the course than in previous mathematics classes. First, Alex changed his study habits and his mentality about the homework. This is revealed with the statement,

> *"The mentality that I think a lot of students, and myself… I catch myself doing it, you will see something in class, like an example, and they think that it is so easy… no problem I really don't have to study much. Do my homework, bam boom, it won't take long. But I have found that you can't really start your homework too early, there is no … you can start it too late but you can't start it too early … and you will find that you think one way … you will perceive it one way … you will think another way when you start your homework. Just because you get your homework done does not mean that you understand. It means your homework is done. In order to understand something you need to go back two or three times and do the problems again."*

When studying, Alex became very conscientious about making sure that he worked the problems completely correct. This is articulated with the statement that he thought "if there was one mistake with it (a written solution of a problem), I would erase it until I am completely happy." His change in mentality about mathematics was also revealed very plainly when comparing his experience in college algebra to calculus. He stated

> *"I think what has made the difference between difficulty in college algebra and not as much difficulty in calculus, is just me sitting down and not thinking okay I have to do this homework as fast as I can, but me sitting down and saying that I have to do this right. I just kind of opened my mind up recently and I am not fighting it."*

Through this experience with worked examples, Alex's mentality about mathematics turned completely around and was very successful in the course (making a B).

The case of Rachel

Rachel is a junior majoring in biomedical science. Rachel was an adult student (around 26 years old) who came back to school and who did not take the ACT exam. Her cumulative G.P.A. was 1.742 and she scored a nine out of twenty-five on the pre-algebra assessment and a thirteen out of twenty-five, on the post-algebra assessment. She stated that she "was not a very dedicated high school student. I never did any homework, ever. I would just go in and take my tests. I would average C's and B's, but did not retain it." After being out of school for years, Rachel "was actually really worried about it (the course)." She knew that she would struggle with the course because of her weak algebra skills and the course's difficulty level. During lecture she "pretty much understood what he (professor) was talking about, but would struggle when I would go and do it on my own, I would struggle." Rachel entered the course with a deflated attitude towards mathematics and a lot of doubt about her probability of succeeding. She thought immediately when she "heard about this discussion group (worked example sessions) and thought well this is going to save me." With this help she believed that she might be successful. The worked example method helped her to understand the material and gradually she gained confidence. At one point during the semester, after being successful on an examination, she called her dad and said, "Wow, I can do this, calculus, so when I can do one after you said do one on your own and I got it right, I was like, 'Wow'. It was a good feeling. I did not think I could even pass this class." In her mind the worked example method was so helpful because "I like to see one and well, I kind of got it, and then when we talk through it (another example), you hear it from other students, and for some reason it clicks in your mind. And then doing it on your own … I can do this."

She went on to express how "in high school I would always cry doing algebra or geometry, I couldn't stand it." Rachel said the worked examples were great because, "for me if I see it, hear it, and write it, then I remember it." Rachel stated when working on homework, "if I didn't understand a problem, I would look for one (worked example) and I would work that one out and come back to my problem and it would help me a lot." That is, she would review what was done during worked example sessions to help her with problems on her homework. Usually she could successfully complete the problem through this procedure. She stated that the worked example method "contributed to my understanding a lot" and she believed that she

"would have failed for sure or would have dropped (the course)." Rachel's attitude changed dramatically during the semester. She realized that she could not just show up, "not do homework, and not study, and get a B or A." She realized that with hard work and a foundation that was laid through the worked example method; she could be successful in the course and obtain a good understanding of the course material. Rachel said that the worked example method had a "direct correlation to my grade." She went on to say, "you can see when I was at the discussion group (she missed some sessions) and (when) I wasn't. My grades were great and I was understanding and comprehending it (the material)." When talking about the worked examples, she commented about the three-step process when she stated "I like to see one and well I kind of got it, and then when we talk you through it then, you hear it from other students and for some reason it clicks in your mind. And then doing it on your own."

The case of Henry

Henry is a freshman majoring in fire protection and safety. Henry decided to go to school here because this university has one of the best programs in fire protection and safety. He took the SAT instead of the ACT and scored a 510 on the math portion, with an overall score of a 980. Henry's high school mathematics experience consisted of taking geometry his freshman year, algebra II his sophomore year, and pre-calculus his senior year. He stated that "my junior year I didn't take math because I hated it so much, I refused to take it." He was so disinterested and bored that he did not pay attention in algebra II and "had no clue what I was doing." This is backed up more from the statement, "I would sleep through class … everyday because I hated it so much. Also you know it was high school and if I pass it with a D then I pass it. Also high school teachers would assign homework that would take hours a night. They would assign 25 to 30 questions a night and it seemed ridiculous."

Before Henry arrived on campus, he successfully completed intermediate algebra at a college in a state near where his parents lived. He successfully passed college algebra and trigonometry when he arrived on campus, earning a C in both. He admits that he hated math in high school and college other than the second semester of his senior year in high school, when he frequently worked on his math skills with his pre-calculus teacher. Henry

said that “it was the only time that I enjoyed math” and his teacher helped him understand math and “got a lot of thinking out of me.”

Unlike Alex or Rachel, Henry’s college G.P.A. was fairly strong with a cumulative G.P.A. of 3.067, but he knew “calculus was going to be rough”. He scored a thirteen out of twenty-five on the pre-algebra assessment and a sixteen out of twenty-five on the post-algebra assessment. Henry stated “I seem to taken a little bit more pride. Especially since it seems realistic for me to obtain a B in this class, where some other classes you just try to get it down. The opportunity is open for people to definitely achieve a higher score if they would realize that they might put a little bit more work into it instead of I am going to get a C or a D and pass it”. He went on to say that the worked examples “has been really useful and probably the single biggest reason that I am passing the class. It has made a huge difference”. Henry through the process of the worked examples developed pride in his mathematical ability and restored a favorable attitude towards mathematics. His pride is a product of being more successful in the course, which resulted from building a better understanding of the concepts in the course, through worked examples and individual work. In high school mathematics classes Henry became frustrated and was turned off by mathematics. He determined that he was not as good in mathematics as he thought he was during elementary and middle school. The worked examples allowed him to see that through hard work in groups and studying individually, he could do well in mathematics once again. The mathematics light was switched on once again. At the beginning of the semester, Henry thought without the worked examples “I would probably be withdrawn (from the course) right now”. Henry would use the worked examples, when he studied by himself, to review himself on problems so that he could work other problems. Anytime that he was stuck on a problem he “would go to the worked examples (that we had went over with the three step method), find a problem like it and see how it is done, and try to apply it to the problem that I am stuck on”. As the semester progressed, he did not have to review worked examples as extensively as he did during the beginning of the semester. He even went to the extent at the end of the semester to say that students should explain the steps of the problem as they solve it with “I thought that it (three step method) was something that we could do on our own (during the worked example sessions). Since the answer (example problem) was right there and if not we could ask you about it; I wasn’t too

fond of it." Finally, Henry commented that "this class got me thinking a little more than any other mathematics class, probably because a lot of it is new".

Results

We will summarize results by including data from all the participants as we answer the research questions. While we discuss each of the questions and viewpoints from the participants, we will analyze the case studies to bring out trends from the viewpoint of the weaker to the stronger algebra students.

Question 1: What are students' perceptions and experience with worked examples in Calculus?

Most all participants were very positive about the worked examples. The majority of students taking calculus across the country expect to see examples during lecture that illustrate the concepts and theory in calculus. Many would say the more the better. Students build understanding of concepts through examples. Sometimes a few examples will be enough for a student to get a clear understanding of a concept in calculus. Other times it takes many different examples before a student gets a clear understanding. Furthermore, understanding builds from students working examples. It is no surprise that students are very receptive of the worked example method. Participants made it very clear that just because they see problems worked in lecture and things seem clear, this does not mean that they understand. It is not until they work problems that they understand. The worked example method is so valuable because it allows students to build confidence and understanding by transitioning from seeing examples to working examples. Participants stressed many times that although they thought they understood the concepts and examples worked in class, things were much harder when they tried to work things by themselves. This would lead to frustration and sometimes to an attitude of surrender. The worked example method helped participants build up confidence that they could understand and work other problems by themselves, especially the weaker students. Most of the participants with weaker backgrounds thrived in this environment. Although, the worked example method tends to emphasize building of procedural schema, it does build a foundation of schema that can be used to

build higher order thinking skills. It is this foundation of mathematical schema that students will use in problem solving situations.

Question 2: What are the participants' perceptions on how the worked examples help them in the course?

Participants' perceptions on how the worked examples help them in the course varied from responses "minimally" to responses "I would have dropped or withdrew from the course." Out of eighteen participants that talked more extensively about how the worked examples helped them in the course, six stated that they believed they would have earned an F or would have withdrawn from the course without the worked example method. Three of them were Alex, Henry, and Rachel. Three other participants believed that they would only earn a D without the worked example method and the other six believed that they would have earned a C or better. Therefore the perceptions of the participants in this study were unequivocally that the worked examples helped them in the course.

Question 3: In what ways, if any, do worked examples build self-efficacy in participants' ability to learn material and instill confidence that they will be successful in Calculus?

We have seen that the worked example method increased Alex, Rachel, and Henry's confidence in their ability to "do" mathematics and be successful in the course. These three case studies showed the dramatic change in students' self-efficacy after using worked examples. These three were representative cases because both Alex and Rachel started technical calculus with a weak background and had dramatic improvements and Henry's dramatic change in mathematics attitude was because he enjoyed mathematics again. Overall, most all participants' confidence rose throughout the semester and participants' overall confidence with the material showed how the worked examples helped students build confidence in their abilities to "do" mathematics. Furthermore, the change in the way participants viewed their ability to do mathematics, were very dramatic.

Discussion

There have been numerous studies that have examined how worked examples have helped studies in a variety of disciplines. Chi et. al. (1989) showed that while students studied worked examples, "good" students generally monitored their own understanding and misunderstanding through self-explanations. Compare this to "poor" students who did not generate sufficient self-explanations or monitor their learning inaccurately. In this study, we examined the perceptions and experiences of participants that used worked examples in voluntary discussion sessions outside of class time. The weaker students in this study relied on worked examples heavily as Chi et. al. (1989) found in their study. The two specific cases of this are with Alex and Rachel. For both of these students the worked examples were very central to their learning. After heavy use with the worked examples, Alex and Rachel built confidence that they understood the topics and would work more problems by themselves. It was through this process that changed their self-efficacy in mathematics. Both of these students were good representatives of other participants that had a weaker mathematics background and lower self-efficacy that they could succeed in math.

Chi et. al. (1989) also showed that stronger students did not rely heavy on the worked examples and self monitored their learning. The stronger students in this study relied less on the worked examples as the semester progressed, because once a student learns things at a level that they do not need to refer to worked examples, they become more confident that they are doing things correct and work problems independently. This can be seen from Henry, who used the worked examples more extensively at the beginning of the semester as he built confidence in his ability to do mathematics and, as the semester progressed, monitored his own understanding more by working many homework problems by himself and referencing the worked examples only when he could not work through a problem. Henry's attitude about mathematics increased dramatically as the semester progressed. Henry also knew that he had the ability to do mathematics but his past experience in mathematics brought him to a point that he really disliked mathematics and his self-efficacy was low. The experience with worked examples changed the way he viewed mathematics. He finally liked mathematics again and his self-efficacy was high. Henry is a good representative of the participants that transition from using worked

examples more to using them less at the end of class and self-monitoring themselves as they learned calculus.

The majority of the students in this study were weaker math students and the worked examples helped the students lower their cognitive load and assist in the transition of procedural understanding of calculus from short-term to long-term memory. Keeping the cognitive load from being overloaded, and hence losing information before it could be transferred, relieved students of math anxiety and, as they were successful learning calculus, helped them build confidence that they could understand the material.

Overall, when examining participants' perceptions and experiences with worked examples, we found that all of the participants liked building understanding of course material through worked examples. The weaker students used the worked examples throughout the course. For them the worked examples helped build confidence in their ability to "do" mathematics and increased their self-efficacy. It is important to note that the worked examples helped weaker students to change their perception on how successful they would be in the course. For each course topic, this success came from a slow transition from relying more on worked examples to build understanding and confidence, to relying more on the their knowledge and understanding after they built up sufficient mathematical schema on the topic. For the stronger students the worked examples helped them first to obtain a clear understanding of previous lectured material, second to build schema of how to work problems, and third to transition (usually earlier than weaker students) to monitoring their own learning. The self-efficacy of the stronger students also increased from the beginning to the end of the semester, however they will attribute less of it is due to the worked examples. The stronger students attributed that the worked examples helped them in the course to a lesser degree than the weaker students because they made the transition to relying less on worked examples and more on their self-monitoring of their own learning.

Finally, Miller (2010) found that not only did the worked examples help students to build understanding and confidence in their own ability to learn calculus, there was a significant difference in course performance between participants and non-participants. All the participants, except two, earned a course grade of C or better and no participant failed or withdrew from the

course. Compare this with almost 40% of the non-participants that earned a D, failed the course, or withdrew.

Implications of this research not only reinforces the results of prior studies (Sweller & Cooper, 1985; Zhu & Simon, 1987; Chi et. al., 1989; Catrambone & Yuasa, 2006, Crippen & Boyd, 2007) that showed that worked examples improved performance, self-regulation, and self-efficacy, but it shows that students' perceptions of worked examples can help students improve their attitude and overall experience in calculus. This more than likely helps them persist in longer in the face of difficultly, apply more effort to be successful, and use self-regulated learning strategies.

Limitations

There are several limitations to this study. The first possible limitation is that students that volunteered were more motivated to be successful in the course than non-participants. Also, since we asked for volunteers by visiting the class and students knew that the non-success rate (the percent of students who earn a D, fail the course, or withdraw from the course) was over 40%, many of the students that attended the discussion session were weaker students. The second limitation is that the author interviewed the students that attended the discussion sessions and used the three-step method and hence could have caused the students to give bias statements. The third limitation is that working through worked examples in the group sessions could have provided a support system where participants' self-efficacy increased, causing students to apply more effort, persist longer in the face of difficulty, and more than likely use self-regulated learning strategies (Tanner & Jones, 2003; Bandura, 1977; Wolters & Rosenthal, 2000). Although worked examples can help students with conceptual understanding, it is more conducive to building procedural understanding, which is another limitation of a student building complete understanding in the course. Finally, some of the participants might have practiced working out more problems through the worked examples (in group sessions or when studying by themselves) than non-participants resulting in an increased self-efficacy of these participants, participants being more comfortable with the course material (less anxiety on exams) and having a more favorable view of worked examples, transitioning quicker to relying on worked examples

less and more on self-monitoring their learning, and an increased performance in the course.

Conclusion

Previous studies in a variety of disciplines (Sweller & Cooper, 1985; Zhu & Simon, 1987; Chi et. al., 1989; Catrambone & Yuasa, 2006, Crippen & Boyd, 2007) showed that worked examples are a good way to help students learn and understanding the material, but never investigated students' experiences and perceptions with worked examples. This article examined the perceptions and experiences in a case study of three students – two weaker students and one average student – working to understand the material in a technical calculus course through the use of worked examples. The results were that worked examples helped the students to perform better in the class, increase their motivation to learn the material, and improve their self-efficacy, especially the weaker students. These students were good representatives of other students in a study by Miller (2010) who used worked examples in a three-step method during voluntary discussion sessions. The average student used the worked examples more extensively during the early part of the semester and transitioned later in the semester to self-regulating their own learning by working problems without referencing the worked examples very much. The two weaker students relied more heavily on the worked examples, only self-regulating themselves to a certain degree. The results of this study reinforced the conclusions of the prior studies, but also showed students' perceptions and experiences of worked examples helped them significantly improve their attitude about mathematics and their motivation to learn calculus. This helped them to persist longer when they faced difficulty and cognitive obstacles, apply more effort to be successful in the course, and had a significant affect on their overall learning of calculus.

This research and previous research (Miller, 2010) has been adapted to help students in large enrollment college mathematics classes where a once a week in-class supplement session focuses on small group learning through instructional materials using worked examples. Future research plans will focus on in-depth research on exactly how students' use worked examples

and differences of learning from worked examples versus traditional homework.

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