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Explanations, Illusion of Explanations, and Resistance: Pre-service Teachers' Thoughts on Models for Integer Operations

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Abstract: This article reports the experience of a group of pre-service teachers when they discussed effective models for integer operations. While the participants proposed various models, analogies, and metaphors, it was difficult to find effective models that explain all the cases of integer operations and that were acceptable to all of them. This experience of discussing the rules for integer operations provided the participants with an opportunity to refine their prior knowledge and rediscover extant findings regarding effective mathematical models. However, their experience also revealed uncertainties about the mathematical and pedagogical nature of integer operations. Those uncertainties include the issues of convention versus understanding, the unnecessary complexity of the models, and the possibility of using models in a rote manner. The opportunity to discuss these models of integer operations helped the pre-service teachers to unpack their perceptions and understanding. For teacher educators, this work emphasizes the value of offering this type of activity in their mathematics teacher preparation courses.

Key words: Pre-service teacher education; Integers; Metaphors; Explanation

Introduction

Mathematics educators generally believe that effective teachers should focus on promoting students' understanding by employing a variety of teaching strategies rather than delivering isolated facts or rules. This is often referred to as "teaching for understanding." In order to achieve this goal, teachers are expected to provide sound explanations that encourage students to engage in inquiry and reasoning. Wertsch (1998) defined "mastery" as knowing how to use cultural tools as it relates to procedure and automaticity, whereas "appropriation" as "the process of taking something that belongs to others and make it one's own" (p. 53). Appropriation refers to fully supporting and understanding the mechanics of the tools in addition to simply knowing how to use them. In mathematics education, teachers need to provide their students with rich opportunities that intend to foster not only mastery of mathematical tools but also the appropriation of those tools. One approach that many mathematics teacher education courses adopt is to help pre-service teachers re-visit their prior mathematical knowledge (e.g., computational algorithms and procedures), offer them the opportunities to represent meanings with various concrete manipulatives or models, and discuss their effectiveness. My K-8 mathematics methods class was no exception. The shift from a concrete, rule-based approach to “teaching for understanding” was accepted well by my students. However, their unhesitant acceptance raised a question: Is it possible that my students merely master and practice the modeling process without appropriation? It was hard to detect how my pre-service teachers really felt about the emphasis of modeling until we encountered an unexpected resistance to the models for integer operations. This article presents the issues addressed in a small group discussion where five pre-service teachers reflected on their learning experiences. This experience raised a number of concerns for both my pre-service teachers in an American teacher education program and myself, a mathematics teacher educator. This article addresses these concerns in a methodical manner.

Concept of Negative Numbers

Historically, mathematicians have had numerous debates and difficulties with negative numbers (e.g., Hefendehl-Hebeker, 1991). It is therefore not uncommon to encounter challenges when introducing students to negative numbers. This may be because the students’ existing understanding of numbers needs to be revised when broadening the set of numbers to include negative integers. Many properties that apply to the set of natural numbers are no longer valid in the set of negative integers. For example, it is no longer true that “addition makes bigger” and “subtraction makes smaller”. To help students better understand negative integers, researchers and teachers have developed several general models that can be extended to integer operations. It has also been claimed that by creating unique contexts/models, students can better apply their previous understanding of whole number operations to this new concept.

The National Research Council [NRC] (2001) of the United States stated that various types of physical metaphors have been utilized to introduce negative numbers and operations of integers, such as “elevators, thermometers, debts and assets, losses and gains, hot air balloons, postman stories, pebbles in a bag, and directed arrows on a number line” (p. 245). The report mentioned mixed opinions regarding the effectiveness of these metaphors:

> many of the physical metaphors for introducing integers have been criticized because they do not easily support students’ understanding of the operations on integers (other than addition). But some studies have demonstrated the value of using these metaphors, especially for introducing negative numbers. (ibid)

Although a set of specific learning activities or models was not indicated, the NRC (2001) strongly implies that posing and solving problems through movements on a number line will be more effective than stand-alone equations (pp. 245-246).

Previous studies have suggested instructional activities based on contextual metaphors or models. Thompson and Dreyfus (1988) utilized a computer-based model (the microworld called INTEGERS) that shows a turtle that walks up and down the number line in order to illustrate the integer addition process. This helps students to conceptualize integers and operations upon integers: integers as transformations; addition of integers as the composition of transformations; negation as a unary operation upon integers and integer expressions. Petrella (2001) criticized the conventional approaches, such as the charged-particle model or the number line model, claiming that they are more of a distraction than an asset. As an alternative instructional approach, Petrella used familiar, nonmathematical language, such as optimist, pessimist, positive and negative thinking (e.g., "To take away a little positive, we could add a little negative"), and reported that students were able to make sense of the subtraction rule for integers when this approach was explained. Reeves and Webb (2004) also had unfavorable opinions for the typical introduction to integer operations, including walking on a number line, temperature model, and chip-models. Instead, they utilized a tug-of-war analogy between gravity and helium balloons (e.g., A toddler weighed 18 pounds and a balloon has -20 assigned to it to explain 18 + (-20) = -2). Gregg and Gregg (2007) pointed out the contrived nature of the balloon metaphor since the rules for calculating with helium balloons cannot be similarly inferred as the deduction from the properties of integers. They suggested using the allowance contexts and giving students the option to solve questions in ways that make sense to them rather than telling them how to use the context to solve integer number sentences. As shown by the variety of models and metaphors proposed, however, there is not one clear effective model. It is still widely believed that some types of models or contexts are capable of explaining the reasons behind the various rules and procedures for integer operations. It should also be noted that there is criticism about the heavy reliance on the use of analogies and metaphors in mathematical reasoning. Wu (2011) expressed concern that these analogies and metaphors often only half-satisfy students' appetite for knowledge due to the absence of precise definitions and logical reasoning. He asserted that mathematical modeling should require the use of precise mathematical language and provide a coherent and logical explanation.

The remainder of this article discusses the resistance of five pre-service teachers (henceforth referred to as "trainees") to the various models of integer operations while simultaneously exploring insights gathered throughout this experience.

Pseudonyms are used. The accounts presented below were taken from their written reflections/logs or recorded/transcribed discussions conducted by the researcher.

The Course and Participants

Course: This mathematics methods course was offered at a Midwestern university in the United States. This four-credit course was required for elementary education majors and was typically taken prior to student teaching. They had successfully completed two mathematics content courses (Numerical Structures and Introduction to Statistical Concepts and Reasoning) prior to this methods course. The classes had 27 trainees. Typical course activities consisted of sharing past learning experiences, brainstorming teaching strategies, examining widely used teaching strategies, and discussing effective teaching strategies. It is common to observe trainees' breakthrough moments during class, revealing that pre-service teachers not only develop teaching strategies but also refine their own understanding of mathematics.

Participants: During initial discussion about integers, most of the trainees revealed that their previous experience with the topic focused primarily on simple memorization and application of the various rules, for example, "a negative times a negative is a positive". After this, the following five trainees were invited to in-depth follow-up small group discussion. The term "traditional" refers to no gap between graduation from high school and college enrollment. In keeping with state requirements, each trainee selected one teaching major or two teaching minors.

- Dan (male) was a traditional student who chose language arts as a major.
- Janine (female) was a traditional student who chose social studies and language arts as her two minors.
- Jen (female) was a traditional student. Her teaching major was mathematics.
- Morgan (male) was in his mid-twenties. He previously attended another college and studied engineering but did not complete his degree there. He was a part-time student who worked for the automotive industry. He chose social studies and language arts as minors.
- Sue (female) was a non-traditional student in her mid-forties. She had an Associates Degree in Computer Aided Design (CAD). She chose language arts as a major.

Past Learning Experiences

The prior experiences stated by the five trainees were as follows:

- *Dan:* I do not remember the particulars of learning integers, but I know that I understand the concept well now so the knowledge was instilled in me.
- *Janine:* As far as learning the integer operations in my previous schooling, I have to admit I don't remember. Once again, math was not…my favorite subject and I basically just did what it took to…pass and would forget the rest.
- *Jen:* I learned to add and subtract integers by picturing a number line and comparing the two numbers to figure out if the answer should be negative or positive. I learned the multiplication and division of integers by saying opposite signs in the problem creates a negative answer, if the problem contains the same signs (either both positive or both negative) the answer will be positive.
- *Morgan:* I do not remember…[but] I knew the rules and how to use them. I wanted to see how this could work on a number line.
- *Sue:* I learned integers…using a number line to count the number of spaces between the integers for the answer…we were mostly taught to turn the subtraction problems into an addition problem. We had to cross the subtraction sign to make it an addition sign and then cross the negative sign to make it a positive number and then add those numbers from there to get the correct answer. I was never taught using manipulatives like chips.

Their prior understanding relied heavily on rules and formulas. Since this reliance did not present a significant obstacle in their learning, they typically did not see the need for other approaches.

Examining Widely Used Models: Emerging Resistance

I prepared two models for class discussion: the chip model (e.g., similar to the "charge model" of Battista [1983]) and the number line model (e.g., similar to Chilvers' [1985] or Thompson and Dreyfus' [1988]). Although these two models are commonly found in American textbooks, only very few trainees remembered *how* they were used.

Addition using the chip model was easily understood by the trainees as they showed no resistance to using zero-pairs. However, for subtraction of negative numbers, see Figure 1 for the case of modeling 3 – (-5), which was based on the ideas of zero-pairs and the preservation of the value of minuend, several trainees started to express their uneasiness about this explanation.

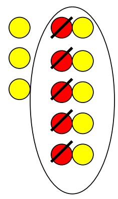	* Positive 3 (minuend) * Wish to have negative 5 (subtrahend) * Use zero pairs (+5 & -5) to keep the value of the given minuend * After taking away negative 5, positive 8 left

Figure 1. Chip Model for 3 – (-5) = 8.

Some stated that they could not understand it, while others mentioned that this model would be very confusing for young pupils, even though they themselves understood it. Whether the opinions were from a learner's or a teacher's perspective, most of them deemed the chip model ineffective. They stated that the process of making zero pairs and working with the minuend was most frustrating. Some comments follow:

- We added multiple chips to the model [5 red chips and 5 yellow chips]. I know that the total value of minuend is still 3. But it would be confusing for many students. Students might expect that something should be changed when we do some actions. It seems very unnatural.
- If you ask this question in the exam, I can just build and write or draw what you demonstrate step-by-step. I think it seems to be another way of enforcing procedures. Also, I am not sure if I fully understand.

At this stage, I stopped the chip model demonstration and invited them to exchange ideas about their interpretations of the rules of integer operations. They made three key suggestions.

- *Dan:* The way that I explained how to solve subtracting a negative number was by using a fast food restaurant as an example. I used Burger King. Let's say three students go to Burger King and order happy meals. They get their cups that go with their meals but have a small problem when they cannot decide what kind of drink they want. So they contemplate what kind of soda they want to drink during dinner. Finally…they decide to mix two kinds of soda together to see what happens. One student mixes two pops that he likes and finds that he like the combination: it makes a positive result as does the combination of two positive numbers in math. The second decides to mix one pop that he does not like and one that he does like. So he pours some of the bad out and the result is much more appetizing. He subtracted some of the negative to get a positive result. After that analogy, I will explain that two positives will always be positive, and a positive plus or minus a negative will change its sign depending on numbers, but subtracting a negative from a positive will always become positive. Unlike pop, which is something that people can taste and change, math cannot be changed and does not make sense to some people; it has to just be explained.

- *Jen:* The method I proposed was that a double negative equals a positive. The saying I came up with was "I am *not not* going to the store." Then asked, "Am I going to the store?" This statement proves [my method] and [applies] my statement to a real life situation. I feel that some of my fellow classmates responded quite well to my statement in our initial class discussion.
- *Janine and Sue:* Our idea was a bank account balance and borrowing money from a friend. This method made sense to us due to the fact that it has to deal with real life and real situations. We feel that the key to understanding a lot of math problems has to do with how it relates to real life.

While most of them shared their opinions and tried to develop applicable models or contexts, Morgan, however, continuously expressed his dissatisfaction with every proposed idea.

- *Morgan:* I had never seen the chip model before. Jen's phrase for the double negative rule confused me the most. I was not following the logic of all the different verbal scenarios to try to make sense of the rules. I am not sure if it is necessary to use such weird scenarios or models to explain the rules. Without those models, I can successfully perform all operations. I can live with that.

For the follow-up small group discussion sessions, Jen, Janine, Dan, and Sue were invited because they proposed different models that they believed to be effective, while Morgan was invited due to his critical opinion of his peers' proposed ideas.

Talking about Proposed Ideas: Surfaced Concerns

The first small group discussion occurred one week after the initial class discussion. The trainees had the opportunity to elaborate on their initial ideas as well as provide feedback to one another. The following accounts illustrate the primary reasons for their dissatisfaction with or resistance to the proposed models.

Loss of mathematical meaning: Resistance to Dan's explanation

Even with Dan's enthusiastic explanation, his initial idea was not favorably accepted mainly due to its non-mathematical approach and imprecision in the use of so many subjective factors, for example, preference of drink flavors.

- *Janine*: I don't think that the context you provided helps clarify the given problem situation.
- *Jen:* What is the chance that I like the taste of drink you like?
- *Morgan:* You could create the context like this because you already knew the rule. However, we cannot lead to the rules through the pure experiment. It seems that you tried to artificially fit the rules in the context, not the other way around. I cannot consider this is a proper explanation even though it sounds like you explained a lot.

Dan also admitted that his example could involve so many subjective factors and could not be generalized. However, he believed that simple, non-mathematical ways to explain negative integers do exist. He also mentioned that some (not all) mathematical rules are similar to naturally occurring phenomena.

- *Dan:* The sky is blue and the leaves are green. We know that these [facts] are true. We could explain why. However, most of the time, it is not necessary because it is commonly and widely observable… Likewise, we know that negative minus negative will result in positive (subtracting a negative number is the same as adding a positive number). It is just like a fact or phenomenon. We could explain but [*sic*] not necessary. The point is how we can use the conventional system when we actually apply these rules.

Although there were some arguments regarding the examples Dan used such as, "some leaves are not green," and "sometimes the sky does not look blue," Dan's assertion led to a different layer of discussion: "Does everything we teach need models or explanations?" The other trainees also supported Dan's point that this might be a mathematical convention:

- *Janine:* I think it is like driving a car. I know how to drive a car, but I do not know well how the car works. Does it matter? Do I need know all mechanics before I drive a car?
- *Jen:* There are so many mathematical conventions. The integer operation rules may be mathematical conventions, something we agreed to use in that way. For example, three to the zero power is one [$3^0 = 1$]. This is a convention. It is not necessary to explain why it is. Likewise, the rules for order of operations are mathematical conventions.
- *Morgan:* I agree that there are many mathematical conventions we are allowed to use without explanation. I am not sure if we can say that "a number to the zero power is one" is a convention, since I can explain it using patterns, although it is not a physical model. But I think the order of operation is a mathematical convention. It is just like grammar in language. It is just for

> effective mathematical communication. I think the integer operation rules can be considered mathematical conventions. I don't like all the weird analogies.

In the end, Dan withdrew his original beverage example due to its subjective interpretations, but he said that he would keep on searching for better, non-mathematical contexts to help students.

Limited usage and disguised real life connection: Resistance to Jen's explanation

Jen reiterated the example she used to explain the situation of subtracting a negative number: "I will not *not* go to the store." Regardless of the incorrect grammatical structure, this approach was well received by fellow trainees in the initial whole class discussion. However, during the small group follow-up discussion, some issues were raised. Sue questioned what the word *"not"* represented in Jen's phrase and Jen was not clear on this:

- *Sue:* Is this model explaining the multiplication of two negative numbers?
- *Jen:* Originally, I tried to explain the problem that needs to subtract a negative number. But I am confused now…I am not clear how I can distinguish the sign of a number (e.g., a negative integer) and the sign of an operation (e.g., subtraction). I am not sure if it matters or not.

Sue also questioned whether or not similar phrases could be consistently used to explain all four operations:

- *Sue:* Can you explain all four operations using this…saying?
- *Jen:* I am not sure. I am still not clear whether the word "*not*" represents a negative number or it represents the operation, subtraction.

Morgan was also dissatisfied with Jen's explanation:

- *Morgan:* Just like Dan's explanation, Jen's does not explain *why*. They just explain how to do it or how to memorize the rules like some other mnemonic devices when we memorize something…it is very artificial. [No one] would say this in real life instead of saying "I will go to the store."

Unnecessary complexity and awkward translation: Resistance to Sue and Janine's explanation

Sue and Janine explained their account balance model. For the question "+3 – (-2)", they created a context:

> I have \$3.00. Jen borrowed \$2.00. Jen is paying me back the \$2.00 she owed. Then, how much money will I have?

It seems to Sue and Janine that the math sentence matches the context. However, it was not clearly conveyed to the other trainees, and some of them were not sure how subtraction was used in this context.

- *Dan:* How is the subtraction context explained here?
- *Sue:* The amount someone owns represents positive integers. The amount someone owed represents negative integers. In this context, I own $3.00. So, it is a positive 3. Jen owes $2.00. So it is a negative 2. Since Jen is subtracting from her debt to pay me back, it represents a subtraction context. (Sue had to indicate which words matched with which parts of the math sentence [Figure 2]).

+ (positive integers): owns amount
– (negative integers): owed amount

(+3) – (-2)
$3 owns ↓ $2 owed

Subtracting from her debt (paying me back)

Figure 2. Sue's Approach.

Morgan seemed to accept this justification as it attempted to explain both the sign of a number and the sign of an operation distinctively in a more systemic way. However, he pointed out that this explanation also had weaknesses.

- *Morgan:* Once again, this explanation, as presented, is very artificial. In real life, how many people can come up the equation: 3 – (-2) = 5 when the context is given? If the word problem was given first, many people might think of "$3 + $2 = $5" instead. It must be an unlikely event to link this word problem to the given math sentence.

Sue, who set up this modeling, could consistently create matching contexts covering various operations and cases. However, during the discussion, her idea was not fully delivered to the other trainees. Dan felt that this context resulted in unnecessary complexity: "I think this explanation would make students more confused. I wonder if it is worthwhile to do."

Revisiting the Widely Used Models: Unresolved Concerns

After examining the models/contexts generated by these trainees to explain integer operation rules, I requested them to refer to the chip model and the number line model (e.g., Van de Walle, 2000, pp. 424 - 432) when they considered issues raised while they critically reviewed each other's proposed ideas. The primary issues they were to focus on included: (a) lack of mathematical meaning, (b) limited usage, (c) disguised real life connection, (d) unnecessary complexity, and (e) awkward/artificial translation. The trainees generally agreed that these models could explain the overall mathematical structure in a more systemic manner (i.e., mathematically meaningful) and that there are consistencies in the system (i.e., generalizable usage). However, the other issues still resonated. Morgan felt that these models still added too much complexity for the learners, while others considered that these would not be practical teaching strategies in an actual classroom due to their complexity.

- *Morgan:* I know that the rules associated [with] integer operations can be explained using the chip model or the number line model and the contexts we discussed in class. We tried to give a certain meaning to operations and numbers to understand the nature of the math concepts. But this process produces unnecessary confusion… We tried to come up with different scenarios to try to make sense of integer operations, but I doubt that my students will follow the logic. I think the various scenarios and models could be obstacles rather than aids.
- *Jen:* [This experience] was not very practical for use in the classroom.
- *Janine:* I am not sure whether [or not] the instructional time I used for this explanation will pay off later for my students and me.

Another unresolved issue was establishing what should be considered as mathematical conventions:

- *Jen:* I think I learned a lot from the process of examining different models. I agree that the models we created had many flaws. They were not perfect. But I think the process of modeling and explanation is important.
- *Morgan:* I agree. But there are different categories of math content. One is the concepts and procedures that require more conceptual understanding. Another is the procedures or conventions that can be used as tools to explain other concepts and procedures. I consider that the integer operation rules belong to the latter. It may be due to my engineering background. I never like to use something other than written symbols and calculators. Everything is simple. The approaches we examined are too complicated.

- *Jen:* I am not clear [about] the link between the concepts … and the conventions. It sounds like a "chicken and egg" situation.
- *Morgan:* I don't know either … I am not against the importance of conceptual understanding. What I dislike is the pedagogic chaos around models that can possibly [cause] more harm than good for students' understanding.

Instructor's Afterthoughts: Gains and Challenges

It is generally believed that the only way to change the teaching practices of teachers is to help them to see their current practices as problematic (Cobb, Wood, & Yackel, 1990). As a teacher educator, I expected pre-service teachers to see their past learning experiences as problematic, but I am not sure that I allow myself to view my own teaching in the same way. The series of discussions on teaching integer operations not only provided pre-service teachers with an opportunity to examine their future methods of teaching, but also gave me an opportunity to examine my own practices of teaching future teachers. Though I gained significant insight from this experience, I continue to ponder several issues that were brought to my attention.

The trainees' resistance to the widely used models for integer operations reveals their perceptions about the role of using models in teaching mathematics, which was not explicitly uncovered when discussing other topics. Considering the fact that the path to "appropriation" is likely to involve tension between the tool and the use we make of it within a particular context (Wertsch, 1998), the resistance revealed can be the evidence of the process of appropriation of individual pre-service teachers.

Acknowledging the difficulties and numerous debates that mathematicians have historically had with negative numbers (e.g., Hefendehl-Hebeker, 1991), the trainees' struggle in devising proper instructional strategies seems to be a natural process. This was the first time these trainees had voiced their confrontation to the external authority (i.e., what other experts said and what the textbook said) showing their personal difficulties in understanding the mechanics of the tool during the path to appropriation. One major source of their resistance was the complexity of the models. That is not to say that these trainees are in favor of procedural knowledge (i.e., just providing the rules) over conceptual knowledge just because of its simplicity. The main issue was that they felt that the risk to benefit ratio was not advantageous for this specific topic as illustrated in some of their comments, such as "pedagogic chaos," "obstacles than aids," or "more harm than good." Unlike the modeling process in other topics, the case of integer operation requires more abstraction or structural understanding than the targeted concept being modeled. Certainly, to these trainees, the major role of a mathematical model or explanation would be to represent the complicated mathematics concepts in an easier and more approachable way. It benefits everyone when they voice their concerns. However, it is uncertain what they had considered the optimal level of complexity in a mathematical model or explanation.

It was a gain that comparing and contrasting the trainees' proposed ideas had sparked a rich discussion. I cannot say that the discussion itself was mathematically rich as many incorrect or non-mathematical ideas were exposed. However, this provides an opportunity to reveal pre-service teachers' perceptions of good mathematical explanation. For example, Dan seemed to believe that teachers' explanation should be student-friendly and easy and he preferred to use a familiar metaphor for that purpose. His idea was criticized due to the ambiguous nature and the loss of mathematical meaning. The criticism of Dan's idea was similar to the concerns voiced by some researchers about using analogy in teaching mathematics. A metaphor is described as "a condensed analogy" (Pimm, 1981, p. 48). Due to its condensed nature, a metaphor demands that, "the interpreter becomes actively involved in searching for meaning" (Ashton, 1994, p. 358). Pimm (1981) viewed analogy and metaphor as uncertain methods of working that are unsuited for use in mathematics education. He went on to state that they could be mere illustrations of structure that do not necessarily preserve the mathematical meaning. From the initial class discussion, there was no one model that made sense to all the trainees, all the time. They continued to gather different interpretations of the same metaphorical explanation. This dilemma is exactly what Ashton (1994) and Pimm (1981) mentioned as the disadvantage of using metaphors. Morgan's comments are especially in line with this perspective as he considered Dan's attempt to be an illusion of explanation instead of a genuine explanation.

- *Morgan:* You could create the context like this because you already knew the rule. However, we cannot lead to the rules through the pure experiment. It seems that you tried to artificially fit the rules in the context, not the other way around. I cannot consider this is a proper explanation even though it sounds like you explained a lot.

Jen's non-mathematical language approach and explanation of the meaning of operations suffer because of their limited scope. That is to say that they are not applicable to a range of instances and there was no motive for extending her model other than using it as a reminder of the rule. Jen, Sue, and Janine had their models criticized due to the unrealistic real life connection or, in Morgan's term, "disguised real life connection."

The preliminary distinctions made by these trainees between *explanation* and *illusion of explanation* or *real life connection* and *disguised real life connection* allowed me to reflect upon my own university teacher education course. As many researchers have noted, the focus on rote memorization of rules and procedures is one of the characteristics of many pre-service teachers' prior experiences as learners of mathematics, and teachers often teach the way they were taught (e.g., Ball, 1998; Brown & Borko, 1992; Goos, 1999; Janvier, 1996). In response to this situation, teacher education programs strive to provide pre-service teachers with opportunities to critically think about the way they were taught and to reshape it. There is an unspoken agreement that "the way they were taught" is an example of "authoritative discourse" (Bakhtin, 1991) or "mastery" (Wertch, 1998). This focuses on the transmission of knowledge from the teacher and/or the official text, and more desirable teaching should focus on the "internally persuasive discourse" (Bakhtin, 1991) or "appropriation" (Wertch, 1998), which focuses on the learner's own understanding. However, it is unclear whether the models, analogies, and metaphors frequently used in a teacher education course would become another means of rote practice rather than tools to promote comprehension. In other words, since mathematical learning can depend on mere mastery or external authority, pre-service teachers might think that they or their students can manipulate the models into rote application instead of using them to support true conceptual understanding. It is, indeed, important to have this experience of sharing different ideas for explaining integer operations and discussing the flaws in each model. The remaining challenge is to determine how to encourage deeper involvement of pre-service teachers in order to shift their perception and attention.

Another point of uncertainty is whether or not the rules for integer operations are a matter of convention or a matter of understanding. In particular, Dan and Jen believed that these rules are already there for us to use for other mathematical work. It is therefore not necessary to "make sense" of these conventions, unlike other mathematical situations. Why did the trainees raise this issue for this particular topic? What kind of mathematics can be passed down within a community of learners as convention for practical use and what kind of mathematics should be explained for understanding? Given this concern of the trainees, one immediate challenge for me is to unpack the distinction of "convention vs. understanding." I will do that by providing explicit opportunities for trainees to engage in reflections and elicit their thinking processes in order to have personally meaningful experiences. Claiming that the rules for integer operations are a mathematical convention is of concern to me. I agree that similar claims were made in the history of mathematics. A relevant example is this: "It *must* be that minus times minus must be plus. After all, this rule is used in computing all the time and apparently leads to *true and unassailable outcomes*" (Stendhal, 1835, cited in Hefendeh-Hebeker, 1991,

p. 27). However, I am not fully convinced that the discussion of integer operations should be placed explicitly in the realm of mathematical convention. Earlier, Janine mentioned that using the integer operation rules is like driving a car, claiming that it is not necessary to know all the mechanics of how the car works in order to drive it. I am still pondering this delicate distinction and how this can be addressed most effectively in my teacher education course.

This descriptive report was designed to provide a snapshot of the experience of a specific group of pre-service teachers, and it is not appropriate to draw strong implications from their views. Nevertheless, the discussion regarding the rules of integer operations should stimulate further conversation concerning the nature of these rules, how they should be taught, and the struggle pre-service teachers have with teaching them. Some readers may be disappointed with the weak pedagogical content knowledge exhibited by these pre-service teachers. For me, this is the point from which they can start developing more reasoned and objective teaching approaches. Teacher educators would not have been aware of these faulty interpretations had they not had these discussions. The drive to change is not a matter of *the way it is* but a matter of *deliberate thoughts*.

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