The Mathematics Educator 2013, Vol. 14, No. 1&2, 49-65

Performance of Balance Scale Tasks of the Challenging Type amongst Singapore Children

Ng Swee Fong

National Institute of Education, Nanyang Technological University, Singapore

Madeline Lee Pe

Department of Psychology, University of Leuven, Belgium

Abstract: Since the pioneering research of Inhelder and Piaget in the 1950, "balance scale tasks" have become synonymous with items used to study cognitive development of young children. This paper reports on the results of two studies about these tasks: study 1 about 137 kindergartners from five kindergartens and study 2 about 122 Primary 2 children from five primary schools in the western part of Singapore. The balance scale tasks were designed by the Curriculum Development Institute of Singapore. Three-quarter of the primary one children tested could compare and identify which was the heaviest object when the information was presented concretely. However, when the weights of the objects were specified relationally as a function of each other rather than in a concrete manner their performance fell dramatically. Primary three children demonstrated similar difficulties. These findings may inform future studies on how to improve children's competencies with such tasks and how to incorporate such tasks into curricular materials.

Keywords: Balance scale tasks; Kindergartners; Primary children; Memory

Introduction

In 2005, the authors and their colleagues conducted a cross-panel longitudinal study to provide baseline data on the relationship between the development of executive functioning and mathematical proficiencies of young children in Singapore (Lee, Bull, & Ng, 2008; Lee, Ng, & Bull, 2011; Lee, Ng, Bull, & Ho, 2009; Lee, Ng, Bull, Pe, & Ho, 2011; Lee, Ng, Ho, & Bull, 2008; Lee, Ng, Pe, Hasshim, Ang, & Bull, 2012). That study tracked the development of children's working memory, executive functions, and algebraic thinking over a four year period. A battery of cognitive (intelligence, inhibitory, switch, updating) and mathematical tasks were administered to the children. Findings of this longitudinal study are reported in the papers mentioned above. This paper will describe findings about children's competencies with the balance scale tasks.

The balance scale tasks were designed by the Curriculum Development Institute of Singapore (CDIS, 1981, 1982, 1992, 1995). In these tasks, the pictorial stimuli present objects placed equidistant from the fulcrum of a balance. Currently there is no Singapore data about competencies with such tasks among lower primary children. These competencies are related to the Measurement topic, as explained below.

In Singapore, the topic of Measurement is introduced at Kindergarten and continued throughout the primary years. At Primary 1, the syllabus focuses on the measurement and comparison of the lengths and weights of two or more objects using non-standard units (Ministry of Education, 2000, 2006). The curriculum specifies clearly that children be taught to use terms such as heavy, heavier, heaviest, light, lighter, and lightest to compare objects of varying weights. Unlike comparison of lengths, comparison of weights of objects is more challenging. Given two ribbons of different lengths, children are able to state which ribbon is longer as the attribute of length is overt and the difference in length is clear. No instrument is needed to ascertain which of two given ribbons is longer. This, however, is not so with comparison tasks involving weights. The weights of objects are not determined by their relative sizes. A bigger box may have the same weight as or is lighter or heavier than a smaller one. An instrument such as a balance is required to determine which of two objects is heavier. When two objects are different in weights, the heavier of two objects can be found by lifting or "hefting" (weighing in the hand) (CDIS, 1982, 1992, p. 64), but a more sensitive tool than the human hand is needed, especially when the difference in weights is not easily discernible. To prepare children to understand differences in weights, the primary curriculum includes various balance scale tasks for lower primary lessons.

Balance Scale Tasks

Balance scale tasks have been used by various researchers to address different research foci. Inhelder and Piaget (1958) designed and used these balance scale tasks to assess how formal reasoning develops in children. Since their pioneering research, many researchers have constructed specific balance scale tasks to study cognitive development of learners. For example, Case (1985) studied infants; Siegler and Chen (1998) pre-schoolers; Boom and Laak (2007), Amsel, Goodman, Savoie, and Clark (1976), and Siegler (1976, 1978) worked with school aged children; Hardiman, Pollatsek, and Well (1986) worked with adults. One reason for the widespread use of these tasks is that, despite their simplicity, they are hierarchically related. They can be used to assess how participants of different ages apply a sequence of related rules to progress on the tasks.

50

The seminal work of Davydov (1962) demonstrates how 1st to 4th grade Russian children, working extensively with authentic tasks, were able to explore the different notions of equivalence, to use schematic representations and then symbols to represent relations exhibited by a selected parameter of concrete objects such as length, volume, weight and composition of sets of objects. For example, when different letters were used to represent two objects with the same weight on the balance scale tasks, the relationship of the two objects placed equidistant apart from the fulcrum can be represented by the symmetric equivalence x = y, where x and y represent the weight of each object. The symmetric equivalence can be broken or "disturbed", either by increasing one quantity (x + e > y, or x < y + e) or reducing one quantity (x - e < y or x > y - e) respectively. Davydov (1962) demonstrated that, with appropriate instructional strategies, it was possible to develop children's capacity to think relationally, abstractly, and algebraically.

Siegler's (1976, 1981) work is noteworthy because it allows for systematic analysis of children's capacity to apply rules to respond to items that require proportional reasoning. Siegler constructed six types of items to assess rule use, and reinterpretations of those items presented by Siegler (1976) are shown in Figure 1. He identified four rules children use to solve these tasks.

- Rule 1: Participants paid attention to the most salient aspect of the task, i.e., the number of weights on each arm. Some 4 and most 5-year olds based their predictions on the relative weight on the arms on either side of the fulcrum. If both arms have equal number of weights, the children predicted that the scale would balance. However, when the numbers of weights were different, children predicted that the side with more weights would go down. Distance was not considered as a variable.
- *Rule 2:* Most participants aged 8 or 9 applied this rule where they considered the distance from the fulcrum when the number of weights is equal on both arms, but otherwise reacted similarly to those who applied Rule 1; i.e., they focused their attention on weights when the number of weights on either side of the fulcrum differed.
- *Rule 3:* Most 12 to 13 year olds applied this rule whereby they considered distance and number of weights simultaneously. But these participants were challenged to resolve the conflicting situation when one side has more weights and the weight on the other side was further from the fulcrum.
- *Rule 4:* Known as the Torque Rule, it is the most advanced of the four rules. Children in the study and many adults failed to apply this rule. The torque on either side is the product of the weight and its distance from the fulcrum.

Title of Task	Pictorial Representation	Short Description of Task
(a) simple- balance		Equal number of weights on both sides placed equidistance from the fulcrum.
(b) simple-weight		Different number of weights on both sides placed equidistance from the fulcrum.
(c) simple- distance		Equal number of weights on both sides placed at different distance from the fulcrum.
(d) conflict- weight		More weight on the shorter arm and this side falls.
(e) conflict- distance		More weight on the shorter arm but the weight on the longer arm falls.
(f) conflict- balance		Differing weight and differing distance but equilibrium is maintained.

Figure 1. Summary of Six Balance Scale Tasks; Re-interpretation of Tasks in Siegler (1976).

Siegler and Chen (2002) summarised the findings emerging from work related to balance scale tasks as follows. Children aged 3 and below rarely applied rules systematically to solve balance scale tasks. However, analysis of participants' correct answers and errors, the proffered explanations, and the relationship between the responses and their explanations suggest that majority of the 5-year-olds and older children were able to consistently apply rules to solve balance scale tasks. This suggests that rule use increases substantially between ages 3 and 5 years. The rules applied are ordered hierarchically with participants basing their predictions *only* on weight, i.e., Rule 1. Then they based their predictions on weight and on Rule 2, i.e., distance if the weights are equal. Then with all other tasks, their predictions were based on guess work. Others based their predictions on weight and distance.

52

However, they did not use specific rules to combine these two variables. Some would add the distance and the weights together and predicted the result with the greater sum (addition rule). Some adolescents and a minority of adults applied the Torque rule which would lead to the correct solution. Thus, rule use is age dependent.

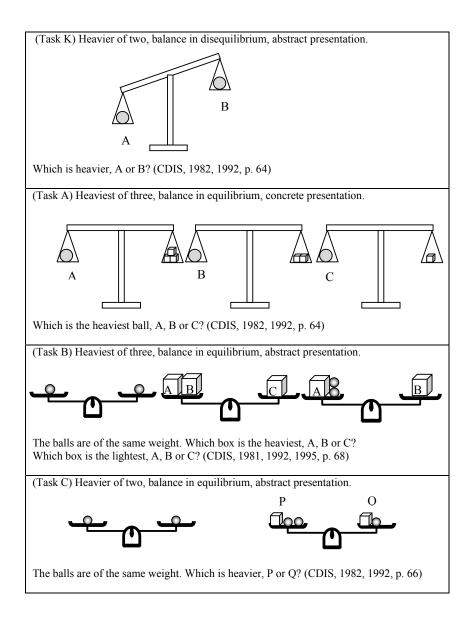
How much children learnt from proffered feedback was contingent upon the variables they used in their rule. Children who used Rule 1 in their predictions but who also attended to distance of the arm from the fulcrum were more likely to integrate the distance variable to help them learn a more advanced rule than those who did not. Thus children who used Rule 1 but factored in distance in their prediction were more often able to construct Rule 2 based on feedback to their responses than those children who only considered weight but not distance in Rule 1.

Balance Scale Tasks Used in This Study

At the time this study was conceptualised, there was no official mathematics curriculum for kindergartners. Thus, the mathematics syllabi for Primary 1 to 3 (Ministry of Education, 2006) were used as a guide. Tasks presented in Figure 1 were found in the Primary 1 and Primary 2 syllabi. The tasks for the kindergartners were designed based on the curricular materials such as worksheets used by the selected kindergartens. These tasks were similar to those presented in the Primary 1 syllabus. The weights on each arm of Tasks (a) and (b) of Figure 1 are equidistant from the fulcrum, whereas this condition is not maintained for Tasks (c) to (f). All the tasks used in the current study and listed in Figure 2 were similar to Tasks (a) and (b) with the weights placed at equidistant from the fulcrum. Theoretically one would infer that children would apply Rule 1 to respond to the six tasks presented in Figure 2.

The terms "concrete presentation" and "abstract presentation" will be used to describe the balance scale tasks used in this paper. A presentation is considered *concrete* when the weight of the object is presented in non-standard units. For example, in Task A of Figure 2, the weight of Ball A is equivalent to three cubes. A presentation is described as *abstract* when the weights of the objects being compared are not specified.

To ensure that responses from the children were not due to guessing, there were three questions for each task, all variations testing the same concept. To save space, only one question per task is given in Figure 2.



54

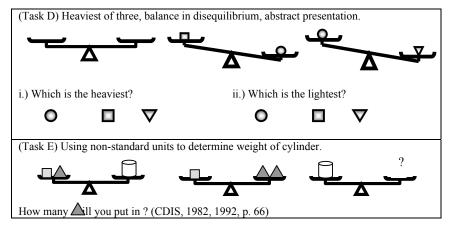


Figure 2. Six Different Sets of Balance Scale Tasks; One Question per Set.

Task K. Given two identical looking balls, "Which is heavier, A or B?" This task was first introduced in the Primary 1 syllabus (CDIS, 1982, 1992, p. 64). The answer can be found by direct comparison of the weights of two objects and is based on abstraction of numerous experiences playing on the seesaw. Because the left and right arms of the balance are equidistant from the fulcrum, the relative up-down position of the left and right arm of the balance will enable one to answer the question. If A is on the lower arm, than A has a weight greater than B, therefore A is heavier than B.

Task A. The weights of the three identical balls are specified in terms of nonstandard units (CDIS, 1982, 1992, p. 64), i.e., in terms of cubes. The objective is to determine "Which is the heaviest, A, B or C?" The heaviest ball is the one balanced by the most number of cubes. Hence, the weight of the ball is a function of the number of cubes. The question can be answered by exploring which of the three balls is balanced by the most number of cubes. A is balanced by 3 cubes so it has a weight equivalent to 3 cubes. Similarly, B has a weight of 2 cubes and C, 1 cube. Hence, A is the heaviest as it requires the most number of cubes to maintain balance.

Task B. This task was presented as a problem solving task in the Primary 2 Teacher's Guide (CDIS, 1981, 1992, 1995, p. 68). The weight of each box is presented relationally. The first balance indicates that the two balls are of equal weight. The middle balance shows that the weight of box C is equivalent to the total weight of two other boxes, A and B. Therefore, box C must be the heaviest of the three boxes. Since box B's weight is equivalent to the total weight of box A and two

56

other balls, then box B must be heavier than box A. Applying the principle of transitivity, A must be lighter than B, and therefore A must be the lightest of the three boxes.

Task C. This Mathematical Thinking task Number 51 was presented in the Primary 1 Teacher's Guide (CDIS, 1982, 1992, p. 66). The pictorial stimuli present two pieces of information: (i) The balance on the left shows that the balls are equally heavy. (ii) The balance on the right shows that the sum of the weight of box P and two identical balls is equivalent to the total weight of Q and one identical ball. The question is, "Which of the two boxes is heavier, P or Q?" The weight of the heavier object is not a function of the number of additional balls needed to maintain equilibrium. Instead, the opposite is true. To restore equilibrium the arm bearing the heavier box requires fewer balls than the arm bearing the lighter object.

Task D. This task differs from Task B in two distinct ways. In Task B, the balances are in equilibrium but the balances in this task are in *dis*equilibrium. In Task B, the three boxes are identical in form. However, in this task, the three objects (a disc, a triangle, and a square) are distinct from each other. It is not possible to discern from the physical attribute of the objects which is the heaviest of three solids and which is the lightest. Transitive property is needed to ascertain which of the objects is the heaviest, and which is the lightest. Because the arm bearing the square is up, and that bearing the disc is down, the square is lighter than the disc. The third balance compares the weight of the disc against that of the triangle. In this case, the disc is lighter than the triangle. The triangle is heavier than the square. Hence, the triangle is the heaviest of the three objects and the square is the lightest.

Task E. This is a variation of the Mathematical Thinking task Number 52 presented in the Primary 1 Teacher's Guide (CDIS, 1982, 1992, p. 66): "What is the weight of the cylinder in terms of triangles?" The three balances are in equilibrium. Hence, each balance gives the weight of the object. Starting from the far left, the first balance shows that the weight of the cylinder is equivalent to the weight of one square and one triangle. The balance in the middle gives the weight of the square as equivalent to two triangles. Hence, the weight of the cylinder is found by substituting the weight of the square in the far left with two triangles. The weight of the cylinder is equivalent to three triangles.

The Present Studies

The objective of the two studies reported below was to track the performance of a sub-sample of the main longitudinal study mentioned earlier. These two studies provide the baseline data on children's initial competencies with the balance scale tasks and their competencies with these tasks one year on.

Study 1: Kindergartners (K2) to Primary 1

Study 1 addressed the following research questions:

- 1. How well did kindergartners perform in selected balance scale tasks?
- 2. How well did they perform in such tasks one year later?

Method

Participants

For logistical reasons, kindergartens from the western part of Singapore were invited to participate in the study. A total of 137 second year Kindergartners (K2) from five kindergartens typically serving families from a low to middle socio-economic status background agreed to participate. In the first year of the study (2005), these K2 (6+) children were tested with tasks K and A, totaling six questions. To address developmental issues, task A was retained and tasks B and C were added to the instrument used in the second year of testing (2006). Thus, at Primary 1, they answered nine questions. The same children were tested one year later.

Procedure

All the intended tasks were piloted with small groups of children from the relevant age groups to ascertain the suitability of the tasks. These children were from schools that were not part of the actual study. The kindergartners and Primary 1 children were tested individually because they may not be proficient readers. The interviewer read the question to the child and recorded the child's response.

Scoring of tasks

To differentiate responses obtained by guessing from those based on the children's knowledge of related concepts, scores were grouped as follows. Children who obtained at least 2 points were described as *confident* about the concepts tested by the task and were classified as *successful* (S). Those who obtained one point or less were deemed to have arrived at the answer by chance, described as unsure of the concepts tested by the task, classified as *unsuccessful* (US).

Findings

Table 1 shows the performance of the K2 children with Tasks K and A and their performance in Tasks A, B and C one year later. About 65% of the children answered at least 2 of the 3 Task K questions correctly. The remaining 35% either gave the wrong answers for all three questions or answered only one of the three questions correctly in set K. Because about two-third of the K2 children were successful with Task K and the pilot study showed a ceiling effect when we tested the same task with P1 children, Task K was not repeated in P1. About 59% of the K2 children were successful with Task A. Because just over one-half of the K2 children were successful with Task A, and the pilot study showed that P1 children found this task challenging, this task was repeated in P1. One year on, 75% of the children were successful with Task A and this improvement in performance was significant with t(136) = 4.12, p < .001. This suggests that after one year, these children were better able to make the connection that a heavier object would require more cubes to maintain equilibrium. However, they found Tasks B and C, which were introduced in P1, extremely challenging as only 12% and 20% of them were successful with these two tasks respectively.

Table 1

Performance (as percentages) for Tasks K and A by Kindergartners and Tasks A, B, and C by Same Children One Year Later (n = 137)

Balance Scale Tasks -		K2		P1	
	US	S	US	S	
Task K: Heavier of two, balance in disequilibrium, abstract presentation.	35	65	NA	<i>1</i> *	
Task A: Heaviest of three, balance in equilibrium, concrete presentation.	41	59	25	75	
Task B: Heaviest of three, balance in equilibrium, abstract presentation.	N.	A	88	12	
Task C: Heavier of two, balance in equilibrium, abstract presentation.	N	A	80	20	

Note. *These tasks were not administered in these grades.

To gain further insights into the learning trajectory of the kindergarteners, their performance in Tasks K and A was placed into the three groups: (i) successful in both tasks, (ii) successful in only one task, (iii) not successful in both tasks. The performance of these three groups in the P1 tasks one year later is shown in Table 2. Children from all three K2 groups had similar rates of success in each P1 task. Chi-square tests were conducted to check statistical significance in success rates on each P1 task by the three groups of K2 children as well as two groups (group (i) versus groups (ii) + (iii), because some of the cells have fewer than five observations). All the chi-square tests were statistically not significant. Thus, K2 performance did not

58

predict performance in P1 tasks. Thus, if a child did not do well in the K2 tasks, it did not necessarily mean that he/she would also perform poorly in the P1 tasks.

Table 2 Performance (as percentages) of P1 Children in Tasks A, B, and C by Groups based on Their Performance in Two K2 Tasks

	Tas	k A	Task B		Task C	
Grouping of P1 children based on	US	S	US	S	US	S
performance in two K2 tasks						
(i) Successful in two K2 tasks $(n = 53)$	25	75	87	13	75	25
(ii) Successful in one K2 tasks $(n = 64)$	25	75	89	11	81	19
(iii) Failed both K2 tasks $(n = 20)$	25	75	90	10	85	15
χ^2 (2) between US and S for each task	.004		.209		1.021	
	(<i>p</i> =	.998)	(p = 1)	.901)	(p =	.600)

Discussion of Study 1

At P1, 75% of the children could compare and identify which was the heaviest ball because the comparison is based on comparison of concrete objects as the weight of each ball is specified by the number of cubes (Task A). However, when the weights of the objects were specified relationally as a function of each other rather than in a concrete manner (Task B), their performance fell dramatically. This difference in performance is not unexpected. When presented with such findings P1 teachers explained that although Task B was in the Teacher's Guide, it was not included in the teaching. In fact those children who participated in the pilot and were successful with Task B explained that they received extra help from their parents. This suggests that young children need inputs from competent adults to make sense of such abstract tasks. A more serious implication of this finding is that children may have even greater difficulties solving such tasks if the weight on each arm is not equidistant from the fulcrum.

Performance in P1 tasks did not depend on success or otherwise of performance in K2 tasks when the children were younger. Thus, children who did not succeed in K2 tasks could catch up with those who were successful when both groups moved to P1. This finding is consistent with the summary by Siegler and Chen (2002), who highlighted that children aged 5 and above were able to consistently apply rules to solve balance scale tasks. It appears that the P1 children in this study were applying the appropriate rules to solve the given tasks. It is possible that the curricular materials may stimulate these children to think about the working of balance scales. More research is needed to account for this change in performance.

Study 2: Primary 2 to Primary 3

Study 2 addresses the following research questions:

- 1. How well did P2 children perform with selected balance scale tasks?
- 2. How well did they perform one year later?

Method

Participants

A total of 122 Primary 2 children from five schools located in the western part of Singapore participated in this phase of the study in 2006. They were tested a year later.

Procedure

They took Tasks B and C in Primary 2 and Tasks C to E one year later. Task B was not repeated in P3 for the following reasons. Both Tasks B and C tested similar reasoning, these P2 children were more successful with Task B than Task C so it was prudent to retain the more challenging of the two tasks, and it was decided to maintain consistent structure of the K2-P1 study, i.e., to have 2 sets of tasks in the first year followed by 3 sets of tasks in the second year of testing.

The children were tested in small groups of three to four children per group at each sitting. They wrote their responses in the test booklets. No time limit was set for the completion of the tasks, and they took an average of 20 minutes to complete the tasks.

Findings

The results are given in Table 3. Task D was the least challenging of the three P3 tasks, and this suggests that children were able to apply the construct of transitivity to solve to identify which of the three objects is the heaviest and which is the lightest. The children may have found the presentation of the balances in disequilibrium useful as this representation was more concrete than the two tasks where the balances were in equilibrium.

60

Table 3

Performance (as percentages) for Tasks B and C by P2 Children and Tasks C, D, and E by Same Children One Year Later (n = 122)

Balance Scale Tasks	P2		P3	
	US	S	US	S
Task B: (Heaviest of three, balance in equilibrium, abstract presentation)	48	52	Nz	A_*
Task C: (Heavier of two, balance in equilibrium, abstract presentation)	53	47	52	48
Task D: (Heaviest of three, balance in disequilibrium, abstract presentation)	NA		32	68
Task E: Using non-standard units to determine weight of cylinder	N.	A	49	51

Note. *These tasks were not administered in these grades.

P2 children's performance in Task B and Task C. As presented in Table 3, 52% of the P2 children succeeded with Task B, while 47% were successful in Task C. Logically Task B seems more demanding than Task C because it requires identifying the heaviest of the three boxes, in addition to finding which of two boxes are heavier. However, their performance was otherwise. Further analysis shows that 84% of the children who were successful with Task C could solve Task B, while about 23% who were unsuccessful with Task C could solve Task B.

Performance in tasks C, D, and E, one year later. Table 4 compares performance in the three P3 tasks based on performance in two P2 tasks. For Task C, there was no significant improvement in the children's performance after one year, t(121) =1.164, p = .247. However, the difference in performance between the three groups is significant, χ^2 (2) = 27.58, p < .001. Proportionally more children from the better group were able to solve this task compared with those from the less successful group. This raises important epistemological questions. It could be that children's performance with Task C did not improve because they did not engage with such tasks and hence did not engage with the relevant mathematical reasoning. This suggests that the primary mathematics curriculum for the early years needs to ensure that young children are provided with such rich learning experiences. That only three-quarter of those who were successful with the two tasks succeeded with Task C a year later suggests that those who were unable to repeat their successful performance did not have a sound understanding of the concepts required of the task. It is important to be mindful that what was taught need not be retained. It is equally important to review the concepts taught, especially with young children who are continually taught new concepts.

•	Task C		Task D		Task E	
Grouping of P3 children based on	US	S	US	S	US	S
performance in two P2 tasks (i) Successful in two P2 tasks $(n = 48)$	25	75	6	94	25	75
(ii) Successful in one P2 tasks $(n = 24)$	50	50	17	83	50	50
(iii) Failed both P2 tasks $(n = 50)$	78	22	64	36	76	24
χ^2 (2) between US and S for each task	27	.58	45	.69	25	.49
	(p <	.001)	(<i>p</i> <	.001)	(p <	.001)

Table 4 Performance (as percentages) of P3 Children in Tasks C, D, and E by Groups based on Their Performance in Two P2 Tasks

Discussion for Study 2

Study 2 shows that P2 and P3 children had difficulties interpreting information presented in an abstract manner. Those who did not perform well in the P2 tasks continued to perform poorly in the P3 tasks, and this suggests that those who failed to perform in the P2 tasks did not catch up with those who were successful with the tasks one year later. One possible reason for this trend is that children did not have experiences working with such tasks in their lessons; indeed, teachers may not be aware that their pupils could be challenged by such tasks.

Conclusions

The objective of the above two studies was to establish K2 and P2 children's competencies with the balance scale tasks designed by the Curriculum Development Institute of Singapore and their progress with such tasks a year later. The tasks used in the two studies differed from those used by Siegler (1976) in that the weights were equidistant from the fulcrum in these studies, whereas Siegler manipulated both variables: weight and distance from the fulcrum. Nevertheless, responses to Tasks B and C were not dissimilar to those reported by Siegler where children used Rule 1 to solve simple-weight task. In his studies, children predicted that the arm with the most number of weights would go down. The children in this study responded in a similar manner to Tasks B and C. However, the findings from these two studies may not be generalised to all children in Singapore as the participating children attended schools located in the west part of the island. More research needs to be conducted to determine whether children from other parts of the country would respond similarly.

62

Can teaching improve children's performance in such tasks? The first author was a consultant to a team of six Primary 1 teachers who were involved in a TLLM IGNITE2 project called "L.I V. E Maths" (Tan & Sri Darmavijaya, 2009). These teachers wanted to improve the teaching of mass in Primary 1. Specifically, they wanted to incorporate experiential learning (Kolb, 1984) into the teaching of mass. As professional development was part of the project, the findings from Studies 1 and 2 were shared with these teachers who then developed a series of well-designed lessons targeted at developing P1 children's competencies with mass and including concepts needed to solve Tasks B and C. A total of 198 P1 children participated in the project. The pre- and post-test results show that the P1 children benefitted from the experience with significant improvement in their performance in Tasks B and C.

Thus, the findings from these two studies add to a corpus of research using balance scale tasks as stimuli to explore children's thinking. Consistent with results in the extant literature, these two studies show that kindergartners and lower primary children find balance scale tasks challenging. Furthermore, these two studies differed methodologically from Siegler's in that the latter study interrogated the rules used by participants to predict the behaviour of the balance scale. Because of the design of the current studies, it was not possible to identify the rules the children used to solve the problems presented in the stimuli. Findings from these two studies can be used as baseline data to inform the design of future studies to understand the rules used by Singapore primary children to solve balance scale tasks. Such studies would add new knowledge to this field of research. Knowledge such as these are crucial as Siegler's study shows that without appropriate teaching, adults continue to apply the wrong rules to solve balance scale tasks. Furthermore, these tasks are found in teacher's guides. Curriculum designers may wish to incorporate such tasks into standard textbooks so that they become more mainstream than in the status quo.

References

- Amsel, E., Goodman, G., Savoie, D., & Clark, M. (1976). The development of reasoning about causal and non-causal influences on levers. *Child Development*, 67, 1624-1646.
- Baddeley, A., & Hitch, G. J. (1974). Working memory. In G. A. Bower (Ed.), *Recent advances in learning and motivation* (pp. 47-90). New York, NY: Academic Press.
- Boom, J., & Laak, J. T. (2007). Classes in the balance: Latent class analysis and the balance scale task. *Developmental Review*, 27, 127-149.
- Case, R. (1985). Intellectual development: A systematic reinterpretation. New York, NY: Academic Press.

- Curriculum Development Institute of Singapore. (1981, 1992, 1995). Primary mathematics teacher's guide, 2A. Singapore: Federal Publications.
- Curriculum Development Institute of Singapore. (1982, 1992). *Primary mathematics teacher's guide, 1B.* Singapore: Federal Publications.
- Davydov, V. V. (1962). An experiment in introducing elements of algebra in elementary school. Sovetskaia Pedagogika, 5(1), 27-37.
- Hardiman, P. T., Pollatsek, A., & Well, A. D. (1986). Learning to understand the balance beam. *Cognition and Instruction*, *3*, 63-86.
- Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. New York, NY: Basic Books.
- Kolb, D. A. (1984). *Experiential learning: Experience as the source of learning and development*. Englewood Cliffs, NJ: Prentice Hall Inc.
- Lee, K., Bull, R., Ng, S. F. (2008, August). Age and individual differences in the development of algebraic thinking: The roles of working memory and executive functioning. Keynote address presented at the Executive Functioning Workshop, University of Oxford, UK.
- Lee, K., Ng, S. F., & Bull, R. (2011, April). The influence of executive functioning on proficiencies in mathematical patterns and algebraic problem solving: From kindergarten to upper elementary. Unpublished paper presented at the biennial meeting of the Society for Research in Child Development, Montreal, Canada.
- Lee, K., Ng, S. F., Bull, R., & Ho, R. (2009, August). Mathematical pattern recognition, working memory and proficiency in solving word problems. Unpublished invited paper presented at the meeting of the European Association for Research in Learning and Instruction.
- Lee, K., Ng, S. F., Bull, R., Pe, M. L., & Ho, R. H. M. (2011). Are patterns important? An investigation of the relationships between proficiencies in patterns, computation, executive functioning, and algebraic word problems. *Journal of Educational Psychology*, 103(2), 269-281.
- Lee, K., Ng, S. F., Ho, R., & Bull, R. (2008, July). Executive functioning and children's mathematical competence: from kindergarten to early adolescence. Poster presented at the 20th meeting of the International Society for the Study of Behavioural Development, Wurzburg, Germany.
- Lee, K., Ng, S. F., Pe, M. L., Hasshim, M. N.A.M., Ang, S.Y., & Bull, R. (2012). The cognitive underpinnings of emerging mathematical skills: Executive functioning, patterns, numeracy, and arithmetic. *British Journal of Educational Psychology*, 82(1), 82-99.
- Ministry of Education. (2006). *Mathematics syllabus: Primary 2007.* Singapore: Curriculum Planning and Development Division.
- Ministry of Education. (2000). *Mathematics syllabus: Primary*. Singapore: Curriculum Planning and Development Division.

- National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: Author.
- Siegler, R. S., & Chen, Z. (1998). Developmental differences in rule learning: A microgenetic analysis. *Cognitive Psychology*, 36, 273-310.
- Siegler, R. S., & Chen, Z. (2002). Development of rules and strategies: Balancing the old and the new. *Journal of Experimental Child Psychology*, 81, 446-457.
- Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive Psychology*, *8*, 481-520.
- Siegler, R. S. (Ed.). (1978). *Children's thinking: What develops?* Hillsdale, NJ: Lawrence Erlbaum.
- Siegler, R. S. (1981). Developmental sequences within and between concepts. Society for Research in Child Development Monographs, 46, (Whole No. 189).
- Tan, Y. H., & Sri Darmavijaya (2009, December). L.I.V.E. Maths! Learning through varied experiences/environments. A TLLM Ignite2 Project. (Available from Curriculum Matters South, an S4 Cluster Publication).

Authors:

Ng Swee Fong [corresponding author; sweefong.ng@nie.edu.sg], National Institute of Education, Nanyang Technological University, Singapore; **Madeline Lee Pe**, Department of Psychology, University of Leuven, Belgium.