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# Textbooks and Cultural Traditions: A Comparative Case Study of Berlin and Hong Kong

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**Abstract:** This study compared the textbooks and their underlying intended curricula and traditions in Berlin and Hong Kong using the Rezat model. The German educational philosophy was characterised by the humanistic-oriented approach, while the Hong Kong education was strongly influenced by the Confucian heritage culture (CHC) and the Anglo-Saxon curriculum tradition. Differences in the textbooks include linguistic characteristics and use of history of mathematics, which are related to the intended curriculum. The findings support the notion of the dependence of the textbook curriculum in mathematics on cultural traditions.

**Keywords:** Mathematics textbooks; Berlin; Hong Kong; Confucian heritage culture; German educational philosophy

## Introduction

Many studies, such as Haggarty and Pepin (2002), Howson (1995), and Park and Leung (2006), reveal a strong dependence of mathematics textbooks on cultural traditions in different countries. Some researchers also assert that textbooks reflect a nation's cultural values. For example, Apple (1992) stated that "texts are not simply 'delivery systems' of facts'. They are the simultaneous results of political, economic, and cultural activities, battles, and compromises" (p. 4). Castell, Luke, and Luke (1989) shared a similar view and suggested that "the school textbook holds a unique and significant social function: to represent to each generation of students an officially sanctioned, authorized version of human knowledge and culture… textbooks form shared cultural experiences, at times memorable and edifying, while at others eminently forgettable and uneducational" (p. vii). In this study, mathematics textbooks in Berlin and Hong Kong were investigated to explore the similarities and differences of their structures and how cultural traditions might influence them. The key research problem was: If mathematics education is culturally embedded, what are the cultural and intellectual underpinnings that influence the junior secondary mathematics textbooks in Berlin and Hong Kong?

## Rationales for Textbook Analysis

Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) considered textbooks as mediating between the intended and implemented curriculum and, as such, textbooks are important tools in today's classrooms. It is a common perception that textbooks play an important role in the daily school activities. Traditional commercial textbooks dominate mathematics curriculum materials and thus have a large influence on teaching practices (Goodlad, 1984). Textbooks basically set out in details suggested teaching contents and practices, which are usually based on the broad principles and objectives stipulated in the intended curriculum, for teachers to follow in classrooms. Textbooks are thus outputs of the authors' interpretation of the curriculum guides of the program of study designed by the education authorities, and textbooks are not necessarily an embodiment of the intended curriculum. On the other hand, textbooks are not straight manifestation of the implemented curriculum. Whilst taking textbooks as reference, teachers always exercise their own discretion in choosing which topics therein are to be covered in their course of teaching and deciding the approaches in teaching particular topics. As such, a more precise exposition of textbooks is that they act as a bridge between the intended curriculum and the implemented curriculum.

Owing to the fact that each country rarely has a very large number of mathematics textbooks, a set of textbook covers a fair amount of the student population (Park and Leung, 2006), whereas there is a wide distribution of implemented curricula since teachers vary enormously in their capabilities (Clements, 2002). Sutherland, Winter, and Harries (2001) suggested that pupils' construction of knowledge could not be separated from the multifaceted external representations of this knowledge which envelope the learning pupils. This implies that textbooks, as one such external representation, can influence and "shape" students' mathematical knowledge (Healy & Hoyles, 1999), and therefore it is important to study them. Some researchers indicated that textbooks affect both what and how teachers teach (Robitaille & Travers, 1992). The TIMSS study (Beaton, Mullis, Marin, Gonzales, Kelly & Smith, 1996) found that textbooks were a main written source that teachers used to plan and conduct their lessons. Textbook is an essential teaching aid for teachers in most if not all classrooms.

## Curriculum Background in Germany and Hong Kong

### *Germany*

There are 16 federal states in Germany, and the educational system is not centrally dominated. In general, the power to make policies in most areas is vested with the parliament (Bundestag) and the central government (Bundesregierung), and these central policies are applied to every federal state. However, federal states are left with some areas in which they are empowered to make their own policies, and education is one of these areas. This leads to a situation that each federal state has its own educational system and its own guidelines for curricula and school forms, though there is commonness in quite many aspects. In this regard, textbook publishers have to develop different textbook series separately for every federal state or at least a textbook series compatible for those federal states with a large degree of commonness in their educational systems.

The German educational system is especially distinguished by a strong and early segregation of students by performance levels based on assessment results. After primary schooling, teachers and parents then decide where to place the students within the tripartite secondary school system, which are called Hauptschule, Realschule, and Gymnasium. In general, students in Hauptschule focus mainly on vocational and technical training, while in Gymnasium academics is much more important, and Realschule falls in-between. Although the core curriculum for mathematics in Germany is similar for all pupils, the mathematics is treated in different ways, with proof and cognitive challenge being important in the Gymnasium and the use of algorithms to apply mathematical ideas being emphasised in the Hauptschule and Realschule (Haggarty & Pepin, 2002).

Haggarty and Pepin (2002) suggested that Germany espouses mainly humanistic views, based on Humboldt's ideal of humanism, combined with naturalistic tendencies (Haggarty & Pepin, 2002, p.588). Humboldt's concept of *Bildung* searches for "rational understanding" of the order of the natural world. Gravemeijer and Terwel (2000) suggested that *Bildung* refers to the ideal of personality formation, and it entails not only simply the transmission of knowledge, but also the development of the knowledge, norms, and values associated with "good" citizenship and/or a membership of the cultural and intellectual élite.

For German educational philosophy in mathematics, Kaiser (1999) suggested that it can be characterised by the development of two approaches. The first one is the humanistic-oriented approach, which aims for a general education. Mathematics is a means for promoting general abilities like logical thinking. Emphasis is placed on the understanding of structures and general principles. Active work through examples is of relatively low importance. The understanding of structures is seen to be more important than deep knowledge in single areas. The second approach is the realistic-oriented approach for the masses. It emphasises a utility principle and preparation for future life. Lui and Leung (in press) noted that the intended curriculum followed a realistic-oriented approach.

***Hong Kong***

Hong Kong, once a British colony, was thus influenced by the Anglo-Saxon curriculum tradition. It is now a Special Administrative Region of the People's Republic of China. The population of Hong Kong is predominantly ethnic Chinese. Although some Hong Kong people have adopted Western lifestyles, a substantial number of them still adhere to traditional Chinese morals in various aspects of living. Leung (1999) wrote: "The Chinese are also known to place high emphasis on education. This can be explained by the Confucian view of education. All Confucian-influenced places such as Japan, Korea, Singapore, Taiwan and Hong Kong share a similar view" (p. 244). Lee (1996) pointed out that the Chinese emphasis on education "rests upon the Confucian presumption that everyone is educable" (p. 28). Confucius acknowledged that there are individual differences in intelligence, but he believed that "differences in intelligence… do not inhibit one's educability" (Lee, 1996, p.29). The aim of education is not the pursuit of knowledge for knowledge's sake, but the development of the character of the learner (Leung, 1999).

The Chinese placed great importance on applicability of mathematics. For example, the aim of the *Jiuzhang Suanshu* or *Nine Chapters on the Mathematical Art*, which is a Chinese classic mathematics book and contains 246 mathematical problems, is to find a general solution or method to solve problems. This may be contrasted to the ancient Greek mathematicians, who tried to deduce propositions from axioms and postulates. Wang and Sun (1988) analysed the development of mathematics in China from prehistoric times to the Yuan Dynasty. They concluded that there are six major characteristics of ancient Chinese mathematics: pragmatic, mystical, algorithmic, numerical and discrete, primitive dialectics, and conformity to orthodoxy.

## Method

This study focused on the textbook curriculum. Textbook curriculum refers to the curriculum materials provided by the schools to teachers and students. The model of Rezat (2006) was adopted because his classification of textbooks covers most, if not all, of the characteristics suggested by previous researchers (e.g., Howson, 1995). Rezat (2006) defined the five microstructural levels of mathematics textbooks to be:

- characteristics in terms of content
- linguistic characteristics
- visual characteristics

- pedagogical functions
- situative conditions

Characteristics in terms of content refer to the structure elements in the textbooks. They include definitions, theorems, procedures, algorithms, and exercises. This paper dealt with the structure of the content rather than the choice of content itself. Linguistic characteristics refer to the level of language that is appropriate for students and for their age group and how concise the language is. Visual characteristics refer to the visual elements in the textbooks. These include the followings: the main theorems are boxed or clearly structured, and the examples are somehow highlighted. Pedagogical functions refer to functions such as "engaging the students actively," "informing," and "practicing" (Rezat, 2006). This general definition is of little help in the comparison of textbooks, as virtually all textbooks purport to have goals such as "engaging the students actively" in mind. The crux of the problem is thus the approach the author has taken, and how effective it is in attaining various pedagogical functions. Pedagogical functions can be achieved in various ways by a combination of different features, and in this study, we will call these ways "pedagogical features". The notion of situative conditions is twofold. According to Rezat (2006), "[I]t comprises expected activities as well as suggested contexts of use such as the introduction of new subject matter or homework" (p. 483). It may include puzzles that help students discover mathematics on their own.

The best-selling textbook series in Berlin and Hong Kong were selected for study. However, sales information is not easy to obtain because the publishers would not like to disclose their market share. There are different textbooks for each school type in Germany, and in this study we looked at textbooks used in the Gymnasium. Choosing a Gymnasium textbook was justified because the content of the Gymnasium covers all the components suggested in the intended curriculum. The textbook series, *Elemente der Mathematik*, was chosen from one of the major German textbook publishers, Schrödel. This textbook series was one of the most popular textbook series used in Berlin and this piece of information was provided by a researcher in Freie Universität Berlin. For Hong Kong, the textbook series *Exploring Mathematics* published by the Oxford University Press was chosen. Its author, who is also the second author of this paper, knew that this textbook series had the largest market share in Hong Kong.

The analysis of the selected textbooks was conducted by the first author and checked by the second author.

## Findings

The results of the analysis are presented below according to the five parts of the Rezat model.

***Characteristics in Terms of Content***

The contents in the textbooks were stipulated in the intended curriculum. The students in both cities were required by their respective education authorities to learn similar topics in Algebra and Number, and Probability and Statistics (Lui & Leung, in press). However, the intended curriculum in Berlin did not place much emphasis on learning geometry through a deductive approach or analytic approach, while the students in Hong Kong had to perform simple proofs related to different geometric figures.

Haggarty and Pepin (2002) found that all mathematics textbooks in Germany were clearly structured into mainly two parts: introductory exercise/s and the main notion, followed by an extensive range of exercises. Similar results were obtained by Howson (1995), who found strong similarity in the way that mathematics was presented in Grade 8 textbooks in those countries in Asia, Europe, and North America that he studied. A standard pattern is:

- introductory activities or examples;
- exploration in detail of a generic example;
- presentation of kernel: definitions, procedures, etc.;
- consolidation through abstract and contextualised exercises and problems.

This pattern was also found in *Elemente der Mathematik* and *Exploring Mathematics*.

*Berlin: Elemente der Mathematik*

Every chapter in *Elemente der Mathematik* has a similar structure. It starts with an artificially posed problem, and then relevant definitions, theorems or information are given. It then illustrates how to solve the problem step by step very clearly, and there are exercises at the end. For example, the following is the introductory example for solving inequalities (p. 32):

> When I multiply a number $x$ by 7 and then add 8 to it, what are the possible values for $x$ when the result is less than 36?

The mathematical meaning that solving inequality is to find its solution set is stressed. The solution is presented clearly. In solving the inequality $7x + 8 < 36$, the importance of the solution set in each step being the same is emphasised. It illustrates, graphically, the reversibility of the algebraic manipulations used in order to preserve the solution set. Then, the algebraic manipulations allowed are justified by the preservation of the solution set:

> The solution set in the three inequalities is the same. The three inequalities are therefore equivalent.

The presentation of the solution set also deserves discussion. The solution does not just stop at $x < 4$. The solution $\{x \in Q: x < 4\}$ is expressed in set notation. This is not only mathematically rigorous but also serves to remind the students the meaning of solving inequality, which is to find the solution set rather than just to work on a series of algebraic manipulations. As real numbers have not yet been introduced in the 8th grade, the solution set is for only rational numbers. The textbook author does not use mathematical terms which have not been defined. After giving the relevant definitions, theorems or information, it gives another example and illustrates how to solve a more specific type of problem step by step, namely, the kind of inequality with negative coefficient (p. 35). Three different ways to get rid of the negative coefficient in $41 - 3x < 35$ are then presented.

To sum up, the presentation of the mathematics content in the Berlin textbook is very concise. In most of the sections, there are two examples followed by a theorem or some information. There is little motivation using daily situations other than the mathematics itself. This may be inconsistent with the aim of the intended curriculum in Berlin, which stresses the applications of mathematics. The mathematical contents in the textbooks are rigorous and presented in a logical way. The techniques used are often explained clearly in detail.

*Hong Kong: Exploring Mathematics*

The basic structure of each chapter is clearly stated in the Preface of this series. This includes *Chapter Opener* (which guides students to appreciate mathematics through a beautiful picture), *Previous Knowledge* (which lists all the basic knowledge for revision purposes), *Content of Sections* (which includes content, class activity, class exploration, class discussion, quick example, example, basic practice, class practice and exercise), *Supplementary Exercise*, *Challenging Problems* (which include problems requiring high order thinking skills), *Chapter Summary*, and *Revision Test* (multiple choices). To cater for learners' differences, the questions are classified into two types. "Level 1" questions are easier and "Level 2" questions are more difficult. The following is an example of a section, taken from "Change of subject of a Formula" in Chapter 4.

This section starts with the formula $A = lb$, which is already familiar to the students. A diagram of a rectangle with length *l*, breadth *b* and area *A* is shown. It states: "*A* is called the subject of the formula. It means the variable *A* stands alone on one side of the equal sign and does not appear on the other side." This definition is not mathematically rigorous but it uses a language that students may be able to understand. It then changes the subject of the formula to *b*, i.e., $b = \frac{A}{l}$. It reminds students that *l* cannot be *0*. This section uses an example to introduce a mathematical idea. Before giving further examples, it notes that "the method of changing the subject of a formula is similar to that of solving equations."

There are five examples. Two examples are "level 1" questions and three are "level 2". The two level 1 questions are (2A, p. 111):

Make *x* the subject of the formula $y = 3x + 4$.

Change the subject of the formula $H = (a - b)x + (a + b)y$ to $a$.

How each step follows the other is explained. For $y = 3x + 4$, students have first to minus 4 from the both sides and then divide by 3. For the second question, each step is worked out clearly and descriptions are written on the right. Students are to "put all the terms containing $a$ on the same side of the equal sign," "(t)ake out the common factor $a$ on the right-hand side" and then "(d)ivide both sides by $x + y$ " to arrive at $a = \frac{H+bx-by}{x+y}$.

The first two level 2 questions are (2A, pp. 111-112):

Make *u* the subject of the formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Change the subject of the formula $c = \frac{b(a+c)}{b+d}$ to *d*.

The lens formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ does not have a subject. The textbook provides a solution without any descriptions. It assumes that students can work on their own after reading the two previous examples. Then it provides an alternative method.

1. Multiply both sides of the formula by *fuv*.
2. Put all the terms containing *u* on the same side of the equal sign.
3. Take out the common factor *u* and divide both sides by $v - f$.

Three steps are written in words and students are required to work on the formula on their own.

For the second question, a solution is provided without explanation. It is then followed by Class Practice to "help(s) students integrate all the knowledge in a whole section" (2A, Preface, viii). The final example in this section is the word problem below:

Suppose Rose is driving a car and she speeds up steadily from $u$ m/s to $v$ m/s in $t$ seconds. Then the distance travelled $d$ m is given by the formula, $d = \frac{(u+v)t}{2}$.

(a) (i) Change the subject of the formula to $t$.
(ii) Change the subject of the formula to $v$.

(b) (i) If Rose speeded up from 20 m/s to 60 m/s in 500 m, how long did this take?
(ii) If Rose was driving at 10 m/s and speeded up over the next kilometer in 1 minute, what was her speed at the end of the minute?

This problem lets students apply the mathematics they have just learnt to an everyday situation. As mentioned in the Preface, it "helps students understand that mathematics is not merely a collection of abstract ideas and complicated formulae, but a practical subject closely related to everyday life". Detailed solutions are given without explanation. In part (b), a note tells students to substitute the numbers in the original formula to solve the equation in $t$ or put $t$ in the formula they get in part (a)(i). The section ends with two pages of exercises. The analysis of the exercises is given in the section "pedagogical features" below.

***Linguistic Characteristics***

According to Haggarty and Pepin (2002), the language demands in the textbooks in Germany were high, both in terms of mathematical vocabulary and symbols. They suggested that the emphasis was on the abstractness of mathematics and its structure to educate pupils' minds. This study agreed with their findings. The language used in *Elemente der Mathematik* is very precise. The mathematical terms used are very rigorous and the logical connections between statements are very clear. For example, on page 14:

Associative law of addition
For all $a,b,c \in Q$, we have $a + b + c = (a + b) + c = a + (b + c)$.
In words: in a sum of three or more terms, the brackets can be put anywhere. This does not change the results.
Commutative law of addition
For all $a,b \in Q$, we have $a + b = b + a$.
In words: the terms in a sum can be changed. This does not change the results.
For example,
$12xy + 48xz + 19xy$
$= 12xy + (48xz + 19xy)$ using associative law of addition
$= 12xy + (19xy + 48xz)$ using commutative law of addition
$= (12xy + 19xy) + 48xz$ using associative law of addition
$= 31xy + 48xz$ adding the like terms.

The use of rational numbers in the arithmetic laws is consistent throughout the book because real numbers have not been introduced in the 8th grade yet. The textbook not only uses mathematical symbols or the set language to state the arithmetic laws, it also presents the statements in words. This is rigorous and no ambiguity would result. In the above example, the laws used in each step are stated so that the students know how each step follows from the previous one. In general, it lets the students be aware of what properties they are using. The laws used are explicitly mentioned and the language used shows rigour in mathematics. Our findings are similar to Haggarty and Pepin's (2002) with respect to mathematical logic and structure.

The languages used in *Exploring Mathematics* are easy for students to understand. However, the ways of defining terms are not always rigorous. For example, on page 49: "In the polynomial $4x^2 - 5x + 6$, 4 is the coefficient of $x^2$, -5 is the coefficient of $x$, 6 is the constant term of the polynomial." It is then followed by examples. There are no formal definitions for "coefficient" and "constant," and students are expected to understand the terms through the examples.

### *Visual Characteristics*

In *Elemente der Mathematik*, there are graphs or diagrams to help students understand the problems. It puts boxes around the theorems, definitions, and information. Sometimes there are pictures which do not add much educationally, but they do "lighten" the presentation. The textbook uses a lot of illustrations and photos, mainly to show some geometric shapes and solids. There are far fewer illustrations and photos in algebra, due to its nature.

In *Exploring Mathematics*, theorems or definitions are also boxed. Different colours are used to highlight different kinds of important information. There are numerous pictures and illustrations showing the applications of mathematics in daily life. A relevant photo is usually given when an application is mentioned. For example, there are photos on daily applications such as the meters used in taxis (p. 151).

### *Pedagogical Features*

This section covers three pedagogical features: *exploration in detail of a generic example, presentation of kernel,* and *history of mathematics.*

#### *Exploration in Detail of a Generic Example*

One pedagogical function of a textbook is to explain how a mathematical idea is derived. We illustrate the differences in how the two textbooks deal with the derivation of the same distributive law.

In *Elemente der Mathematik*, the distributive law (p. 40) is introduced by asking students to work on an example:

> Expand $3(4x + 2y)$ using the distributive law $a(b + c) = ab + ac$.

The solution is given directly, and the law is then explained in words:

> One multiplies each term in the bracket by the factor.
> The law is written in two ways:
> $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

In this case, the textbook assumes the students can work on $(a \pm b)c = ac \pm bc$ because in the exercises that follow, similar questions are asked. For example, question 4$g$ (p. 41) requires students to expand $(1- u^2)$•4 .

In contrast, *Exploring Mathematics* has an introductory activity asking students to cut a rectangle with length $a + b$ and width $c$ into two rectangles with width $c$, of lengths $a$ and $b$ (2A, p. 58). The sum of the areas of the two small rectangles is equal to the area of the original rectangle. This activity is not a mathematical proof but is used to illustrate the distributive law graphically. The law is presented in two different ways:

> $c(a + b) = ca + cb$ and $(a + b)c = ac + bc$.

This illustrates the distributive law for only positive numbers because the lengths in the rectangle should be positive. That the distributive law also holds for negative numbers is not mentioned. In the examples or exercises, the laws are used for polynomials, and students may not know under which circumstances the law holds.

*Presentation of Kernel*

The "grammar" of mathematics is the language of proofs or mathematical reasoning. It could be implicit in a simple formula or explicit as in deductive geometry. On the dimension of mathematical rigour, it ranges from intuitive, heuristic arguments to rigorous mathematical arguments with no leaps of faith. The treatment of mathematical proofs in the two textbooks is different due to discrepancies in the intended curriculum.

In *Elemente der Mathematik*, the chapters on algebra focus on solving equations or working on polynomials rather than proving identities. The chapters on geometry stress the construction of diagrams and the calculation of the areas of figures and volumes of prisms rather than on an axiomatic or deductive approach. Nonetheless, in explaining how a geometric figure is constructed, mathematics is not just taught procedurally. For example, on page 79, how a parallelogram is drawn is explained. After the construction of a parallelogram, it mentions that a parallelogram can be divided into two congruent triangles. It also gives a proof on why the two triangles are congruent. The proof is formal but students are not required to work on formal proofs in the exercises. The chapter stresses geometric construction rather than mathematical proof. Formal proof is not required in the intended curriculum, and so students are not expected to learn formal proof.

Under the intended curriculum in Hong Kong, students have to use a deductive approach to work on geometry problems covering the topics below:

- Develop an intuitive idea of deductive reasoning by presenting proofs of geometric problems relating with angles and lines.
- Explore and justify the methods of constructing centres of a triangle such as in-centre, circumcentre, orthocentre, centroid, etc.
- Pythagoras' Theorem.
- Extend the idea of deductive reasoning in handling geometric problems involving quadrilaterals.
- Deduce the properties of various types of quadrilaterals but with focus on parallelograms and special quadrilaterals.
- Perform simple proofs related with parallelograms.

In *Exploring Mathematics*, there are proofs of theorems, and students have to write their own proofs. For examples, they have to know the properties of different kinds of triangles such as equilateral triangle and isosceles triangle, and the relationship of angles in a figure with parallel lines (Chapter 8, 2B). Students not only have to find an angle in a figure, they also have to prove, for example, whether a line is a straight line or whether two lines are parallel (Chapter 9, 2B). The logical arguments are stressed in each step. In the discussion, the story of Euclid and his work *Elements* are introduced (pp. 56-58, 2B). The limitations of using an intuitive approach to study geometry are explained and the deductive approach is introduced. Motivation is given and mathematics is taught using logical arguments.

*History of Mathematics*

One of the key pedagogical functions that differentiate the two textbooks is the treatment of the history of mathematics. Introducing the history of mathematics in textbooks enables students to know the cultural value of mathematics and from where a mathematical idea is originated. It can also be considered a motivation device to introduce certain topics. Indeed, Howson (1995) noted that most texts gave "a very limited view of the nature of mathematics: of its history and multicultural origins, and of its existence as a living and expanding discipline dependent on human endeavour and on two main driving forces — the solution of societal problems and the meeting of intellectual challenges" (pp. 87-88).

There is no formal treatment of the history of mathematics in *Elemente der Mathematik*, but it includes 16 classical problems as exercises and the introduction of the method of Gaussian elimination in solving system of linear equations in more than two variables (p. 161-163). All of them are problems on algebra, and they provide an opportunity for students to work on some aspects of the history of mathematics.

In *Exploring Mathematics*, the history of mathematics appears under different proofs of Pythagoras' Theorem and the historical attempts to construct different geometric figures. Pythagoras' Theorem (2B, p. 110) is taught with historical notes. The notes first introduce the belief of the Pythagoreans, i.e., all quantities are rational. Then they tell the story about a Pythagorean question: what is the length of the diagonal of a unit square? Finally, they mention that in the 19th century, mathematicians gained considerable understanding of irrational numbers. Besides telling students that irrational numbers are the set of all infinite non-repeating decimals, background on the existence of irrational numbers is discussed. In the appendix, different proofs of Pythagoras's Theorem from different cultures are given, including the proof by a US president and one from ancient India. The multi-cultural nature of mathematics is clearly presented.

*Exploring Mathematics* also mentions how ancient Greek mathematicians used only minimal tools to construct various geometric figures (2B, p. 36). In the appendix, the problem of trisecting an angle with a pair of compasses and straight-edge and the historical attempts to construct regular polygons are introduced (2B, A7). It also mentions that Gauss proved a number of theorems identifying which regular polygons are constructible by using these tools. Although the discussion is brief, it enables students to have an idea of the origin of a mathematical idea. This fosters an appreciation of "the cultural aspect of mathematics" as suggested in the intended curriculum.

***Situative Conditions***

Situative conditions comprise expected activities, suggested contexts of use such as the introduction of new subject matter or homework, and puzzles to help students discover the mathematics on their own.

The introductory sections in *Elemente der Mathematik* begin with artificially posed problem. They do not begin with a situative exercise or a modelling problem as intended by the curriculum developer. Here is an example. Chapter 7 begins with an experiment about forming the image of a candle with convex lens. There is a picture showing the experiment. At first glance, it may be regarded as a modelling work: finding different image distances with different given object distances. However, the formula for image distance $e = \frac{5x}{x-5}$ is given without letting the students know how it is deduced. There is no modelling example in the book, and this is not in accordance with the intended curriculum which places emphasis on modelling. What students have to do is to substitute the numbers given into the formula, and after a few trials, they will find that for $x = 5$, the image distance is undefined.

Four chapters (Chapters 1, 6, 9, and 10) in *Exploring Mathematics* begin with a "chatty" problem illustrated in cartoon style. For example, in Chapter 1, *Approximation and errors*, basic concepts of significant figures are illustrated through a picture showing two girls wanting to join a tour to Europe costing \$12 350. The first girl approximates the amount to \$12 000 and thinks it is expensive, whereas the second girl approximates it to \$10 000 and thinks it is not. This introduces the concept of significant figures.

## Cultural Underpinnings of Textbooks

The aims of comparative studies include highlighting the relationships between education and society and recognising the strengths of one approach by looking critically at others. This section deals with our key research question: If mathematics education is culturally embedded, what are the cultural and intellectual underpinnings that influence the junior secondary mathematics textbooks in Berlin and Hong Kong?

### *Berlin*

It seems that the underlying beliefs on "what is mathematics?" of the two textbooks are similar. The textbook authors in Berlin and Hong Kong both agreed that mathematics is a human activity and hence they introduced the history of mathematics in the textbooks. However, the approaches that they introduced it are different. The 16 classical problems introduced in *Elemente der Mathematik* are not "big" problems that students cannot solve. Indeed, they should be able to solve these historical problems themselves and will understand the mathematics behind them. The textbook invites students to walk the full steps of the mathematicians in the past. As mentioned before, the main goal of education in Germany is to help the students to acquire *Bildung*. It is a state in the process of the acquisition of and the dealing with cultural objects and personality development. In the Berlin textbook, working on the historical problems may be a way to enable the students to have a glimpse that mathematics is multi-cultural in origin (Howson, 1995, p.88).

The Berlin textbook is very rigorous and logical, and mathematics is presented in a formal way. Real numbers are not used before a formal introduction in higher grades. This is probably the practice of German mathematicians such as Gauss and Hilbert where the axiomatic approach influences the presentation of the textbooks. Everything, including theorems, structures and symbols, has to be defined formally before it is used. While not denying the cultural and historical roots of mathematics, the textbook authors present mathematical concepts in a mature and rigorous form. Mathematics is a means for promoting general abilities like logical thinking. Emphasis is placed on the understanding of structures and general principles. Our findings agree with those of Haggarty and Pepin (2002): in German textbooks, mathematics is seen as a structured and pre-discovered body of immutable and "true" knowledge, a static discipline developed abstractly.

The topics and the contents in *Elemente der Mathematik* are consistent with the intended curriculum. However, the intended curriculum also stresses the applications of mathematics and this can be considered as a realistic-oriented approach in teaching and learning mathematics. There is not much motivation in the Berlin textbook on daily situations other than the mathematics itself. As mentioned earlier, the examples in the textbook are not situative; instead they emphasise the mathematics itself, for example, substituting values into variables. In this aspect about applications of mathematics, the textbook does not agree with the intended curriculum. The belief in *Elemente der Mathematik* is that mathematics has a life of its own and is best developed through procedures, as illustrated by the derivation of the distributive law. There is no detailed explanation or proof. Students have to follow the instructions in the examples.

***Hong Kong***

With a Confucian heritage culture (CHC), Hong Kong has adopted the Anglo-Saxon curriculum. The author of the Hong Kong textbook expects students to understand mathematics not just for its applications but also as part of a culture. As mentioned earlier, Wang and Sun (1988) concluded that one major characteristics of ancient Chinese mathematics was its pragmatic nature. On the other hand, Wong (2004) explained that one general description of the CHC classroom is that teachers see "the moral responsibility of providing individual care, including those not directly related to learning (e.g., personal growth and transmission of cultural values)" (p. 7). Learning the history of mathematics is a transmission of cultural values even though it does not directly relate to the contents. *Exploring Mathematics*, similar to *Elemente der Mathematik*, also emphasises the cultural aspect of mathematics but uses different approaches. *Exploring Mathematics* introduces different proofs of Pythagoras' Theorem in different cultures and the historical constructions of geometrical shapes. Historical remarks serve almost purely as motivations for the subject that might be historically significant but are not logically essential to the development of the ideas in the textbook. Students are therefore not required to understand or work on the problems. Nonetheless, the historical background of certain topics is given and it enables students to understand the multi-cultural nature of mathematics.

In *Exploring Mathematics*, activities are sometimes given to lead to "proofs" or justification of the laws concerned. This is illustrated by the activity used to introduce the distributive law described earlier. This may be influenced by the Western ideas that laws have to be proved or justified. Hong Kong was influenced by the Anglo-Saxon curriculum tradition, and the textbook author has grown up under this influence.

## Conclusion

The two textbooks in Berlin and Hong Kong are found to be influenced by their cultural traditions and the intended curriculum. Both traditions seem to share the humanistic orientation. The two textbooks include the history of mathematics so that the students may appreciate the aesthetic nature and cultural aspects of mathematics. While the Berlin textbook places strong emphasis on logical and rigorous, the Hong Kong textbook aims to justify mathematical laws at students' level of language and rigour. To conclude, the findings of this study support the notion of the dependence of the textbook curriculum in mathematics on cultural traditions.

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