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The Development of Number Sense Proficiency: An Intervention Study with Year 7 Students in Brunei Darussalam

Palanisamy K. Veloo
Wawasan Open University, Malaysia

Abstract: This paper discusses the effect of an instructional intervention on the development of number sense of a sample of 210 Year 7 students. Before intervention, the level of number sense among the students was low. Students were highly rule bound in their approaches and were extremely inflexible in applying these rules when attempting to answer the test questions. However, after the instructional intervention, *Treatment* students' number sense was significantly higher than that of the *Control* students. The *Treatment* students were beginning to use their existing knowledge of numbers and operations in more flexible ways. Several students' misconceptions were due to learning of rules by rote and over-generalization of partially understood algorithms in solving test items. The findings will inform teachers and curriculum developers to better appreciate how students learn and think during school mathematics lessons.

Keywords: Number sense; Brunei Darussalam; Mental computations; Rasch model

Background and Literature Review

Reforms in the school mathematics curricula in many countries in recent years have emphasised the need for teachers to provide instruction that leads to conceptual understanding, in particular, for students to develop number sense (Australian Council of Education, 1991; Hong Kong, 2000; National Council of Teachers of Mathematics, 2000).

Number sense refers to a pupil's general understanding of numbers and operations, along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments (McIntosh, Reys, Reys, Bana, & Farrell, 1997). The NCTM document *Curriculum and Evaluation Standards for School Mathematics* (2000) defines number sense as the ability to decompose numbers naturally, use the relationships among arithmetic operations to solve problems, understand the base-ten number system, estimate, make sense of numbers, and recognise the relative and absolute magnitude of numbers. Number sense results from a whole range of activities that permeate the entire approach to the teaching of mathematics (Greeno, 1991).

Many researchers have lamented the fact that students in general demonstrate little to no number sense when called upon to do so (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980). The TIMSS video study (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, et al., 2003) reveals that the type of instruction that leads to meaningful learning and development of number sense was rarely found in the participating countries. Students in these countries experienced what is termed traditional instruction which is characterised by teaching for skill efficiency. Teachers often resorted to drill and practice leading students to learn rules by rote and without knowing why or how the rule works. Rote learning of these traditional paper-and-pencil algorithms can interfere with the development of number sense and can lead to the formation of a defective schema (Case, 1989; Thompson, 1999; Sackur-Grisvard & Leonard, 1985). Instruction that leads to conceptual understanding is needed for the development of number sense (Hatano, 2003; Hiebert and Grows, 2007). This paper discusses the effects of an instructional intervention on the development of number sense proficiency of a sample of Year 7 students in Brunei Darussalam.

For this Year 7 class, the mathematics content areas that served as the basis for identifying number sense and mental computation proficiencies were *whole number and operations*, *fractions*, *decimals* and *percents*. These topics were taught and reviewed in the first semester of the school term at Year 7 to ensure that students were well grounded in their basics. This made it easier for teachers to comply with instructions from the researcher on how the intervention lessons should be conducted in their classes.

Methodology

Sample

The subjects of the study were 210, Year 7 students (12 and 13 years old) from four co-educational government secondary schools. The four schools were randomly chosen from "average performing" government secondary schools in Brunei Darussalam. Two schools were randomly assigned as the *Treatment* schools and the other two as the *Control* schools. The top two intact classes of students from each of these schools were selected for the study. All participating students were considered of "average mathematical ability" based on personal and academic background data obtained from each student as part of the number sense investigation. Table 1 shows the distribution of students in the *Treatment* and *Control* groups.

Table 1
Distribution of students involved in the study by gender

School	Boys	Girls	Number of Students
Treatment	59	59	118
Control	42	50	92
Total	101	109	210

Students from each group were divided into *High, Middle,* and *Low* sub-groups based on their total score on Number Sense Test 1 (*NST1*), administered as a pretest. Students in the *High* group scored above 60% in *NST1*; the *Middle* group between 45% and 60%; the *Low* group below 45%. This classification was made in order to study the effect of intervention on each group of students.

Instructional Approaches
Teachers in the *Control* classes were required to teach the content areas as agreed to. They taught the content areas as they had usually done in the past using the textbook (Leong et al., 2003). No assistance of any kind was given to these teachers.

Five instructional units were prepared for use in the *Treatment* classes. These were (a) review lessons on mental computation and exercises, (b) whole numbers and operations, (c) fractions, (d) decimals, and (e) percents. Detail about the teaching of these units is described below.

Mental Computation Unit
The mental computation unit in the *Treatment* class comprised 30 10-minute lessons building up to and including adding and subtracting two 2-digit numbers, multiplying by powers and multiples of 10, and multiplying by 2, 4, 5, 10, 20, 25, and 50. There were also lessons on halving a number and dividing a number by 4, and finding 5%, 10%, 15%, 20%, 25%, and 50% of a given quantity.

Whole Number Unit
This *Treatment* unit comprised six lessons. It focused on whole numbers and structure of numbers and operations. The students were challenged to use their existing knowledge of numbers in more flexible and intuitive ways; for examples, 357 = 2H 15T 7O; 87 + 54 = 90 + 51; 67 + 29 = 67 + 30 – 1; 86 – 38 = 86 – 40 + 2; 6 × 49 = 6 × 50 – 6. The lessons focused on reinforcing students' conceptual understanding of place value, representation of numbers in different equivalent forms, number structure, and the four operations.

Fraction Unit
This *Treatment* unit reviewed the part-whole meaning of fractions using geometric shapes and discrete sets. Students were shown visual representation of a series of unit fractions on fraction charts and asked to write their own conclusions about the size of fractions and to explain their answers. Using visual representations, students were led to see the size of each fraction in terms of its distance from one whole. Similar reasoning was also used to determine size of fractions near a half. The use of benchmarks in comparing fractions was introduced and its use in estimating results of simple additions, multiplication, and division was discussed. Students were also encouraged to mentally add and subtract two like fractions or related fractions.

For the *Control* class, this unit began with the meaning of fraction as part of a whole. This was followed by a comparison of unit fractions and non-unit fractions. The idea of equivalent fractions was introduced before non-unit fractions with different denominators were compared by first expressing the given fractions into equivalent fractions having common denominators and then comparing the numerators. The density of fractions, the use of benchmarks, and comparing non-unit fractions using benchmarks were not explicitly stated in the Brunei mathematics textbooks, and it was left to the teacher to develop these ideas and concepts on their own.

Decimal Unit
This *Treatment* unit focused on the meaning of tenths, hundredths, and thousandths using decimal pieces. Materials were used to help students compare decimals such as 6.7 and 6.49. Decimal numbers involving tenths, hundredths, and thousandths were compared. Decimal numbers between two given decimals (density of decimals) were discussed using a number line. The effect of the operation of multiplication and division when the multiplicand or the divisor was less than one was discussed. The relationship between decimals and fractions such as 0.5 and ½ was explored. Estimation of answers involving decimals and the four operations were discussed.

The Lower Secondary mathematics textbooks developed the meaning of decimals starting from fractions, for example, $^1/_{10}$ is written as 0.1 or a tenth, and $^1/_{100}$, and so on. To compare decimals with unequal decimal places, students were encouraged to annex zeros to the decimal number with fewer places to obtain the same number of decimal places as the other decimal number; for example, to compare 0.2 and 0.12, students were to annex a zero to 0.2 to make it 0.20 and then compare 0.20 with 0.12. The idea of the density of decimal numbers was implied in the local textbooks but not explicitly dealt with. To add decimal numbers with different number of decimal places, students were to annex zeros to obtain decimal numbers with the same number of decimal places so that when the number were arranged vertically, the decimal points fell in a line. To multiply decimal numbers, students were taught the rule: when you multiply two decimal numbers, first drop the decimal points; multiply the numbers as whole numbers, and then insert the decimal point in the answer to give the same total number of digits after the decimal point as there were before multiplication. The textbooks do not deal specifically with the estimation of answers when a number is multiplied or divided by a decimal number smaller than or greater than one.

Percent Unit
This *Treatment* unit used a "100 square" to discuss the meaning of percent. Connections between tenths, hundredths, and their percentage equivalents as well as relationships between fractions, decimals, and percents were discussed; for examples, ½, 0.5 and 50% are different ways of representing the same quantity.

Instrumentation
Three number sense tests, *NST1*, *NST2,* and *NST3* were constructed from a number sense item bank which was constructed and validated by the writer using the Rasch model. The item bank consisted of 113 items, and it was part of a larger study that investigated number sense and mental computation proficiencies of Year 7 students in Brunei Darussalam. Researchers have posited various characteristics or dimensions of number sense (Resnick, 1989; Sowder, 1991). The number sense tests for the study comprised the following five dimensions of number sense:

1. *Understand the meaning of numbers and operations*. This implies an understanding of the base ten number system (McIntosh, Reys, & Reys, 1992).

   Which of the numbers below is another way of writing 2 hundreds + 14 tens + 9 ones? Circle your answer. A. 249 B. 259 C. 2149 D. 349 *(Q2/NST1)*

2. *Recognise the relative effect of operations on numbers*. This refers to the ability of a student to recognise how each of the four basic operations affects computational results.

   Without calculating the exact answer, circle the best estimate for 59 × 0.09. The answer would be:
   A. A number which is very much less than 59.
   B. A number which is a little less than 59.
   C. A number which is a little more than 59.
   D. A number which is a very much more than 59. *(Q15/NST1)*

3. *Judge the reasonableness of computational results*. This refers to the ability of an individual to apply estimation strategies to problems without using written computation (McIntosh, et al., 1992) and at the same time judge the reasonableness of a result of computation. See Table 5, Q14.

4. *Use of benchmarks in computation*. This includes the ability to recognise 0, ½, and 1 as benchmarks. By using 1 as a benchmark, for example, a student might recognise that the sum of $^7/_8$ and $^9/_{10}$ should be slightly less than 2, because each fraction is slightly less than 1. See Table 5, 9.

5. *Understand and use equivalent expressions in computations*. This implies the ability to recognise that 28 × ½ and 28 ÷ 2 are equivalent, or finding 50% of a quantity is the same as finding half of the quantity.

   0.5 × 840 is the same as: A. 840 ÷ 2 B. 5 × 840 C. 5 × 8400 D. 0.50 × 84
   (circle your answer) *(Q1/NST1)*

A 15-item written computation test (WCT), with items that paralleled the number sense items was also constructed. The following are some examples of questions from the WCT.

7. Add: $\frac{7}{8}+\frac{12}{13}$ *(Q20/NST1)*

11. Add: 715.347 + 589.2 + 4.553 = *(Q3/NST1)*

12. All goods in a shop are being sold at a discount of 20%.What is the new price of a shirt that costs $55? *(Q26/NST1)*

Administration of Tests

All students were administered the 30-item MCQ number sense pre-test (*NST1*) and the 15-item written computation test (WCT) a couple of days later. On completion of 10 weeks of instructional intervention, they were given the 25-item number sense post-test, *NST2*. Six weeks after the instructional intervention, all the students were given the 30-item number sense retention test, *NST3*. They were each given a test booklet containing the number sense items. The test administrator read an item to the students once, and they were given about 60 seconds to respond to that item. Students responded by circling one of the four options given for each item. When answering the number sense questions, students were asked to do all their working and reasoning in their head. For the WCT, they were given sufficient time to complete the test. Both the *Treatment* and *Control* classes were given the tests in the same week. Forty-five students from the *Treatment* group were interviewed before and after the instructional intervention.

Analysis of Data

The Rasch model was used to compute students' number sense proficiencies in logits. Mean scores in logits of the *Treatment* and *Control* groups were used to compare whether there were any differences in number sense proficiencies of the two groups before and after the instructional intervention.

Results

Entries in Tables 2, 3 and 4 show the mean number sense proficiencies in logits, standard deviations, effect sizes, *t*- and *p*-values of students' performance of the *Treatment* and *Control* groups by the *High*, *Middle* and *Low* sub-groups on the three tests. Entries in Table 2 show that there were no significant differences in the pre-intervention number sense proficiency between the *Treatment* and *Control* groups and all sub-groups. Thus, the two groups were comparable in their number sense proficiencies before the start of the instructional intervention.

Table 2
Mean scores (logits), standard deviations, t- and p-values with respect to NST1 for High, Medium and Low students from the Treatment and Control groups

NST1 (Pre-test)	Treatment Group Means and SD (*N = 118*)	Control Group Means and SD (*N = 92*)	*t*	*p*
Overall	– 0.63 (0.63)	– 0.72 (0.56)	1.10	0.27
High *(n_t=26; n_c=19)*	0.22 (0.50)	0.03 (0.51)	1.23	0.28
Middle *(n_t=46; n_c=38)*	– 0.59 (0.31)	– 0.68 (0.30)	1.36	0.18
Low *(n_t=46; n_c=35)*	– 1.15 (0.33)	– 1.18 (0.26)	0.40	0.69

Table 3 shows that the number sense proficiency of the *Treatment* students was statistically higher than that for the *Control* students in general and all sub-groups, and the effect sizes were large for all sub-groups. For the retention test, *NST3* (Table 4), both groups showed a slight decline over their post-test. However, the *Treatment* group still performed statistically significantly higher than the *Control* group.

Figures 2 and 3 show the changes in the students' performance from the pre-test to the retention test. Overall, the number sense proficiency of the *Treatment* students improved significantly after the instructional intervention and they held on to most of their gains 6-weeks after the intervention ended. The number sense proficiency of the *Control* students did not change in significant ways during this period. Thus, the instructional intervention was effective in improving *Treatment* students' number sense proficiency.

Table 3
Mean scores (logits), standard deviations, t- and p-values and effect sizes with respect to NST2 for High, Medium and Low students from the Treatment and Control groups

NST2 (Post-test)	Treatment Group Means and SD (*N = 118*)	Control Group Means and SD (*N = 92*)	Effect Size	*t*	*p*
Overall	– 0.09 (0.61)	– 0.64 (0.45)	1.03	7.49	<.001***
High (n_t=26; n_c=19)	0.37 (0.60)	– 0.06 (0.39)	0.85	3.08	0.002**
Middle (n_t=46; n_c=38)	– 0.09 (0.52)	– 0.64 (0.28)	1.32	5.82	<.001***
Low (n_t=46; n_c=35)	– 0.35 (0.72)	– 0.96 (0.32)	1.09	6.18	<.001***

*** statistically significant, $p < 0.001$

Table 4
Mean scores (logits), standard deviations, t- and p-values and effect sizes with respect to NST3 for High, Medium and Low students from the Treatment and Control groups

NST3 (Retention test)	Treatment Group Means and SD (*N = 118*)	Control Group Means and SD (*N = 92*)	Effect Size	*t*	*p*
Overall	– 0.13 (0.58)	– 0.78 (0.33)	1.38	8.29	<.001***
High (n_t=26; n_c=19)	0.32 (0.53)	– 0.04 (0.33)	0.82	2.63	0.005**
Middle (n_t=46; n_c=38)	– 0.17 (0.55)	– 0.84 (0.31)	1.50	6.61	<.001***
Low (n_t=46; n_c=35)	– 0.35 (0.49)	– 1.11 (0.40)	1.70	7.43	<.001***

*** statistically significant , $p < 0.001$, **$p < 0.01$

Note: (n_t represents number of Treatment students in that sub-group and n_c represents the number of Control students in that sub-group)

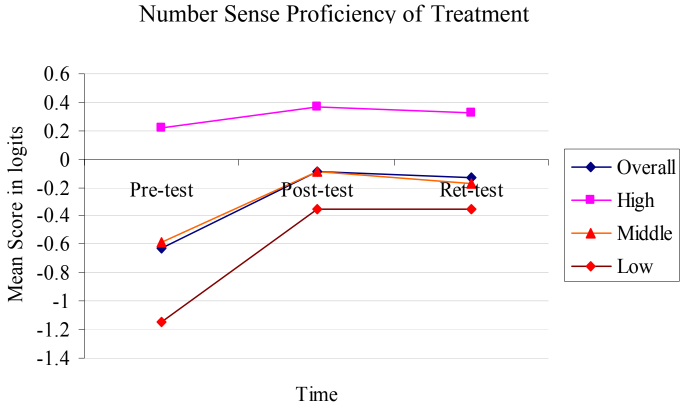

Figure 2. Number-sense mean scores of pre-test, post-test, and retention test: *Treatment* group

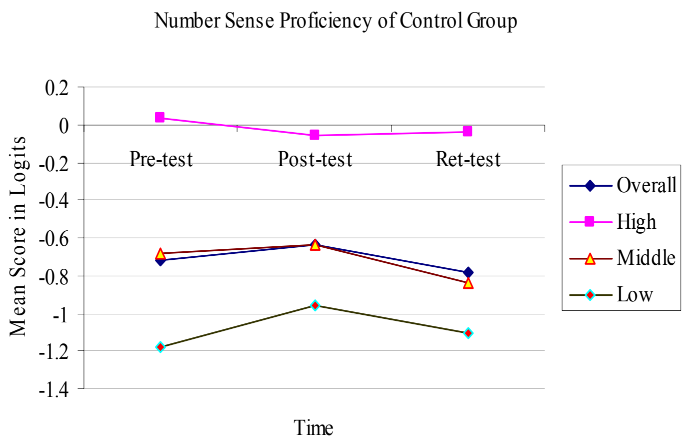

Figure 3. Number-sense mean scores of pre-test, post-test, and retention test: *Control* group

Students' Performance on a Selection of Items Common to NST1 and WCT

Item analysis of students' performance on common *NST1* and *WCT* items was conducted. Table 5 shows the frequency distribution for the whole sample (n = 210) and percentage of correct and incorrect responses of students on selected items common on *NST1* and *WCT*. A majority of these Year 7 students obtained significantly higher scores on the *WCT* than on the common items on the number sense tests. This shows that being able to apply rules and algorithms to correctly compute the answer to a question does not necessarily imply mathematical understanding or strong number sense.

Question 2 was designed to see if students were able to flexibly use different representations of numbers to estimate the product of two decimal numbers. About 77% of the students answered this question correctly on the *WCT*, but only 20% could estimate the answer on the *NST1*. This item could have been solved by first thinking of 0.25 as ¼, the product as one quarter of about 4000, and arrive at option D as the correct answer. The most popular choice was (B. 10.58), chosen by about 36% of the students, because they focussed on the number of decimal places that should be in the product. In their mathematics classes they had been taught this rule for decimal multiplication: "the total number of decimal places in the product should equal to the sum of the decimal places in the numbers being multiplied" (multiplicand and multipliers). This explanation was confirmed by the interview data. When asked to choose the correct answer to the multiplication: 15.24 × 4.5 from four given options (6.858, 68.58, 685.8 and 0.6858), most of them chose 6.858, which relied on the multiplication rule mentioned above. The application of such a rule requires no understanding and this often leads to incorrect answers.

Table 5
Frequencies, and percentages of correct and incorrect responses on common item on NST1 and WCT ($n = 210$)

	Pre-test Freq.(%)	WCT Freq. (%)
Q2 What is 0.25 × 4232? Circle your answer.		
A. 1.058	41 (19.5)	Correct: 162 (77.1)
B. 10.58	75 (35.7)	Incorrect: 48 (22.9)
C. 105.8	53 (25.3)	
*D. 1058	*41 (19.5)	
Q4. Circle the row in which the numbers are arranged in order of size, starting with the smallest to the largest.		
A. 0.595 ; $^3/_5$; 61% ; 0.3 ; 30.5%	23 (11.0)	Correct: 91(43.3)
*B. 0.3 ; 30.5% ; 0.595 ; $^3/_5$; 61%	*60 (28.6)	Incorrect: 119 (56.7)
C. 0.3 ; 0.595 ; $^3/_5$;30.5% ; 61%	86 (41.0)	
D. $^3/_5$; 0.3 ; 0.595 ; 30.5% ; 61%	41 (19.5)	
Q9. Without calculating the exact value, decide which of the possible numbers is closest in value to the sum of these two fractions. $\frac{7}{8}+\frac{12}{13}$		
A. 1	16 (7.7)	Correct: 99 (47.1)
*B. 2	*19 (9.0)	Incorrect: 111 (52.9)
C. 19	75 (35.7)	
D. 21	100 (47.6)	
Q10. Without calculating the exact answer, circle the best estimate for 81 ÷ 0.09		
A. A number which is very much less than 81.	64 (30.5)	Correct: 70 (33.3)
B. A number which is a little less than 81.	78 (37.1)	Incorrect: 140 (66.7)
C. A number which is a little more than 81	39 (18.6)	
*D. A number which is a very much more than 81.	*29 (13.8)	
Q14. The following addition has been correctly carried out except for placing the decimal point: **715.347 + 589.2 + 4.553 = 13091** Which of the following numbers show the decimal point in the correct place?		
A. 1.3091	26 (12.4)	Correct: 176 (83.8)
B. 13.091	89 (42.4)	Incorrect: 34 (16.2)
C. 130.91	54 (25.7)	
*D. 1309.1	*41 (19.5)	

Note: The option with an asterisk (*) indicates the correct answer.

In Question 4, about 60% of the students chose either option C or D. The interview suggests that these two answers were chosen because the students considered percentages to be “like” whole numbers and not fractions, that is, 30.5% is like 30.5 and 61% is like 61.

For Question 9, over 47% of students were able to correctly compute the answer to $\frac{12}{13}+\frac{7}{8}$, but only 9% knew its estimated value of 2 on the number sense test.

Question 10 tested whether students could recognise the effects of dividing a whole number by a decimal divisor whose value is close to zero. About 33% answered this item correctly on the *WCT* and 14% on the *NST1*. Over 67% of students chose either *A* (*very much less than 81*) or *B* (*a little less than 81*), and the interview suggests that many students thought of division as "making a number smaller." This misconception was so well entrenched especially among students in the *Low* sub-group and it was difficult to change it even after the instructional intervention.

Question 14 tested the use of estimation to determine the sum of three decimal numbers, two of which had 3 decimal places and one with 1 decimal place. About 84% of the students correctly computed the exact answer to this question on the *WCT*, but on the *NST1*, only 20% could make the correct estimation. About 42% chose the incorrect option (*B:* 13.091). From the interview, it was clear that many students chose this option by focusing solely on the number of decimal places in the addends. They reasoned that if decimal numbers with 3 decimal places are added then the sum should also be a decimal number with 3 decimal places. This result once again shows the negative effect of learning rules without understanding.

Discussion

Rule Learning, Estimation, and Number Sense

The finding that Year 7 students performed significantly better on written computation test items than on parallel items on the number sense test suggests that they may have learned the algorithm for computation but may not necessarily have acquired an appreciation of the value of the fractions or decimals involved, which is a characteristic of number sense. Thus, fluency in written algorithms does not necessarily imply number sense proficiency. Many students, especially those in the *Middle* and *Low* sub-groups, were unsure that in computations one could move freely (or switch) from one form of representation to another. There is also evidence from research studies that emphasis on standard paper-and-pencil procedure can interfere with the development of number sense (Markovits & Sowder, 1994; Narode, Board, & Davenport, 1993; Thompson, 1999). As Sowder, (1988) pointed out, "correct answers in written computation are not a safe indicator of good thinking" (p. 227), and this was evident from this study. This could result from the strong emphasis on students obtaining correct answers through standard paper-and-pencil procedures as practised in Bruneian secondary mathematics classes.

Effectiveness of Instructional Intervention

The effectiveness of the instructional intervention was demonstrated through statistical significance and the large values of effect size between the *Treatment* and *Control* groups; see Tables 3 and 4. It is argued here that the improved performance of the *Treatment* students was brought about by the differences in content emphases between the instructional units designed for this study and the more traditional classroom instruction found in the *Control* classes which were usually textbook-oriented.

The instructional units on fractions, decimals, percentage, and mental computation did not contain contents that students had not studied in their primary schools. The pre-intervention number sense proficiency scores inform us that the exposure to these topics in primary schools had not been very successful for most of these students. Their responses to the number sense items during interview indicate that prior to instructional intervention, they were very dependent on application of rules and algorithms that they had memorised and internalised.

Demonstrating an understanding of the effect of operation on whole numbers and decimals is an indicator of number sense proficiency (McIntosh, Reys, & Reys, 1992). This study found that many students continued to accept and apply faulty or incorrect reasoning despite having been specifically taught the defining characteristics of the relevant concept. This feature can be described as “fossilised misconception” or an “absence of cognitive change over time” or even “resistance to change over time.” Cognitive inertia persists, despite the individuals having been taught the “proper” view of the concept (Vaiyavutjamai, 2004). The persistence of ideas such as, “multiplication makes bigger,” “division makes smaller,” and “when multiplying two decimal numbers insert the decimal point in the product to give the same number of digits after the decimal point as there were before,” was quite striking among the students in the *Low* group. The reason could be that the students relied heavily on the domain of whole numbers, perhaps treating the whole numbers as a paradigmatic model for any set of numbers (Graeber & Tirosh, 1990). Although some of the students in the *Middle* group were relatively quick in recognising that in an extended domain of numbers their assumptions about division did not hold, those in the *Low* group continued to hold on to the conviction that characteristics of the division with whole numbers should also apply in the domain of rational numbers. Even though they could perform the algorithm for division by a decimal less than one and a fraction less than one, during classroom instruction, some of them still believed that “division makes smaller” more firmly than the result of their calculation.

Concluding Remarks

This study set out to investigate if students who had completed their primary schooling under traditional instruction could be led to use numbers in more meaningful and flexible ways.

At the beginning of secondary schooling, the number sense proficiency of students was relatively low. The emphasis on developing standard written algorithms in primary schools did not bring about practical understanding for dealing with whole numbers, decimals, fractions, and percentages in meaningful ways. Their understanding of fractions, decimals, and percentage was limited to surface features of the concepts (Ma, 1999). Many of the students could not compare and order fractions, failed to demonstrate an understanding of the density of decimal numbers, and were unable to apply benchmarks in estimation and computation problems. These are consistent with results obtained in other number sense studies (McIntosh, et al., 1997). Furthermore, the students were very much dependent on memorised procedures. These methods are not only tedious, but they can be performed with little or no understanding of the underlying concepts and principles (Markovits & Sowder, 1994).

The three sets of number sense measures and effect size values suggest that the instructional intervention was effective in raising the number sense proficiencies of the *Treatment* students. The amount of changes seen among the *Treatment* students in this study, after limited instructional intervention, was not because of learning of new concepts and ideas, but rather existing knowledge was used in different ways, namely, intuitive notions of number were called to the surface and new connections formed. This hypothesis is in keeping with Markovits and Sowder's (1994) view that number sense is a well-organised conceptual network of number information that enables one to relate number and operations to solve problems in flexible and creative ways. Their view of strengthened conceptual networks could be used to explain why retention was generally high among the *Treatment* students and why some of the concepts were at times resistant to change. When strong links are formed it is far more likely that information will be retained and accessible. When the topics was taught with only brief, perhaps cursory coverage, as may be the case during the instructional intervention in this study involving the *Low* group, there is less likelihood for strong links to be formed (Porter, 1989). On the other hand, where robust knowledge and procedures already exists, such as "multiplication make bigger," and "division makes smaller," there is less likelihood of change because the prior knowledge is already accessible. According to Hiebert (1988), it is more difficult for students to acquire the semantic-based processes (conceptual understanding) once they have routinised syntactic processes, that is, they have memorised and practised rules until the rules are automatic and can be executed with little cognitive effort.

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Author:

Palanisamy K. Veloo, School of Education, Languages and Communications, Wawasan Open University, Level 3 Menara PGRM, 8 Jalan Pudu Ulu, Cheras, 56100, Kuala Lumpur, Malaysia; pkveloo@gmail.com