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Exercising Sociomathematical Norms in Classroom Discourse about Data Representation: Insights from One Case Study of A Grade 6 Lesson in Indonesia

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Abstract: This paper examines a research study to foster mathematical discourse about data representations among Indonesian students. It was situated in the context of implementing an Indonesian version of Realistic Mathematics Education, labelled as PMRI, in primary schools. A case study of one lesson involving Grade 6 students on the choice of data representations in Yogyakarta will be discussed. The analysis focused on the enacted social norms and sociomathematical norms during a whole-class discussion and their impacts on students' knowledge of data representations. The need for constant effort to enact these norms in classroom mathematical discourse is highlighted.

Keywords: Discourse; Reasoning; Communication; Data representation; PMRI

Introduction

A call to put more attention to encourage explicit communication of thinking in learning has been advocated over the past few decades. Extensive studies have documented a strong interest on accounts of discourse to foster mathematical thinking in the classroom (Barwell, 2003; Cobb, Boufi, McClain & Whitenack, 1997; Ryve, 2011; Sfard, Nesher, Streefland, Cobb, & Mason, 1998). This growing interest is often attributed to the influence of the Vygotsky's idea of language as a cultural tool and the increased awareness of the role that social interactions play in learning. Having a strong emphasis on explicit communication of thinking in mathematical learning is quite often associated with the "reform-inspired pedagogy" whereby the negotiation of mathematical meanings is co-constructed by students and a teacher in supporting students' mathematical development (Sfard, 2000). In this paper, the analysis of classroom discourse will be grounded in interaction and exchanges between students and the teacher.

Establishing and maintaining a productive mathematical discourse is challenging and demanding for both teachers and learners (Brodie, 2007; Chazan & Ball, 1999). White (2003) noted the importance of valuing students' ideas, and building on them in shaping the classroom discourse is critical. Martino and Maher (1999) pointed out that teachers need to be active listeners to incorporate students' ideas and to stimulate students' further thinking during a classroom discourse. Similarly, Chazan and Ball (1999) underscored a critical role of teachers in guiding and managing classroom discourse by attending to and respecting students' varying viewpoints and ideas including their disagreements. In general, classroom discourse is classified as either productive or unproductive. Discourse that revolves around personal opinions or disputes is regarded as unproductive in mathematical learning. In contrast, a productive mathematical discourse entails active listening, critical examination of each other's reasoning and arguments, and articulation and reflection of own mathematical solution and thinking (Moschkovich, 2007; Sfard, 2000).

The notions of social norms and sociomathematical norms (Yackel & Cobb, 1996) are valuable in understanding and interpreting mathematical learning process, especially when students are engaged in classroom discourse. Yackel and Cobb (1996) introduced the terms social norms and sociomathematical norms to refer to regularities in patterns of social interaction in learning. Social norms refer to the normative expectations of classroom interactions in any subject matter. Actions such as challenging others' thinking, justifying students' own interpretations to the whole class are considered social norms as these norms are not unique to mathematics (see p. 460). In contrast, sociomathematical norms refer to more specific norms pertinent to the normative aspects of discussions on students' mathematical activity. What counts as mathematically different or acceptable mathematical explanation and justification or mathematically efficient solutions are considered as sociomathematical norms.

In Indonesia, there has been an increased awareness to create an engaging and meaningful learning process by establishing and practising sociomathematical norms in learning mathematics. One of the reform initiatives in Indonesia known as PMRI was implemented a decade ago as an adaptation of *Realistic Mathematics Education* (*RME*), a seminal approach inspired by Freudenthal's (1983, 1991) notion of mathematics as a human activity. PMRI calls for a pedagogical approach with an emphasis on communicating the thinking process in learning mathematics using real-world contexts (Sembiring, Hoogland, & Dolk, 2010). Students are guided by the teacher to solve mathematical problems in multiple ways and to share their ideas and strategies in class. Group work and whole class discussion are utilised to engage students in a collaborative work to construct mathematical ideas together. This mode of learning requires a different set of classroom norms and culture. Communication of thinking and making sense of each others' thinking are valued as a critical aspect in learning mathematics and students are expected to exercise this in the mathematics learning process. This paper covers a study of classroom mathematical discourse of Grade 6 students in one lesson on various ways to represent data. It will examine how the dynamics of classroom mathematical discourse with respect to the enacted sociomathematical norms affects students' mathematical knowledge on data representations.

Literature Review

Classroom Mathematical Discourse

Many reform documents advocate students' active participation in communicating their thinking to support their mathematical development (Cobb et al., 1997; Kazemi & Stipek, 2001; Khisty & Chval, 2002; Sfard, 2001). However, it is simplistic to assume that participation in the classroom mathematical discourse will directly imply learning. Communication and exchanges of ideas during a classroom discourse do not necessarily result in learning. Cobb et al. (1997) highlighted the importance of reflection after participating in classroom discourse. They argued that reflection enables students to re-examine and re-organise their ideas and their learning experience which contributes in mathematical learning. Chazan and Ball (1999) pointed out the role of alternative ideas and disagreement among students as a catalyst for students to re-visit and re-examine their ideas. In line with Cobb et al. (1997), Chazan and Ball (1999) noted the essential function of reflection in disagreement for productive classroom discourse. Disagreement without reflection is not considered to support learning in a productive way.

Teachers play a critical role in creating "a context for argument" whereby classroom norms such as communicating solutions to others and justifying arguments, and challenging others' reasoning are exercised (Chazan & Ball, 1999; Cobb et al., 1997). Schoenfeld (2002a, 2002b) also underscored the significant role of teachers in shaping students' understanding of mathematics through a carefully crafted design and plan. Furthermore, Schoenfeld argued that productive exchanges among students are not a spontaneous act but rather a reflection of consistent practice of a classroom discourse community. Similarly, Sfard (2000) called for teachers to take an active role in guiding and managing the focus of the classroom discourse on mathematics. In mathematical discourse, the main message of the lesson should not be that every opinion or argument is equally valid. She concurred with Yackel and Cobb (1996) that mathematical discursive habit and the norms underlying classroom discourse are constantly being negotiated and re-shaped by the students and their teacher.

Sociomathematical Norms

The social norms guide the class on the normative expectations of interactions such as the need to not only provide an answer but also explanations that lead to the answer. In mathematical learning, "sociomathematical norms" signify what counts as mathematically different or acceptable mathematical explanation. Yackel and Cobb (1996) claimed that engaging students in the discussion about "what constitutes mathematically different supports higher-level cognitive activity" (p. 464). Social norms and sociomathematical norms are accepted as a useful tool in analysing and mathematical learning in classrooms (Gravemeijer, 2010; Sfard, 2000).

Graveimejer (2010) related sociomathematical norms to the tenet of "vertical mathematizing" in RME, a didactical principle that concerns with progressions from the less to the more sophisticated mathematical processing. The enacted sociomathematical norms guide students to exercise responsibilities in deciding what can be considered as mathematically different solutions and also what counts as a more sophisticated solution. Furthermore, he argues "that sociomathematical norms lay the basis for the intellectual autonomy of the students, as it enables them to decide for themselves on mathematical progress" (p. 49).

Translation of Norms into Indonesian Classrooms

In the context of implementing PMRI, a classroom norm is one of the critical aspects in classroom pedagogy that needs to be established at the beginning. A practice of having students to voice a chorus answer is commonly practiced in Indonesian classrooms. As noted in Gijse (2010), when a teacher asks a question in Indonesian primary school classroom, it is common that students will shout out "Me, me, me!" (p. 20). This practice creates a "lively" classroom atmosphere but it does not place a responsibility for students to communicate their understanding or lack of understanding. Cultivating a simple norm like raising hands before answering a question enables students to listen to each other better. Students learn to take turns in sharing their ideas and in listening to each other (Sembiring, Hoogland, & Dolk, 2010). Another challenge in Indonesian classrooms is to change the common practice whereby the teacher gives final judgement to students' answers as right or wrong. In PMRI classrooms, teachers are encouraged to invite students to take an active role in making this decision by means of teacher's prompts and other students' questions; they are guided to determine if their answers are acceptable. The detailed analysis of a few classroom episodes in the later section will illustrate this practice in a Grade 6 classroom.

Changing these social norms and sociomathematical norms in Indonesian classrooms requires a strong commitment from both the teacher and students to participate differently as a community. Sociomathematical norms are translated in the classroom by having students to solve problems in multiple ways. A whole class discussion is normally used as a medium to discuss and compare different solutions and to work out which solution is the most efficient one. It is a challenging practice for both teachers and students because in many classrooms these norms are not yet accepted as part of the classroom culture. An example reported by Dolk, Widjaja, Zonneveld, and Fauzan (2010) revealed that the teacher's intention to be kind by complying with students' constant reliance did not result in the development of thinking in students. In this case, group work became less effective as students trusted only the teacher to confirm whether they were on the right track or not. Students should work and discuss their ideas with their peers and be less dependent on teachers. It is realised that fostering such changes is not easy for teachers and students as it creates a tension in the classroom. To date, the collaborative work with some Indonesian teachers reveals the capacity of teachers and students to engage in interactive mathematical classroom discourses that support the growth of students' mathematical thinking and communication skills (Widjaja & Dolk, 2010; Widjaja, Dolk, & Fauzan, 2010). In this paper, I will discuss a research study resulting from a collaborative work with an experienced PMRI teacher in Yogyakarta. It should be noted that in this classroom, the social norms and sociomathematical norms were practiced by the teacher and the students for about one year prior to this lesson.

Method

Setting and Participants

The case study involved one lesson with 27 Grade 6 students from a private school in Yogyakarta. This lesson was carried out on 29 August 2009, part of a sequence of lessons from mid-August to early September 2009. The school is a small private school, one of PMRI pilot schools in Yogyakarta that had worked closely with Sanata Dharma University since 2001. The teacher, Ms. Hanna, was a young teacher and she had been involved with the PMRI project since 2002 when she first joined the young talents program as a teacher trainee from Sanata Dharma University. She had consistently implemented the PMRI approach in her lessons. She gave feedback to the design of activities including the phrasing of the problem, arranged student grouping, and participated in regular research meetings. The researcher was a non-participant observer during the lesson, assisted by two technical assistants who video-recorded the lesson.

The Lesson

This lesson was selected because it featured an interactive classroom discourse during a whole class discussion whereby students shared their ideas and made sense of each others' ideas. Prior to this lesson, the students collected data based on an experiment, conducted outside the classroom, of measuring the amount of water flowing through two plastic bottles with different holes in 10 seconds. For each bottle, they collected the water three times. During data collection, the teacher assigned two groups to demonstrate the process of pouring and measuring the amount of water flowing over two plastic bottles in 10 seconds and one student to measure the time using a digital stopwatch (Figure 1). Meanwhile, the rest of the class recorded data for their own groups. Having collected the data, the students worked in five groups (3-5 students per group) to devise posters that recorded and represented the data. At the end of the 1-hour lesson, students were expected to explore the rate of the water flow (known as "debit" in Indonesian). Videotapes of episodes of students' interaction during the gallery walk and the whole class discussion were taken and students' work presented on posters was recorded using a still digital camera.

Figure 1a *Figure 1b*

Figure 1. Hands-on experiments of measuring rate of water flow

Research Questions and Data Analysis

The research questions were:

1. How do students and the teacher enact social norms and socio mathematical norms during classroom discourse about data representations?
2. How does the classroom discourse about data representations affect students' interpretations of what constitute different data representations?

The research dataset consisted of the videotapes, the transcripts of classroom discourse, and copies of group work and posters. The dialogue was translated from the Indonesian language by the author to English. In order to check the accuracy of the translation, a double translation process was carried out by the author. The dataset was searched to obtain evidence to answer the two research questions.

Findings and Discussions

Gallery Walk

Each of the five groups offered more than one way of representing the data about the water flow as observed in group posters during the gallery walk at the beginning of this lesson. Table 1 presents the list of representations from the different groups. Representations such as tables and bar graphs were among the most common representations found in the posters as illustrated by the samples in Figure 2. These representations are taught starting from Grade 4 in Indonesia; hence, it is not surprising that students at Grade 6 are familiar with them.

Group 1 gave four representations consisting of a table and three different variations of bar graphs (see Figure 2a and Figure 3). Groups 2, 3, and 5, offered different picture graphs. Group 2 presented their data using a picture of water drop whereby each water drop represented 10 millilitres of water as shown in Figure 2b. The pictures of water drop were arranged in groups of 10 to denote data collected in three experiments of 430 millilitres, 470 millilitres, and 370 millilitres.

Table 1
Representations offered by various groups

Group (number of group members)	Representations
Group 1 (4 students)	• Table • Bar graphs (3 variations, see Fig 2a, Fig 3)
Group 2 (4 students)	• Table • Picture graph of water symbol (see Fig 2b)
Group 3 (4 students)	• Table • Picture graph of water symbol (see Fig 2c)
Group 4 (3 students)	• Table • Pie charts
Group 5 (4 students)	• Table • Bar graph • Picture graph of tree diagram (see Fig 4)

During the whole class discussion, some students raised questions about the extension of the water drop symbol to represent data involving larger numbers, which will be discussed later. One of the representations proposed by Group 3 appeared to look like a curved line graph. The author classified this as a picture graph instead. Group 3 divided their graph into two parts: the first top three rows on the left hand corner for data of water flowing through the small hole and the three last rows on the bottom right hand corner represent data of water flowing through the big hole as shown in Figure 2c. However, the reasons behind the choice of a curve line to represent the data were not clear. Group 5 chose to represent the data using a tree diagram (Figure 4); this will be discussed in detail later.

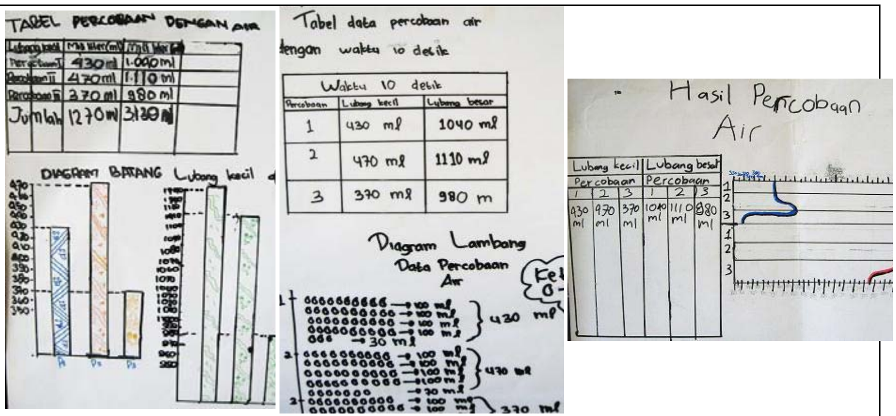

Figure 2a *Figure 2b* *Figure 2c*

Figure 2. Samples of posters from Group 1, Group 2, and Group 3

At the beginning of the gallery walk session, the teacher explained that each group should have a representative to stand by the group's poster and explain group's choice of representations and reasoning to others. The class was given 15 minutes to examine each poster during the gallery walk. Having finished with one poster, the students were expected to move on to the next poster so in average the expected time to spend on one poster was about 3-5 minutes. The first social norm that the teacher articulated prior to the gallery walk was that everyone was responsible to understand other groups' choice of representations as they visited the poster. The second social norm was that the visitors were supposed to ask clarifying questions when they did not understand the posters. The third classroom norm expected that the representative of each group standing by the poster was responsible to answer visitors' questions by explaining the reasons behind their choices of representations. Communicating these social norms explicitly encouraged students to engage actively with each other during this gallery walk session. The teacher also took part in the gallery walk session but she made it clear that the authority to explain and to ask question rest on the students. She made deliberate attempts to focus her attention in understanding students' posters and in listening to students' interactions.

A variety of comments and questions were recorded during the gallery walk session. The comments and questions covered a variety of responses from informative questions to clarify missing information on the graphs to more evaluative questions about connections between data and choice of representations. As noted earlier, Group 1 presented three variations of bar graphs, one of which was shown in Figure 2a and the other two bar graphs were shown in Figure 3. A comment on the superficial feature of the representations such as "This graph does not include any description or label of your experiments" was pointed out to Group 1. This comment highlights the importance of labelling data to help others in reading their graphs. Group 1 followed up by adding labels to record their data. Another remark examined the mathematical feature of representations in terms of the accuracy of the proportions of the graph elements, and it was recorded during an interaction involving students from Group 5 with the Group 1 representative. One student questioned the unconvincing proportion of the size of the bars used to show the difference in volumes of the first experiment (430 millilitres) and the second experiment (470 millilitres); see the circled part of Figure 3. However, the Group 1 representative only explained by a simple and short comment "for variations" without further elaboration. One possible explanation for this response is that this group interpreted the teacher's instruction to represent data in multiple ways without sound mathematical basis behind their choice of representations. In this example, the question posed to Group 1 highlighted an inaccuracy in proportions of the representations. Yet the response was not mathematically sound. Lack of reference to mathematical ideas by the Group 1 representative concurred with the point made by Chazan and Ball (1999) that "mere discussion does not necessarily generate learning" (p. 7). After the gallery walk, the teacher started the whole class discussion. This whole class discussion is a vehicle for the teacher to guide students' development of mathematical ideas and arguments.

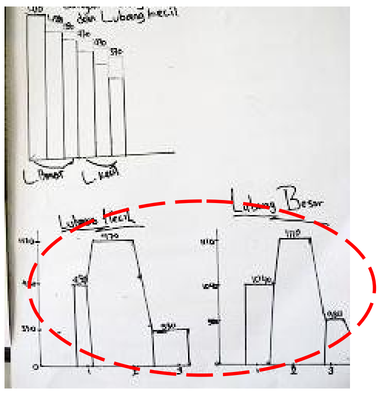

Figure 3. The other two representations from Group 1

Whole Class Discussion

In opening the whole class discussion, the teacher noticed that not everyone managed to observe all the posters due to time constraint. She invited students to ask questions based on unresolved questions or confusions they had during the gallery walk session. In this case, the teacher set another social norm for students to participate by bringing forth any unresolved problem in comprehending data or choices of data representations.

Two episodes are discussed here. These were selected because they featured an evolving understanding initiated from the exchanges during classroom discourse between students and the teacher. All student names below are pseudonyms.

The first episode was initiated by a question to clarify the water drop symbol representing the unit in Figure 2b. The teacher invited other students to explain what they understood and Deni volunteered his explanation as follows: "Every unit here (referring to the water symbol) represents 10 millilitres of water so 10 times of units here represents 10 times 10 and it adds up to 100 millilitres of water." However, Andi was not satisfied with this explanation and questioned what the unit would look like if the data were not multiples of 10. This was a valid question which highlighted the importance of understanding what the unit represents in a picture graph.

Andi: *What if the data of water flows through the small hole was 433 millilitres, how would we divide the figure, teacher?*
Teacher: *Why did you address the question to me? Who did the work?*
Andi: *Deni*
Teacher: *Then you should ask Deni instead.*
Andi: *Deni, what is your idea?*

The teacher clearly tried to set the tone for the social norm in the class by positioning students at the centre of the discussion (lines 3-5). Her response to ask Andi to address the question to Deni instead of to the teacher showed a deliberate attempt to remind Andi about the social norms of the class. Deni had difficulty answering Andi's question and he kept silent for a few minutes. Some students offered their ideas such as "do a division" but Deni just did not respond to this suggestion and kept quiet. Aware of Deni's difficulty, the teacher rephrased the question differently by taking into consideration a proposed idea for division given a fixed amount of water flow: "There is a suggestion to divide but what happened if the data is 1300 millilitres, does it mean that we need to draw so many water drop symbols?" With the help of this prompt, Karina offered her idea of changing the value of the unit, instead of representing 10 millilitres, each unit of water drop symbol represents 100 millilitres to deal with data of larger value (lines 7-8 below). Karina's proposed idea suggests that she understood that the value of the unit can *vary,* and as long as the value of the unit of the representations is noted in the picture graph, other people will be able to read and interpret the given information.

Karina: *To represent 1300 millilitre, each symbol could represent 100 millilitres instead.*
Teacher: *Oh so Karina says that the number is big like 1300, each symbol could represent 100 millilitres of water instead. Yes Rangga?*
Rangga: *That is okay for water runs through the big hole. For the small hole, we might have ones. Do we divide the unit?*
Teacher: *How about the rest, what do you think?*
Andi: *Teacher, initially the unit represents 10 millilitres. Are we allowed to change the unit to represent 100 millilitres when the amount of water is 1300 millilitres?*
Teacher: *Do you think it is acceptable?*
Andi: *It is acceptable but what if the amount of water flow is 135?*
Teacher: *So it is not multiples of 10. How many of you have seen a diagram like this before? [referring to the picture graph in Figure 2b]*
[The class was split in their responses, some said yes, some said never]
Teacher: *Some of you have seen this before, some of you have not. If Karina changes her notes, a symbol of water now represents 100 millilitres instead; can you still understand her poster?*
Class: *Yes, we can*
Teacher: *If we add a note saying that a unit represents 1 millilitre, can we understand it? You can right? How about her idea? This is a different way of representing data. We have learnt about line graphs, circle graphs and now we learnt about a picture graph.*

The interaction between the teacher and some of the students showed how the social norms were exercised during the whole class discussion. First of all, Karina was offered a chance to explain her idea to the rest of the class. However, Rangga and Andi were not satisfied with this explanation and posed further questions. In particular, he challenged Karina's idea by presenting a scenario whereby the amount of water flow is 135 millilitres (line 19). The teacher practiced the social norm by inviting the rest of the class to participate instead of giving a direct answer to the questions by Rangga and Andi to encourage interactions amongst the students. Andi clearly voiced his concern about whether the value of the unit of representations had to be fixed. The action of posing a challenging scenario by Rangga and Andi illustrates how a sociomathematical norm was being enacted (lines 12-19). These students made a valid point by presenting a different situation to check whether the proposed idea by Karina could be extended to another situation. The fact that Karina and her group (Group 2) used the picture graph to represent only part of their data might lead to these questions. Furthermore, Andi pointed out the fact that the data might not always be multiples of 10 and 100 (line 19). This question expanded the discussion to a concept of place value of a number. The idea of place value was brought up by Jose (lines 32-34), which again shows evidence that students came up with a different mathematical idea, an example of sociomathematical norms. The teacher practiced a social norm by giving a room for another student, Jose, to contribute his idea in response to Andi's question:

32 33 34	*Jose:*	*We can write it in rows, for instance if the data is 135 then we can draw 5 units of ones, 3 units of tens and 1 unit of hundred. Can we do that teacher?*
35 36 37 38 39	*Teacher:*	*From Jose's explanation, I understand that if a unit represents 100 millilitres, then 10 millilitres we can draw it like this [the teacher draw a smaller picture graph to represent a unit in proportion to the original unit – See Figure 4a].*
40	*Jose:*	*No, that is not what I mean*
41	*Teacher:*	*No? Okay then what do you mean?*
		[Jose came to write his idea about representing 135 as reproduced for clarify in Figure 4b]
42 43	*Jose:*	*For 135, then we can draw like this, 5 ones, 3 tens and 1 hundred*
44 45	*Teacher:*	*Oh okay so Jose use columns to represent data. Do you think this is acceptable class?*
46	*Class*	*Yes, it is.*

Initially the teacher predicted that Jose's idea was to adjust the unit in proportion to its value (lines 35-39). The teacher drew a large water drop to represent 100 millilitres, a smaller water drop to represent 10 millilitres and she was about to draw a smallest water drop to represent 1 millilitre while interpreting Jose's idea as shown in Figure 4a. However, before she drew the smallest water drop, Jose commented that it was not what he had in mind; he was invited to share his idea (redrawn in Figure 4b) which drew on the place value column idea. Jose's idea was clearly different from the one proposed by Karina or predicted by the teacher. This episode shows another example of enacted sociomathematical norm, whereby students engage in discussion of different mathematical ideas. The fact that Jose was comfortable in disagreeing with the teacher indicates that this practice was accepted and valued as part of the social norms in that classroom. The teacher again invited the rest of the class to decide whether Jose's idea was acceptable, enacting a social norm whereby the class shared a responsibility to decide the reasonableness of others' idea (lines 44-46). However, no further discussion was observed. Hence, the positive response by the class "Yes, it is" could not be interpreted simply as a sign that the class could follow Jose's idea.

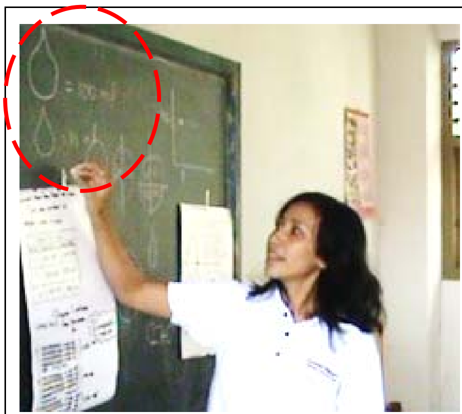

Figure 4a. Teacher's prediction of Jose's idea

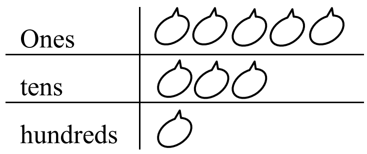

Figure 4b. Representing 135 using place value idea (re-drawn)

Figure 4. Different ideas for the unit of representation for 135 (millilitres)

The second episode underlines the central role of whole-class discussion to guide students in revisiting an inappropriate choice of data representations. Group 5 opted to use a tree diagram as one of data representations (Figure 5a). Some students were puzzled with this choice of representation and tried to make sense of the link between tree diagram and the data. Sinta posed the question to the teacher for the whole class discussion. Rather than answering Sinta's question directly, the teacher invited anyone in the class who understood the choice of tree diagram as a representation to explain it to the rest of the class. This action by the teacher showed her consistent effort to build on the social norm that students were responsible to respond to each other's questions. However, no one in the class could offer an explanation so the teacher asked a representative of the group, Andi, to give its explanations:

47 *Andi:* *This is just copying a model of the tree diagram that is commonly*
48 *used to factorise numbers.*

Sinta: What do you mean by 1, 2, and 3?
Andi: These numbers 1, 2, and 3 refer to the order of experiments, data from experiment 1, experiment 2, and experiment 3.
Jose: How do you differentiate the data from the small and the big holes?
Andi: The ones below are data of experiments using the big hole.
Hari: I don't understand.

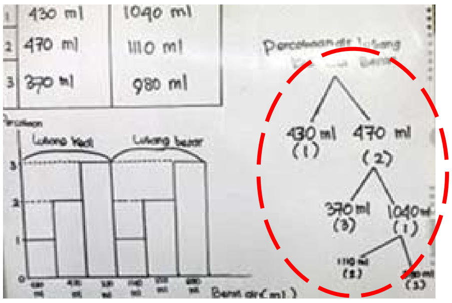

Figure 5a. Representing data using a tree diagram

Figure 5b. Factorising 60 using a tree diagram

Figure 5. Different ideas for the unit of representation for 135 (millilitres)

The class had difficulty following Andi's explanation as indicated by Hari's comment (line 55). Andi's explanation revealed that the tree diagram was chosen to show another variation of representations. Apparently Andi and his group members overlooked the fact that the tree diagram had a special mathematical association to prime factorisation of a number. Hence, the choice of tree diagram to represent the data is not mathematically appropriate. In helping Andi and the rest of the class, the teacher asked Andi to use the tree diagram to factorise 60 (see Figure 5b). As Andi went through his explanation, he noted that in the tree diagram, the product of two factors of 60 namely 2 and 30 would result in 60. Similarly, he noted that multiplying 2 and 15 would give a product of 30. He re-stated that "In my diagram, it is just a copy of the tree diagram as a model." The teacher invited others to give comments if the tree diagram was easy for them to understand and clearly many of Andi's classmates found it confusing as the class muttered "we think it involves a division." Apparently in this episode, the class exercised the social norms of asking questions to make sense of others' strategy and giving feedback to others' explanations. The teacher's attempt to scaffold Andi's explanation by presenting a case to factorise 60 is an instance of sociomathematical norm. This prompt allowed the class to reach a conclusion about the legitimacy of Group 5 solution, which was represented by Andi. This episode shared a similar feature to the earlier episode during the gallery walk session with Group 1 whereby another representation was chosen in order to come up with more representations. Hence, it is critical to set the sociomathematical norm that coming up with a different representation for the sake of being different without valid mathematical justification is not mathematically acceptable.

In summary, the above analysis of the lesson provides evidence of enacted social norms and sociomathematical norms. The teacher and students enacted these norms in the classroom by constantly articulating their ideas and negotiating their interpretations during the classroom discourse which addressed the first research question. The teacher played a central role in positioning student's ideas and their exchanges at the centre of the whole class discussion. The second research question looked into the influence of classroom discourse on students' interpretations of what constitutes different data representations. Some students held incorrect interpretations that mathematically different representations mean simply representations that "look different." By carefully prompting students to clarify their ideas with illustrative examples, the teacher helped the class to realise what counts as mathematically different representations.

Conclusion and Implications

Both the teacher and students in this lesson engaged in the process of constituting understanding of data representations. They exercised social norms by asking questions to understand others' ideas, providing explanations to help others understand their own strategies, evaluating others' ideas, and challenging others' explanations with different cases. The teacher constantly made a conscious effort to practise a social norm of sharing responsibility for students to address questions to each other rather than to the teacher. Moreover, the teacher built her prompts on students' questions and ideas. Brodie (2007) and Wood (1999) highlighted the importance of building on students' own questions in classroom discourse to create a sense of ownership for students in order to motivate them to stay engaged in the discourse. The teacher also balanced her actions of letting students grapple with their own ideas by asking students to listen to each other and to guide students with her prompts. Freudenthal (1991) referred to the role of teachers in facilitating learning as balancing the tension between "the force of teaching and the freedom of learning" (p. 55).

The exercise on sociomathematical norms is critical to build an understanding of what constitutes as a different solution and an acceptable explanation. The analysis of this lesson provides evidence that some students interpreted different representations as any representation that "look different" without sound mathematical basis. Both Group 1 and Group 5 acknowledged that their choice of representations was driven by the intentions to be different. The whole-class discussion enabled the class to re-visit an inappropriate choice of data representation and to realise that a representation should have a sound mathematical basis. Hence, coming up with different representations just "for variations" was not acceptable.

The analysis in this paper only focussed on one lesson which did not present a whole picture of the lesson sequence. This is a limitation of this case study. Yet, the narrow scope of the analysis enables us to learn about specific features of classroom discourse, especially the enacted social norms and sociomathematical norms and their impacts on students' learning. Further studies to examine key aspects of classroom discourse that afford or constrain mathematical learning in classrooms are needed. The author concurs with Yackel and Cobb (1996) who noted the critical role of teacher's beliefs in establishing social norms and social mathematical norms with students in classrooms. The teacher revealed that it took her approximately half a year to establish these norms in her class before her students felt comfortable in participating to ask questions, to challenge other's ideas, to communicate their reasoning, and to justify their thinking. Implementing this classroom practice is time consuming and requires consistent effort and patience by both the teacher and students to exercise the norms. Presently, these factors are recognised as challenges for implementing social norms and sociomathematical norms in mathematical classrooms.

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