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The Third Wave Studies of Values in Effective Mathematics Education: Developing Students' Mathematical Autonomy in Classroom Learning

Huk Yuen Law Ngai Ying Wong Ngar Yin Louis Lee
Faculty of Education, The Chinese University of Hong Kong, China

Abstract: Several decades in the past saw the change of focuses in mathematics education research community from cognitive approaches to affective aspects (the second wave), and then the recent trend (the third wave) on the discursive approach. This project uses qualitative tools to investigate effectiveness in mathematics lessons from both the standpoints of the learners and the teachers. The findings have identified what learners value in effective mathematics learning, and how these findings can help teachers and mathematics educators to develop students' mathematical autonomy in classroom learning.

Key words: Third wave; Effective mathematics lessons; Beliefs; Values; Mathematical autonomy

Background

Several decades in the past saw the change of focuses in mathematics education research community from cognitive approaches (we call it the *first wave*) to affective aspects (the *second wave*), and then the recent trend (where the term *the third wave* comes from) on the discursive approach. The present study is part of a large scale collaborative project from which the data will be compared and shared among the participating regions, including Bangkok, Chiangmai, Delaware, Hiroshima, Macau, Hong Kong, Melbourne, Penang, Shanghai, Singapore, Taipei, Umeå, and WaggaWagga. Adopting the socio-cultural research perspective, this project will explore the contextually-bound understanding and meaning of what counts as effectiveness in mathematics lessons from both the standpoints of the learners and the teachers. This paper reports on the Hong Kong study that involves three Hong Kong secondary schools from which multiple data sources, such as questionnaires, lesson observations, interviews, and journal writings, are used for triangulation.

The Hong Kong context is of particular interest since it situates somewhere between Eastern and Western cultures. On the one hand, over 90% of its residents are Chinese and it is deeply rooted in Chinese culture, and on the other hand, it has experienced two centuries of Western governance as a British colony. It is generally perceived that Hong Kong can serve as a guide to understanding the Confucian Heritage Culture[1] Learning Phenomenon.

By comparing beliefs across regions, we can examine social-cultural values on mathematics education. Prior research studies on mathematics teachers in Hong Kong and the Chinese mainland reveal that, very much in line with students' conceptions, teachers generally regard mathematics as a subject of *calculables*; mathematics is logical, widely applicable to daily life and involves mathematical thinking for solving problems through logic. And a major goal of teaching mathematics to students is to develop among them a mathematical way of thinking (Wong, 2002; Wong, Marton, Wong, & Lam, 2002).

Besides general understanding towards mathematics teaching, views on effective teaching among expert elementary teachers have also been investigated (Wong, 2007). To these teachers, effective mathematics teaching is one that sets a path of mathematization for the students to go from the concrete to the abstract, one that enhances understanding, and one that helps students acquire a flexible use of rules. To achieve these outcomes, well-organized practices (repetition with variation) might serve as scaffolding that leads from the basics to higher-order thinking skills (Wong, 2006). Teachers must have strong professional knowledge, including the mastery of teaching skills and the ability to understand students (Wong, 2007).

Although the responsibility of teaching primarily rests on the teachers, students' views on effective teaching can be as important as those of the teachers. It requires collaborative efforts of both the teacher and the students to develop a discursive space in the mathematics lessons for enhancing students' active involvement in the forms of question asking and communicating results of their exploratory work (Stephen, Bowers, Cobb, & Gravemeijer, 2004). Thus, it is worth further investigation by looking from the perspective of the students of what constitutes effective teaching and how effective teacher should act.

Research in Hong Kong has revealed what local students think of as effective mathematics teaching (Wong, 1993, 1996; Wong, Lam, Leung, Mok, & Wong, 1999a, 1999b). During lessons, good mathematics teachers will explain clearly to

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[1] Generally this refers to regions like China, Japan, Korea and some regions in South East Asia which are believed to be influenced by Confucianism (see, Wong, 2004 for more details).

ensure proper understanding by offering clear and step-by-step explanations of how to approach problems. They will allow time for students to think and check frequently to see if they understand. They are conscientious and well-prepared in the sense that they not only design a variety of classroom activities, but also provide students with adequate exercises for practice. They are able to generate a lively atmosphere while keeping good order. Proper classroom management is deemed important because students believe that a good learning environment is one which is quiet but not boring, and has room for after-lesson discussion with fellow classmates. After the lesson, the ideal mathematics teachers will answer students' queries. They will also act as a supportive friend of their students and show concern toward them. They do not penalize weaker students (Wong, 1993, 1996; Wong, Lam, Leung, Mok, & Wong, 1999a, 1999b). In short, it appears that the students expect to have something to learn from the teacher in a lively but unthreatening learning atmosphere.

What makes *The Third Wave Study* different from previous similar studies is that both teachers' and students' views are investigated in a single study. This study does not downplay the significance of personal factors of the mind and of the heart. Rather, it recognises the efforts made by students and teachers to make sense of mathematical ideas and concepts. Such efforts will regulate the negotiation of cultural border crossings (see Aikenhead, 2001) while teaching and learning of mathematics are taking place in the classroom. This study demonstrates innovation in gaining better understanding of the effectiveness of mathematics pedagogy. And the present study focuses on the socio-affective variable of values and valuing, rather than purely cognitive (the first wave) or affective (the second wave) ones. It looks at what a teacher and his/her students in the class co-value as the lesson unfolds, rather than what a teacher or student alone values.

The present study adopts a research framework that is shared among all the participating regions. However, as undertaken by the Hong Kong team, it is "self-contained". This means that we are responsible for not only collecting data but also analyzing our own findings to empower our teacher community to enhance the effectiveness in learning and teaching of mathematics.

Participants and Procedure

As an exploratory study, three schools were sourced. There were six participating teachers and 36 junior secondary students from six classes (one class from each teacher, and six students for each class). Each school was asked to select and nominate two teachers who were considered as particularly effective teachers of

mathematics to participate in the project. In this study, we realise that in the process of identifying the potential candidates of the "effective teachers" there was a lack of students' participation, and we acknowledge this as a methodological limitation. With the comparison of the practices and norms of teachers within the same school, we hope to tease out common traits valued in effective and not-as-effective classrooms. We also looked into the teachers' process of negotiation (with their respective students) within a similar school context.

We adopted qualitative research tools with the use of multiple data sources for triangulation. Multiple data sources have been accessed not just to optimise data validity through the cross-comparisons afforded, but also to capitalise on the diversity of data formats to stimulate the identification and discussion of relevant contextual factors. These data sources included written documents, verbatim transcripts, visual documents (photographs) and research field notes. Each teacher was associated with one teacher journal, three lesson observations, and three teacher interviews. Similarly, each group of students was associated with three focus group meetings and three photograph sets of "moments of effectiveness". Teacher journals serve as tools for gathering background contextual information of the teachers, and for capturing what each of them values and emphasises in facilitating effective mathematics lessons. The teachers were asked to document their observations and perceptions of their teaching activities over a three-week period. Upon the first visits of the schools, we collected and examined the teacher journals before we began the three-day lesson observations (roughly a week apart between each lesson) in each of the schools.

During lesson observations, which were videotaped, we asked the students to take snapshots of moments (see the notion of "critical moments" of Tripp, 1993) in the class when they felt that they were learning (mathematics) particularly well. Each of them had earlier been given a digital camera with which these snapshots could be captured. The photographs as taken by the students served as tools for stimulus recall purposes in the subsequent focus-group interview meetings. We also conducted the semi-structured interviews with the teachers, during which we solicited not only the teachers' views of the lessons they conducted but also the teachers' responses to the students' perceptions of the lessons as well as the snapshots of moments as taken by their students.

Data Analysis Procedures

The various sources of data were analysed qualitatively. We first transcribed the recordings of teacher interviews and student focus-group interviews, and then went

through the analytical process in which we uncovered the patterns and identified the themes from scanning the pages of transcriptions. The analysis of the findings reveals that both the teachers and the students shared commonalities in what they both valued about effective mathematics lessons. These include teacher's clear and detailed explanation and a lively classroom atmosphere. Nonetheless, close examination of the minor differences between the teachers and the students discloses that the students' voice plays a vital role in enhancing the effectiveness of mathematics lessons.

Categories of Effectiveness

From individual scanning of the transcribed scripts, we identified the key terms and met to categorise the themes. Through negotiation, the categories were as follows:

- Clear and step by step explanations (C)
- Liveliness and an enjoyable environment (L)
- Teacher student Interaction/rapport (I)
- Order kept (O)
- Teacher involvement/led (T)
- Use of Problems (P)
- Opt for Understanding/concept (U)
- Monitoring Students progress/listening to students' voice (S)
- Student Engagement (E)
- Learning Difference (D)
- Help available (H)
- Memorisation/get things remembered valued (m)
- Time constraints (not directly related with value) (t)
- Motivating the students (incentives/rewards) (M)
- Teacher's character/ image/teaching manner (W)
- Student learning attitudes/feeling (R)
- Understanding of students' needs (J)

These themes were counted, and Table 1 shows the frequency for each of the two teachers (A and B) and their students (as groups) under the labels of School A, School B, and School C. These frequencies are shown in Figures 1 to 3. The students in all the schools had put their first priority on teachers' *clear and step by step explanations*, whilst those from School A and B also valued highly *liveliness and an enjoyable environment*. These views were shared and echoed by the teachers. In addition to these two values, other salient categories valued by the students were *Teacher student interaction/rapport, Order kept, Teacher involvement/led, Use of problems* (especially for the students of School B), and *Opt*

for understanding/concept. It appears that students prefered to have fun *and* something to learn in mathematics lessons. They wanted something taught in a tangible way to enable them to arrive step by step at the anticipated results without bafflement.

Table 1
Frequency of qualities by teachers and students within schools

Qualities	School A			School B			School C		
	TA	T B	S	TA	TB	S	TA	TB	S
U	4	6	1	8	4	9	7	2	19
S	4	5	1	7	3	5	3	5	9
I	5	2	10	11	1	2	8	5	22
E	7	5	7	5	6	0	3	3	12
L	4	2	32	1	4	19	1	0	20
C	6	9	31	14	12	23	39	22	108
O	2	1	12	1	3	1	0	0	1
D	3	6	6	0	2	0	3	3	3
H	3	1	5	0	0	2	3	1	10
T	0	5	8	1	1	1	0	0	1
m	1	1	0	4	2	5	0	0	1
t	2	0	0	0	0	0	2	4	10
M	1	0	3	1	6	2	3	1	0
P	0	0	0	2	2	19	2	3	3
W	0	0	0	0	1	2	5	1	13
R	0	0	0	0	0	0	4	0	9
J	0	0	0	0	0	0	0	1	0

Note: TA=Teacher A; TB=Teacher B; S= Student

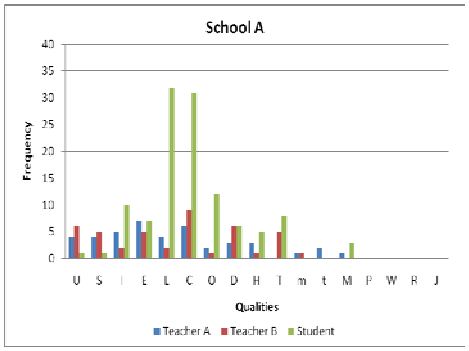

Figure 1. Data for School A

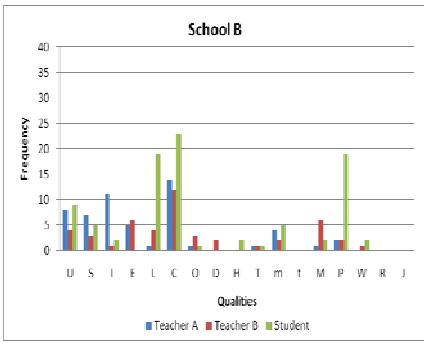

Figure 2. Data for School B

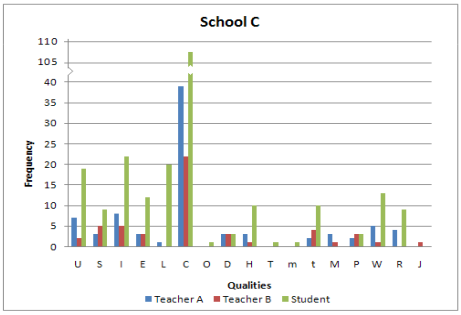

Figure 3. Data for School C

Perceptions through Interviews

Teaching involves risky and complex behaviours (Edwards & Protheroe, 2003). Not a single trial of some isolated teaching strategies or methods adopted will form the cumulative impact of effective teaching behaviours that make a difference to students' learning. If we are to consider whole-class interactive teaching to be effective, we will have to examine any explanations that constitute the "vector sum" of indicators as valued by teachers and displayed in their teaching behaviours (see Brophy, 1986). Hofstede (1997) stated further that any such indicators as valued by teachers serve as "feeling with an arrow to it" that guides their classroom actions in justifying what sorts of cognitive structuring tools (for instances, asking open questions versus allowing only standard solutions) they might use.

Though this study shows some compatibility between students' valuing and teachers' beliefs of effective mathematics lessons, minor alternative anticipations of either party will trigger intriguing tensions that underline the relationship between

classroom organisation and students' learning efficacy. For instance, the students want "Order kept" in the classroom to ensure that they will have something "gained" from the teacher's "Clear step by step explanations"; whereas the teacher under the pressure of time constraints might not be aware of the over-reliance on the use of worksheets for facilitating a delivery of orderly-presented teaching materials for the students at the expense of the "Liveliness and an enjoyable environment" (the post-lesson interviews of the teachers in School A reflect deeply the concerns of such an issue). Based on the further analyses of the interview transcriptions, we will discuss three reflective questions:

(1) What actually happens in a "teacher-led and yet student-centred" classroom?
(2) What is the use of student voice for enhancing teaching effectiveness?
(3) What constitutes a successful valuing process for developing students' learning autonomy in mathematics lessons?

The scenario in a "teacher-led and yet student-centred" classroom. In a teacher-led pedagogical approach, the teacher's role is to create the cognitive demands for the learners. But how can a balance be effectively monitored on the proportion of the lesson spent between the whole-class teaching (teacher-led activity) and the individual or group practice? What is the role of student voice in shaping the teaching strategy as targeted for such a balance? To illustrate, we consider teacher Ben (not his true name) from School A. He exemplifies the usual practices of an average experienced teacher.

As a head teacher in mathematics, Ben was competent in handling the classroom discipline and was very capable in delivering mathematics contents smoothly. It was a Grade 7 class and the topic of the lesson was algebraic simplification of polynomials. After the lesson, the first author interviewed the six students who captured the "moments of effectiveness" before he had the following conversation with Ben.

Researcher: What do you think of the lesson you have just conducted?

Ben: As usual, I would observe whether the students have already mastered what being taught yesterday (the last lesson). The strategy is to give the students chances telling me the concepts they have through group questioning. Usually, I started using examples as a testing ground. It would not be easy to know whether the students really understand how the problems could be solved in some particular ways. Thus, I always demand them writing down the steps and the explanations as well. When I find the students still have difficulty in

explaining their work, I would do some more elaborations before I would introduce a new topic.

Researcher: What features can you identify in the lesson that characterise the notion of effectiveness in your teaching?

Ben: As the class size is small, we can have more teacher-student interactions. The *most effective* way (of teaching) is to give students chance to think by *asking* more—more questions of different variety. When I pick a student to answer my question, I won't let her go even if other students give clues to help answering it. In order to make the student think more deeply, I would ask her again some other questions.

Researcher: Does it mean that you *value* the use of classroom questioning?

Ben: That's right!

Ben was confident in using questioning as a means of "forcing" the students to think more rigorously of what they were doing with the problems being asked to solve. Thus, he valued question-asking as a vital tool in his lesson. Ben ***led*** the lesson by monitoring the flow of classroom discourse through questioning-and-answering and he saw the cognitive demand on the individuals as the *centre* of students' achievement need in catching up with *that* flow of classroom activity.

In the second part of the interview with Ben, the researcher showed him some of the photographs taken by his students during the lesson. Among them, one picture depicted the scenario in which some other students pointed out a mistake made by a student on the blackboard whilst he tried to determine whether the terms $4x^3y^2$ and $3y^2x^3$ belong to the polynomial of the same kind. Ben appeared puzzled by the student's motive of taking such a picture. He was only concerned with the reflection of how to lead the students to adopt the right approach of looking into the problem when *cognitive* conflicts would have turned up in the class discussion. It seems that he did not even attempt to make a hypothesis for explaining the student's photo-taking action in terms of empathetic sharing with the peer's embarrassment as arose from the amendment discourse by the other peers (Lampert, Rittenhouse, & Crumbaugh, 1996). The girl, in School C, who had also taken a picture of a similar nature, voiced her concern during the interview that "I think the classmate as called out by the teacher should *not* be laughed at for the mistakes he made (in the board work)." In the "subjective I" of the students, sociability can be an essential element in building up classroom environments that are conducive to the development of communicative relationship as emerged from intellectual discourse (Sáenz-Ludlow,

2006). Thus, the teacher-led classroom discourse, as seen from the teacher's "objective eye" *alone*, may not be able to include "interpretation games" for developing mathematical autonomy in the lesson as claimed to comply with student-centred ideology.

The "use" of student voice for enhancing teaching effectiveness. Muijs and Reynolds (2000) argued in the book, *School Effectiveness and Teacher Effectiveness in Mathematics*, that "high levels of whole-class interactive teaching are beneficial…overuse of a particular effective technique becomes harmful" (p. 301). Would student voice serve as a mediating factor in unfolding the relationship between teacher behaviours and teacher beliefs and attitudes? As mentioned above, Ben was confident in using questioning as a powerful technique for monitoring the flow of the classroom discourse. It was so powerful that it frames strongly the contextual message of what would be regarded as a legitimate form of communication as entailed in the pedagogic practices without being aware of the need in creating space for establishing the voice that constitutes the students' social identity in their place of learning (Bernstein, 2000, p.204). Bruce, one of Ben's students, voiced his views on the elements that should be embedded in teaching effectiveness:

Researcher: What should an effective mathematics lesson look like?

Bruce: I have no ideas about this. Because (I regard) my mathematics is so good that (it) can be absorbed no matter how.

Researcher: Do you mean as you regard yourself as good in mathematics makes you think that you are not concerned with how the teacher would teach in the classroom?

Bruce: Yes.

Researcher: Do you have any ideas to help the teacher enhance his effectiveness in the mathematics lesson?

Bruce: Teacher should not lecture. The (teacher's) continual speaking would make the lesson getting ***mouldy***. (He) should talk less and (allow us to) do more activities. That will make the students have a deeper understanding.

Researcher: To sum up, can you use a few words to describe what a good mathematics lesson should be?

Bruce: The most important thing is that it will be a good world if (the lesson) can be finished in jig time.

Researcher: How did you feel in today's lesson? Would that fit what you just mentioned about how an ideal mathematics should look like?

Bruce: Today's lesson was really boring, because what was taught today had been taught in last week. And *I already knew it!*

At the beginning of the interview, Bruce told the researcher that he had deleted the pictures he took during the lesson. Upon the request of depicting that scenario, he said, "(The details of it) cannot be exactly recalled. Nonetheless, I still remember that the teacher was working on a problem and he told us that it would be a difficult one, but to me it was not and so I captured it (with the camera)." To Bruce, the lesson was a daily ritual (a time passing activity) in which he made himself physically present without actual participation in the classroom discourse. The absence of discursive space had deprived him of the *agency* necessary for soliciting the meaningful learning experiences from attending the lesson.

Nonetheless, constructivist learning requires a more active student role in learning (Levin, 2000). The effect of using a questioning technique as highly endorsed by Ben would have been greatly hampered without being aware of putting the students back into the centre in the learning discourse. As argued by Cook-Sather (2006), the *student voice* would help us "to connect the sound of students speaking not only with those students experiencing meaningful, acknowledged presence, but also with their having the power to influence analyses of, decisions about, and practices in schools" (p.363). The "over-practice" of revising the learnt contents as used by the teacher was to ensure that most, if not all, of the students would know what he was teaching. But Bruce's voice of "I already knew it!" seems to tell us that he not only knew what was being taught but knew too well what would *exactly* happen in every mathematics lesson he had been attending. While reading and re-reading the transcripts, the researcher was particularly impressed with Bruce's metaphorical use of "mould" signifying his emotional state of dullness as experienced in the lesson. It was so impressive that the utterance had inspired the researcher to remember the idiomatic saying "A rolling stone gathers no moss". If we are to adopt the voice of Bruce, we may perhaps use the metaphorical clue in the making of the effective mathematics lesson: Let the students do the rolling of stones rather than just put the sheer focus on gathering moss. The teacher might take "gathering moss" as the desired learning outcome, but it is the *student engagement* in the "stone-rolling" that makes the outcome meaningful. Without the agency of rolling, what being gathered could be the mould rather than the moss.

Valuing students' learning autonomy in math lessons. Despite the teachers' display of some effective teaching behaviours (such as interactive teaching through questioning, reasonably good classroom management), what constitutes the "unreasonable ineffectiveness" in developing a democratic classroom through which the students' learning autonomy for the learning of mathematics can be fostered? Perhaps, we may have some clues from the reflections that Ben made during the post-lesson interview:

Researcher: Let's recapitulate the students' views about the lesson. These include that some students said you taught too fast but some consider the teaching pace as slow; some said boring but not so boring for some others… What do you think of their opinions?

Ben: Basically I agree with them. As I said in our last conversation, on one hand, the lesson was conducted just after the lunch, and on the other hand, (I did) not know how much the students have mastered what have been taught. Thus, I started the lesson by raising some simple questions but not many showed active responses to what I asked. So, the lesson turned out to be somewhat more teacher-dominating… I also agree that some of the explanations were bit clumsy and as I re-examined the way I taught I should have made them more refined.

Researcher: How would you depict the general scenario for the case of a teacher dominating lesson?

Ben: I would talk for almost 60 to 70% of the lesson time and would also encourage the students to do some board works… Because mathematics is somewhat special (in its learning) and mathematics is a kind of thinking which involves a lot of concepts. Without the acquisition of some known knowledge, it is hard to teach them anything. Thus, there must be something we have to teach in order to make them clear of what they have learnt before (I) could give them some problems to think.

Ben adopted the teacher dominant role as the one who does the explanations with which the students are expected to find mathematical truths. Nonetheless, from the perspective of critical constructivism, the content in the curriculum should be exposed by taking it as "contingently constructed, contextualized, and value-oriented" through which the democratic ends of teaching and learning could be conveyed (Bentley, Fleury, & Garrison, 2007, p.10). To constitute teaching as a democratic exercise, the teacher has to look into the ways in which the students *use* the mathematical object rather than the object itself. This is simply because the

meaning of learning is not found in the object itself but in the social practices that are regulated by that particular mathematical object. Thus, in making sense of the mathematical object, the students would have to engage or participate in *that* practice as suggested by Dewey's views about "communication-as-participation" (Dewey, 1980 [1916], as cited in Biesta, 2009, p.70).

Conclusion

In response to the latest pressures on school accountability at both the local and global levels, understanding effectiveness in education in general and in mathematics education in particular becomes increasingly vital. Nonetheless, such an understanding which is embodied in the actual classroom, both as a cognitive and social-cultural phenomenon, inevitably involves intriguing and complex inquiry. Hence, there cannot be simple answers for the reflective questions that we have raised in our discussion about the interviews. In discussing the teacher-led and student-centred scenario, we do not mean to treat it as a dichotomy. It is possible that a teacher leads the class without him/herself dominating everything. This could be regarded as typically Chinese (we can know more if we contrast our findings with results from other regions) though we are not saying that all Chinese lessons are so tactfully treated. The above is done through the solicitation of student voice and teacher's valuing of student autonomy. It might offer a scenario and explain why Chinese classroom environment can be so effective even though it seems contradictory to what is perceived as conducive to learning.

This research has justified itself, above all, in providing prospective teachers with a practical and substantial agenda for reflections on developing effective mathematics teaching strategies for students' learning in their teaching careers. Future research based on the further cross-regional studies will throw more light on such a mathematics education agenda.

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Authors:

Huk Yuen Law [corresponding author; hylaw@cuhk.edu.hk]**, Ngai Ying Wong** and **Ngar Yin Louis Lee,** Faculty of Education, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong.