The Mathematics Educator
2010, Vol. 12, No.2, 25-50

"Special Segments of Triangles" The Use of Paper Folding for the Purpose of Terms Understanding by Middle School Female-students

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Abstract: Geometry studies in middle school constitute a bridge between geometry learnt intuitively at primary school and the studies of deductive geometry in high school. Due to this transition, students are required to know term definitions, to reason and present demonstrations only in a formal and abstract way. This is a meaningful change which embodies a difficulty for both teachers of this subject and for students. The present study explored the extent of influence of learning abstract terms, such as "special segments of triangles", using illustrations of paper folding. The research population comprised middle school female-students in the 8th-grade. The research findings indicate that the students improved their definition capability and their comprehension of terms learnt in this way, demonstrating a change for the better in their attitude towards the geometry studies.

Key words: Paper folding; Special segments of triangles; Angle-bisector; Altitude; Median; Triangle

Theoretical Background

The Cognitive-Developmental Stage of Middle School Students

The relation between teaching-learning and the cognitive development of children at any age is, frequently, a central and important issue. The professional literature presents three main approaches, which attempt to address this question: the constructivist approach advocated by Piaget, the data processing approach according to Strauss and the social approach according to Vygotzsy. Furthermore, another approach is prevalent among those engaged in mathematical education, for determining the levels of development in geometry perception according to Van-Hiele.

The constructivist approach

Piaget, who conceived this approach, argued that cognitive development is characterised at each age by a new progress from a qualitative and not only quantitative point of view. Learning is a process of conceptual development, built by children themselves from the learning opportunities offered to them from the outside (Even & Tirosh, 2003). Piaget (1969) divided the development levels into four stages: the senso-motor stage (0-2 years), the pre-operational stage (2-7 years), the concrete operations stage (7-11 years) and the formal operations stage (12 years and above). According to Piaget's stages approach, adolescence is the formal operations stage. At this stage, children can think in a more abstract and systematic way than in the past. This stage – formal operations – is not manifested at once, but rather, adolescents develop it gradually. One can distinguish between the level of specialty in formal operations, which is common during early adolescence and those prevalent during the late adolescence period. Early adolescence, the age of middle school with which the present study deals, is called by researchers "forming formal operations" and one can see that children at middle school age are not consistent in their use of formal thinking (Stroufe, Cooper & DeWart, 1996).

The data processing approach

Theories advocating data processing relate to the human memory structure as a factor affecting learning. This approach views cognitive development as a series of gradual changes which transpire in our attention, memory and thinking, leading to greater competence of event interpretation and to a wider range of problem-solution strategies (Strauss, 1993).

The research of adolescents' data processing indicates a consistent improvement in attention and recollection skills. Since childhood, improvements in the short-term memory and long-term memory are also displayed. The improvement of memory mainly stems from improved recollection processes as a result of greater sophistication in the use of recollection aids. Adolescents understand better when and how to employ suitable recollection aids and they use more external recollection aids, such as notes. Another factor in adolescents' advancement is an enhanced cognitive processing capability. The more adolescents are trained in attention and memory assignments, the more automatic and less effortless these processes are. Thus, adolescents have more cognitive resources to invest in more complex cognitive assignments (Strauss, 1993).

The social approach

Vygotsky (1978), who conceived this approach, maintained that cognitive development occurs mainly by social interaction with others, who are experts in a certain area of knowledge. The learner's participation in an interaction-by-dialogue with experts creates a high level of learning. According to this approach, cognitive development is a lifelong process.

Adolescents do not invent by themselves the logical thinking. Rather, discussions with others - one person or in groups - foster the emergence of high level thinking skills. This improvement can transpire through direct teaching, for example when a teacher or parent suggests a strategy for learning material before a test. However, it mainly occurs through social interaction, providing a neighbourhood whereby one can try ideas, respond to contrasting opinions and obtain demonstrations from other people's arguments (Karpov, 2005).

Van-Hiele's theory

This theory was developed by a couple of Dutch mathematics educators, Dina and Pierre Van-Hiele. This theory advocates that thinking in geometry develops in several levels (Fuys, Gedds & Ticshler, 1988; Van Hiele, 1986, 1999). Partial mastering of a specific level is a necessary, but not sufficient condition for mastering a higher level and students cannot function at a specific level if they have not fully learnt the previous ones. Moreover, students can simultaneously function at various Van-Hiele levels, depending on the understanding they have developed with regard to various terms.

The above approaches illustrate that middle school age, namely the beginning of adolescence, is characterised as the transition from the concrete thinking level, relating to tangible objects, to the abstract thinking level and the logical procession and deduction level.

Acquisition of Mathematical Terms

The difficulty in teaching the subject of "special segments of triangles" stems from the fact that they are not only new terms for the learner but also terms perceived as similar. Many middle school students find it hard to define these terms accurately and, as a result, the differences between them become confusing and blurred.

Cognitive psychology deals extensively with the essence of a term. An important aspect of the discussion relates to the question whether this term can be unambiguously characterised by means of properties. When debating the question, "what constitutes a term", we are on solid ground as far as mathematical terms are concerned, since in mathematics there is a definition for any term which is not fundamental. The definition determines the meaning to be attributed to a new expression by means of previous ones. It determines the scope of the term (namely, the section which includes all the examples belonging to it) and how it relates to other terms.

As to the logical aspect of the term, the situation in mathematics is unclear. This does not apply to its psychological aspects, i.e. the questions accompanying the acquisition of the term. Acquisition of a term, one of the key elements in learning, has numerous facets: knowing the scope of the term, knowing the content of the term, understanding the name of the term (Tennyson & Cochiarella, 1986). Being a deductive and abstract discipline, mathematics sets rigid requirements for an accurate and unambiguous expression (Zaslavsky, 1994). Consequently, throughout their years at school, students tend to develop not only difficulties in learning the subject but also deep fears of it.

According to the constructivist theory, one of the elements for constructing mathematical knowledge is the linguistic field. That is, building mathematical knowledge must pass through the language. Difficulties in mathematics do not stem only from lack of knowledge but also from failure to acquire the language of mathematics (Perkins, 1992).

A study conducted by Patkin (1994), investigating students' mastery of geometrical terms, found that students do not accurately formulate terms nor give correct definitions. It can be assumed that knowledge disabilities are due to lack of verbal distinction. Moreover, the study shows that students make mistakes when asked to identify differences between shapes. This supports the assumption regarding lack of logical skills for distinguishing between terms.

An interesting way for teaching terms in geometry has been presented by Patkin (1996) – from experiencing to definition and from definition to experiencing. Thus, students manage to define a term by themselves when a large number of examples and non-examples of that term are shown to them. Furthermore, based on this approach, students relate to crucial and non-crucial properties of a shape or a term.

The well-known Dutch mathematics educator, Hans Freudenthal (1973, in Friedlander & Lappan, 1987), described geometry as an "experience and interpretation of the space in which the child lives, breathes and moves". From this point of view, we can think about children who start learning geometry from the moment they see, feel and move in space. When children grow up, they start absorbing the properties of objects in space, such as: shape, size, location, movement, order and growth. Our goal, as geometry teachers, is to provide students with experiences, which will expand their comprehension of the space surrounding them (Friedlander & Lappan, 1987).

In a sense, the teaching and learning of geometry comprises two major aspects: viewing geometry as the existing space science and viewing it as a logical construct. These two aspects are interrelated, since students cannot cope with the logical construct without understanding space on a certain level (Hershkovitz, 1998).

Difficulties in Geometry encountered by Middle School Students

When relating to all the subjects learnt during mathematics lessons in middle school, there are students who consider geometry as a difficult and incomprehensible subject. Although geometrical terms and arguments are based on reality, reality does not check the validity of a mathematical theory. This distinguishes mathematics from all the other sciences, whereby many axioms are examined, for example, through laboratory experiments. It is not always clear to mathematics students to what extent and when it is forbidden to use what one sees in the solution or demonstration process (Zaslavsky, 1994).

Secondary school students are supposed to start deductive, systematic and formal geometry studies. According to Van-Hiele's levels, these studies require mastery of learning skills which do not exist at the visual level. Hence, the main difficulty in geometry is encountered in middle school where, for the first time, students are asked to employ logical and deductive thinking. The first stage of geometry studies is done at primary school, through the students' active experiencing which includes drawing, measuring and cutting, namely the intuitive level. At the second stage, starting in middle school, students begin organising the material as a deductive system. At this point, students get acquainted with the terms: definition, theorems and axioms. They learn to prove arguments and refute others which are wrong, by means of a counter-example (Kinard & Kozulin, 2008).

These abstract terms make it hard to understand geometry in middle school. At this age, students experience mainly basic cognitive difficulties of geometrical term acquisition or visual perception. Many teachers are unaware of these difficulties or are not familiar with tools for coping with this difficulty (Gal & Linchevski, 2002).

Teaching "Special Segments of Triangles"

The customary way for teaching the subject of special segments of triangles in Israeli middle schools is frontal[1]. In textbooks acceptable in most schools, the chapter "special segments of triangles" starts with term definitions, followed by a

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[1] Frontal teaching is predominantly thematically orientated and is conveyed orally and visually. Communication between teacher and pupils is at the forefront - co-operation of the pupils with each other is limited. With frontal teaching new fields of knowledge are introduced, work results are ensured and performance is monitored.

collection of gradual exercises, from the easy to the difficult. The present study, dealing with the teaching of terms called "special segments of triangles", proposes a different approach to teaching them, i.e. using paper folding. In order to consolidate the validity of paper folding activity as a means of geometry teaching, we are going to examine the relation between the two.

At the end of the 19th-century, a merchant in Yoshima district in Tokyo, imported coloured papers from Europe, cut them into little squares and sold them under the name "origami". This was the beginning of origami as we know it today. Origami in itself is much more ancient but, until then, it had different names and was made of different kinds of paper. The engagement in origami also spread to the west and it is accepted both as a simple children's game and as a mature and complex art. The name of the origami art derives from the combination of the two Japanese words: ori – to fold and kami – paper (Kasahara, 1973; Gray & Kasahara, 1977, 1985).

Levenson (n.y.m.) maintains that the engagement in origami greatly benefits the students, due both to the cognitive skills they develop and to their relation to mathematics and geometry. It can facilitate the learning of an abstract term such as symmetry and demonstrate geometrical exercises. Moreover, Levenson stipulates that working with origami is an example of a situation, whereby students' success depends on them and not on the teacher. Only students who meticulously follow the instructions can succeed. Learning by means of origami is active learning through practice and experience and it has an advantage over passive learning, based solely on listening to words. Achieving the final outcome may be rapid and lead to an experience of success and accomplishment. None of the students can remain inactive while the class works with origami or fail to draw conclusions and discover meanings of the new terms while engaging in origami (Hull, 2006). Using hands while folding enhances students' ability to monitor the instructions, their recollection and comprehension. According to Olson (1975), all mathematics teachers understand that learning mathematics through active experiencing is the most effective learning method. Paper folding allows such active experiencing, the formation of straight lines by paper folding being an interesting way for discovering and presenting relations between straight lines and angles. Once students have experienced and discovered new facts by means of paper folding, the formal use, the later one, will not be unfamiliar to them.

Golan & Jackson (2004) report students' satisfaction with the "Origametry" - teaching geometry by means of paper folding. They found that the programme is very successful and popular when teaching plane geometry and solid geometry. Students like and praise this programme. They are fascinated with the model preparation and enjoy learning about the geometry embodied naturally in the process of folding. Moreover, the findings indicate that they recall geometrical names better when they learn according to the Origametry method. The physical process of creation is certainly the best process for learning any subject; the more so for learning the principles of geometry (Golan & Jackson, 2004).

Paper folding allows description of all the geometrical axioms similarly to constructions made with a ruler and calliper. Hence, all the constructions of the Euclidean geometry in the plane (except for the issue of circles) can be performed by origami (Alperin, 2000). Many students, not necessarily those with learning disabilities, sometimes encounter difficulties with abstract mathematical terms. By folding instructions and hand activity we can help them to solve the "mystery" of abstract terms.

Origami, as ancient art, employs both previous knowledge and active learning. Thus, by gradual instructions (step-by-step), spatial comprehension, right schemes as well as a logical and correct mapping of terms are being constructed (Sze, 2005a, 2005b).

Paper folding is not a trivial and one-time project. This is a rich field in mathematics, having the advantage of immediate accessibility, availability and benefit for middle school students. It presents the geometrical notions to the students and encourages assessment of convincing demonstrations and explanatory demonstrations. Paper folding is attractive and appealing and it encompasses an effective benefit which traditional geometrical demonstrations greatly lack (Coad, 2006).

The present study explored to what extent paper folding affected the comprehension of geometrical terms.

The research questions are:

1. To what extent teaching by means of paper folding affects middle school female students' learning and comprehension of geometrical terms?

2. To what extent teaching by means of paper folding affects female-students' motivation for and pleasure from geometry classes?

The research hypothesis is grounded on the fact that middle school female students who, in primary school have been accustomed to use tangible means for learning, will improve their geometry comprehension through paper folding activities. The use of this aid will facilitate their assimilation of new terms and will enhance their knowledge of intelligent and accurate definitions. Moreover, the students will develop a more positive attitude towards deductive geometry classes following these activities.

Methodology

The Research Population

The research population was small and consisted of eleven female-students learning at the 8^{th}-grade (ages 13-14) on a medium tracking level in an academic school[2] at the centre of Israel.

The Research Procedure

The research lasted four weeks, one hour per week. At the beginning of the study, the students had basic knowledge of the terms: median, altitude and angle-bisector of triangles. Previous learning was done by the usual frontal way. They practised with the textbook and the mathematics teacher reported that the students have failed to master the terms, which tended to confuse them. In the first week of the study the students were given a preliminary test dealing with the special segments (pre-test). Immediately following the test, the students participated in another activity (a sports lesson). This was deliberately in order to prevent them from comparing the results and learning from their mistakes. The next two weeks constituted the experience process, during which the students had two lessons about special segments and, this time, these lessons were based on teaching by means of paper folding.

During the lessons the students performed directed folding assignments, which presented medians, altitudes and angle-bisectors in various triangles, not necessarily isosceles or equilateral triangles. This activity was performed during a mathematical discourse about correct or incorrect folding in order to obtain each of the above segments. Bisecting a segment is achieved by putting two ends of a segment one upon the other, the straight line of folding is the middle perpendicular of that segment and the meeting point of the straight line with the segment is the centroid of the segment. Finding the median requires two folding stages: first, finding the centre of the side and marking it and, then, folding around a straight line, whose ends are the centre of the side and the third vertex. An angle-bisector is achieved by folding the sides of an angle one upon the other. The angle-bisector is located on the straight line of the folding. The altitude of a triangle side is located on the straight line passing through the vertex and is perpendicular to the opposite side or on the continuation of that same side. In order to obtain that straight line, on which the

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[2] Teaching theoretical subjects as opposed to a vocational school

altitude is located, the side should be folded upon itself, the straight line of the folding passing through that vertex.

At the end of the third week of the experience, an interview with seven out of eleven students was conducted (Appendix B). In the fourth week, namely one week after the lessons, the students were given another test which was identical to the first one (post-test).

The Research Method

The present study used both quantitative and qualitative methods, thus allowing a more thorough and valid research[3].

As indicated, first the students were given a test consisting of six questions (Appendix A), some close-ended and some open-ended. Questions No. 1-3 were open-ended, the answer given by means of drawing. The students were asked to draw medians, altitudes and angle-bisectors of triangles.

These questions investigated the students' visual comprehension of special segments. Question No. 4 dealt with a common mistake, namely confusing an angle-bisector with a median. Question No. 5 examined the ability to identify and match drawn representations and the term name. Question No. 6, an open-ended question, was divided into three items and, in each item, the students had to define one of the special segments of triangles. This question aimed to examine the students' extent of comprehension and accuracy of term definitions. The post-learning test was identical to the previous one. Both the pre- and post-tests were quantitatively analysed.

The interview with the students was open-ended, designed to check how they felt following their experience with paper folding. The interview was qualitatively analysed.

The Research Limitations

The present study was conducted with a small number of participants, over a short period of time and only on a particular topic. Hence, it is recommended adopting this approach when coping with a larger sample and additional geometrical contents.

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[3] The use of several data sources enables triangulation and validation of the data (Payne, 1999).

The Research Findings and Analysis

The present study aimed to explore to what extent paper folding activities enhanced female-students comprehension of terms associated with special segments of triangles.

The research findings showed that the average score (in %) in the test conducted prior to the paper folding activity was 23. In the test conducted following the paper folding activity the score increased to 62. Moreover, the findings illustrate improved knowledge in each of the questions.

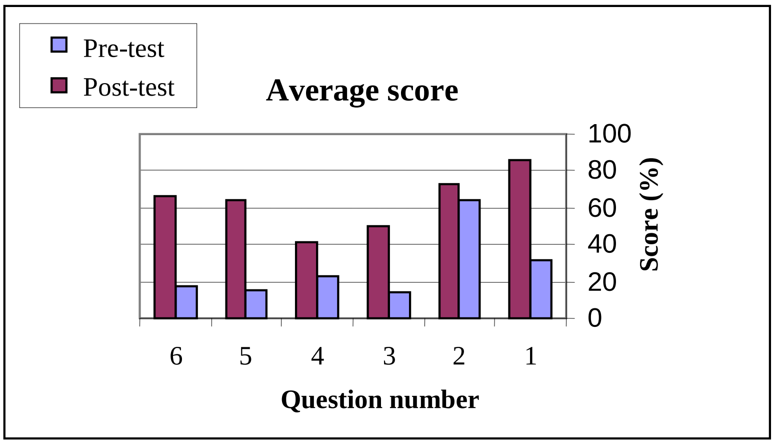

Figure 1. Test scores divided by questions (in %)

Figure 1, describing the test score distribution, shows that in certain questions the improvement was considerable while in others it was smaller. However, we can clearly see the improvement in all the questions.

In order to discuss more in detail the research findings, we will relate to the change in the students' scores from different aspects of term comprehension in accordance with the characterisation of the questions included in the test. Questions Nos. 1, 2, 3 and 5 examined visual comprehension of the special segments; hence we will discuss them as a unit by itself. Question No. 4 investigated a typical mistake, whereby students tend to attribute to a specific segment a property which does not belong to it. Question No. 6 explored the students' ability to define learnt terms. The following figures will specify the research findings in each unit separately.

Research findings in questions examining visual knowledge

Questions Nos. 1, 2, 3 and 5 examined the extent of identification and knowledge of terms from a visual point of view. In questions Nos. 1-3 the students were asked to draw by themselves special segments and in question No. 5 they were asked to identify drawn segments. In the pre-test, the average score (in %) of all the visual questions was 29.3. In the post-test, the students' average score (in %) was 67.7, namely an improvement of 38.4%.

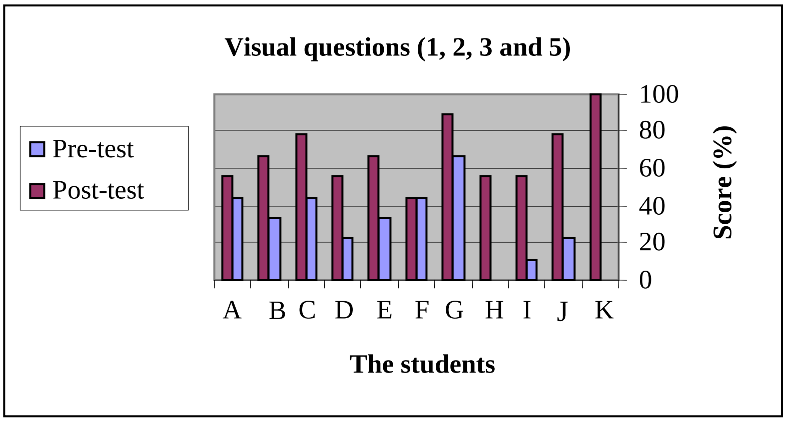

Figure 2. Scores (in %) obtained in the questions checking visual comprehension

Figure 2 illustrates that all the students improved their attainments in these questions. Two students received 0 in the pre-test, that is, they had absolutely no comprehension of the terms checked from a visual point of view whereas in the post-test these students improved their scores. One of them even scored the full number of possible points.

Research findings in the question checking mistaken perceptions

Question No. 4 examined typical mistakes of confusing the properties of special segments of triangles. This question presented a triangle with an angle-bisector and the students were asked about the size of the angles formed by the angle-bisector as well as about the parts of the side which it reached. In this question, the average score in the pre-test was 22.7% while in the post-test it increased to 40.75%, namely an improvement of 18.05%.

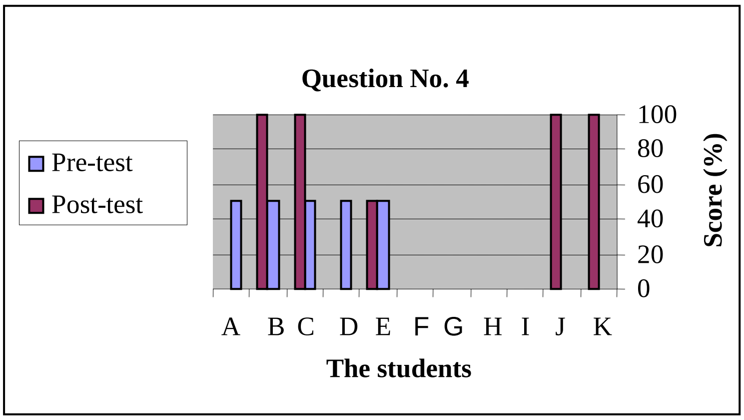

Figure 3. Scores (in %) obtained in question No. 4

The improvement attained is relatively smaller than the improvement in the other questions. However, checking the improvement of each and every student, figure 3 illustrates that four students achieved a score of 0 in the pre-test and got the same score in the post-test. These students demonstrated no improvement in the typical mistakes and the confusion between the terms. Two students manifested a regression, so that in the pre-test they scored only half of the points in percents whereas in the post-test both of them scored 0. Four other students greatly improved their attainment. Two scored 0 in the pre-test and two others attained half of the possible points. However, in the post-test, all four of them scored the full number of possible points.

Research findings in the question checking definition ability

In question No. 6, which checked the students' definition ability, the average score in the pre-test was 17.2%, whereas in the second test, the average score was 66.6%, namely an improvement of 49.4%.

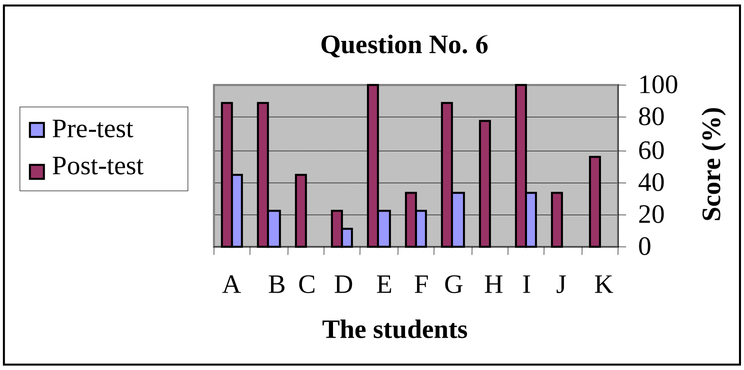

Figure 4. Scores (in %) obtained in question No.6

Figure 4 illustrates that all the students improved their attainments in this questions. The four students, who were entirely unable to define the terms in the pre-test and whose score was 0, improved their definition ability. Tables 1-3 present the definitions given by the students prior to and following their experience with paper folding.

Table 1

Definitions of the term "median" as written by the students (translated verbatim)

Definitions / Students	Pre-test	Post-test
A.	From the median, the line goes straight down to the base resulting in 90°	The line goes downward from the vertex, dividing the base intotwoequal parts
B.	Less than 90°	Starts from the angle
C.	Bisects the side	A segment going out of the vertex, bisecting the opposite side
D.	----	A segment going out of the vertex and bisects the middle
E.	A side-bisector line	A segment going out of the vertex, bisecting the opposite side

Definitions / Students	Pre-test	Post-test
F.	A line indicating 90°	Segment of a triangle going downward from the vertex
G.	A line going out of the vertex until the base side, creating 90°	A segment going out of the vertex, reaching the opposite side and bisecting it intotwo equal segments
H.	-----	A segment bisecting the angle
I.	-----	A segment bisecting the base side of the triangle
J.	Bisecting the side	A segment going out of an angle, bisecting its opposite side
K.	A segment bisecting the side into two parts	A segment going out of the vertex, bisecting the side

Perusal of the definitions of the term "median" from Table 1 indicates that:

1. In the pre-test, three students did not answer at all the question whereas in the post-test, all the students answered the questions.
2. In the pre-test, only one student used the term "segment" whereas in the post-test, nine out of eleven students used this term.
3. In the pre-test, four out of eleven students knew that a median bisects a side. On the other hand, in the post-test, eight out of eleven students knew that it bisects a side (although not all of them gave an accurate definition).

Table 2
Definitions of the term "angle-bisector" as written in the tests(translated verbatim)

Definitions / Students	Pre-test	Post-test
A.	Bisects the base in an equal manner	It bisects the angle (and it's impossible to know whether it's equal)
B.	A segment situated at the middle of a triangle	Bisects the angle into two equal parts
C.	Divides the angle into two angles equal to each other	A segment going out of the vertex, bisecting the angle

Definitions / Students	Pre-test	Post-test
D.	An angle bisecting the triangle into two parts	A segment going out of the vertex, bisecting the angle into two equal parts
E.	A line bisecting some angle, the two sides being equal	A segment bisecting an angle into two different halves
F.	A line bisecting the triangle angle	A segment of a triangle which bisects the angle
G.	A line bisecting the vertex angle	A segment dividing the vertex angle intotwoequal parts
H.	Bisecting the triangle angle	A segment going out of the vertex and bisects the triangle in the middle
I.	-----	A segment going out of the vertex and bisects it
J.	Divides the angle into two	A segment going out of an angle exactly in the middle – bisects the angle
K.	A segment bisecting an angle into two parts	A segment going out of the vertex and bisects the angle

Perusal of the definitions of the term "angle-bisector" from Table 2 indicates that:

1. In the pre-test, one student did not answer at all whereas in the post-test all the students answered the question.
2. In the pre-test, two students used the term "segment" whereas in the post-test, nine out of eleven students used this term.
3. In the pre-test, eight out of eleven students knew that the property of the angle-bisector is bisecting an angle of a triangle whereas in the post-test all the students knew this property.

This finding is not surprising since the angle-bisector property is embodied in the name of the examined term. Hence, most of the students are not mistaken in the definition. Please note that student "I" added the following words to the definition: "…and it's impossible to know whether it's equal". In these words, she probably wanted to emphasise what had been said during the lesson, namely that the angle-bisector does not necessarily bisects the side.

Table 3
Definitions of the term "altitude" as written in the tests(translated verbatim)

Definitions / Students	Pre-test	Post-test
A.	The line goes down diagonally so that an equal base is achieved	A line going perpendicularly out of the vertex, creating a 90° angle
B.	More than 90°	Bisects the triangle into 90°
C.	------	A segment going out of the vertex, creating 90°
D.	------	Altitude going out of the vertex, creating 90° with the base of the altitude
E.	A line forming 90° in a triangle	A segment going out of the vertex, forming 90° with the opposite side
F.	Going out of the vertex perpendicularly	A segment of a triangle which equals 90°
G.	Going out of the vertex, reaching the base and creating two equal parts and 90°	A segment going out of the vertex until the opposite side, forming 90°
H.	The triangle altitude	A segment going out of the vertex and bisects in a straight way (90°)
I.	-----	A segment forming 90° in a triangle
J.	From the middle of the side and upward	A segment going out of the angle, forming 90° with the base of the altitude
K.	-----	A segment going out of the vertex, forming 90°

Perusal of the definitions of the term "altitude" from Table 3 indicates that:

1. In the pre-test, four students did not write any definition whereas in the post-test, all the students defined the term.
2. In the pre-test, none of the students used the term "segment" whereas in the post-test eight out of eleven students used this term.
3. In the pre-test, two out of eleven students knew that the altitude forms a right angle with the side, whereas in the post-test all the students knew that it forms a right angle (although not all of them gave an accurate definition).

Analysis of the definitions shows that most of the students started the definition with the word "segment" in the pre-test, while in the post-test, almost none of them wrote it. The findings indicate that the students realised they should begin the definition with a general and accurate word (segment) and then describe the segment's property. Moreover, the findings illustrate that most of the students realised that in a definition they should write a clear and unequivocal property of the term which they were defining.

Findings obtained from Analysing the Discourse in the Interview

The interview was conducted as a free conversation with the students, after having attended the two paper folding lessons and prior to the post-test (Appendix B). The students responded willingly to the interview and it was apparent they were pleased to share their feelings after the lessons. Only seven students participated actively in the interview. The interview analysis showed five key indices: pleasure, comprehension, positive attitude towards the paper folding method, a wish to study regularly according to this method and a sense of self-confidence.

Examples:

- **Pleasure***:* "It was fun. We want more classes like that" (No. 2); "I wish all the classes were like that" (No. 3); "It was really fun and I also understood much more" (No. 4); "It was more fun than a regular lesson and I too understood (No. 5); "it's fun to have diversified lessons and it's not so boring" (No. 10).
- **Comprehension**: "Yes, I really understood the material well. When the teacher explains on the board I understand nothing" (No. 13); or "I am the stupidest student in the class and even I managed 'to make the folding'" (No. 15); "Now this subject seems really easy to me" (No. 16); "I feel that now I understand the material because I actually saw it" (No. 21).
- **Positive attitude towards the paper folding method**: "True, when we folded I actually understood" (No. 14); "we actually saw these segments and all their properties" (No. 19).
- **A wish to study regularly according to this method**: "But if we have already done an angle-bisector, median and altitude by means of paper folding, so what can't we do with it? I believe everything can be done in this way" (No. 30); "If this is how we understand, I believe we have to continue studying in this way. Perhaps in medium tracking level it is suitable because, in fact, we understood better" (No. 35).
- On the other hand, some students thought that "it is good in order to make us feel that we understand but, obviously, we can't learn the entire subject of geometry in this way" (No. 29); "I don't think so. High school geometry is much more difficult" (31).

- **Sense of self-confidence:** "I am sure I will do better" (No. 23); "Me too. It is really easy now" (No. 24); "I believe I will manage better" (No. 39). Conversely, students expressed also the following assertion: "Don't be sure that we will succeed. We are not that smart" (No. 38), or "We are learning on a medium tracking level so don't raise your expectations too high" (No. 41).

Summary of the findings indicates a progress in the students' attainments in paper folding, term comprehension and the ability to define them correctly. Moreover, they demonstrate a sense of experience and pleasure from the lesson as well as an enhanced motivation to study geometry.

Discussion

The present study focused on middle school female students, who are at the beginning of their way in deductive geometry studies. Elementary school students are required to comprehend geometry on the two first levels according to Van-Hiele. When they reach middle school, however, geometry studies require enhanced thinking skills, i.e. formal and informal deduction level (Fuys, Gedds & Tischler, 1988; Van Hiele, 1986, 1999). This transition makes it difficult for the students, who are accustomed to learning tangible geometry and using measuring tools in order to reason and prove arguments (Gal & Linchevski, 2002). By using paper folding, many geometric activities can be performed. For example: bisecting an angle, drawing a perpendicular, bisecting a segment, etc. However, the activity is tangible and the students "create with their hands" the learnt term or the geometrical demonstration (Gray & Kasahara, 1985, 1997).

In the present study, a group of female-students studied geometrical terms in a tangible way, by folding by themselves the altitudes, medians and angle-bisectors of triangles and, thus, actually "felt" the terms. The students learnt in practice the difference between a "bisector" and a "divider". By placing both parts of the side one upon the other, they noticed whether a bisection or division of a segment has occurred. The research findings illustrate that this activity reinforced their comprehension of the segment properties. The use of non-isosceles triangles during the lesson, emphasised the differences between the special segments and enhanced the special property of each of them in a tangible manner.

When asked to fold an altitude of a triangle, all the students folded a middle perpendicular due to the tendency to fold the page at its middle. The mistake made them go back to the altitude definition and fold the paper correctly, leaving the folding mark of the middle perpendicular. This activity highlighted the differences between the two segments. Moreover, folding the altitude upon itself, helped the students comprehend that the altitude does not bisect the angle from which it comes out. All the students found it difficult to fold the median because it necessitated two stages in the action of folding: finding the middle of the side and marking it and then folding the median. This experience, too, left its impact on the students, facilitating their recollection of the median property. A conversation conducted after the lesson showed that the students greatly enjoyed the lesson. Some of them told that, generally, they do not like mathematics lessons, are afraid of them and do not understand the material. Similarly, Zaslavsky (1994) maintains that there is a considerable interaction between mathematics anxiety and the student's preferred learning method. Most of the students indicated that the paper folding lessons were "fun" and it was obvious they enjoyed them. The students themselves felt that they had actively participated in the lesson, contrary to regular lessons in which the students did not actively work and did not feel part of the lesson because they failed to understand or were bored. They reported that they understood the subject of special segments. Some of them even defined the subject as "easy", explaining that the understanding stemmed from the practical activity which made the terms more concrete. Another positive comment expressed in the conversation was "the teacher helped each of us". The paper folding activity entailed participation of all the students and, at the same time, made the teacher pass among the students, helping each of them to make the correct folding. Thus, every student "experienced" the terms by herself.

During the conversation, some of the students said they were not sure paper folding was indeed geometry. Consequently, we have to conclude that when teachers use paper folding in order to teach geometry, they must emphasise to the students that the activity is geometry and support this argument with demonstrations.

When asked about another test, some of the students displayed self-confidence, which resulted from the activity they had experienced while some of them were still unsure of themselves. It should be pointed out that the activity was conducted only once and, hence, it has not yet promoted the confidence of all the students. We can surmise that a more extensive activity, resulting in higher attainments, will increase the students' self-confidence.

The findings illustrate that the paper folding activity greatly enhanced the participating students' term comprehension of special segments of triangles. This was manifested by their achievements in all the questions. Moreover, in the overall score, a constant pattern of the students' higher scores was found. The greatest advancement was made in the question relating to the definitions, whereby an improved comprehension of definitions in general was manifested, rather than only in the specific studied subject. The students themselves folded the triangles and, thus, related in particular to the property typical of each segment. This activity taught the students that these properties were unique to this segment and it had no other properties.

The smallest improvement occurred in the question checking mistakes and confusion between terms. The findings show that the students understood and learnt the properties of each segment. Nevertheless, when they were looking at the drawing in which the angle-bisector looked similar to the median, they found it difficult to avoid sticking to "what we see" and to concentrate only on the definitions and data given in the question.

Conclusions and Recommendations

The findings of the present study show that manual experience did improve the students' comprehension and recollection of the terms: median, altitude and angle-bisector of a triangle. Olson (1975) argues that active experiencing is the most effective learning. Similarly, the present study indicates that the students' experience with paper folding was indeed effective. This can be due to the fact that, by using one's hands while folding, following instructions, recollecting and comprehending becomes much easier. This conclusion is in line with Benjamin (1995), who illustrated his words with a Chinese proverb: When I see - I identify, when I hear – I remember, when I do – I understand.

The study was conducted with a small group of medium tracking level female-students learning in a middle school. However, based on the test results analysis and the conversation after the lesson, we can conclude that paper folding activity might enhance and improve term comprehension and geometrical mistakes in middle school. Middle school students are not yet competent in the formal demonstrations system and the use of practical tools can lead them to understand and even formulate arguments by themselves.

It is recommended taking chapters in plane geometry, in which students encounter difficulties to understand abstract terms, and let them experience paper folding in connection with these terms. Not only do such lessons expand the students' comprehension; they diversify mathematics lessons and stimulate more interest and motivation of the students. In such lessons, active participation of all the class students can be expected. The teacher should prepare the lesson in advance and get organised for the mathematical discourse and the geometric reasoning of each step of the paper folding activity.

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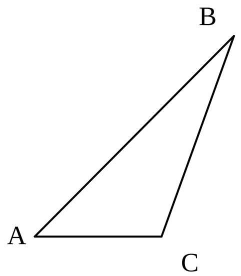
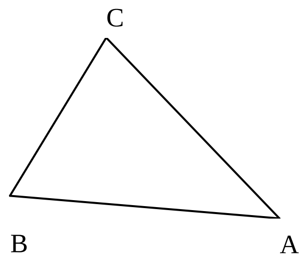
Appendix A – The Test (Pre and Post)

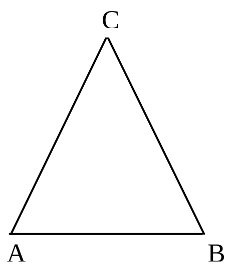
Test on the subject of: Special Segments in Triangles

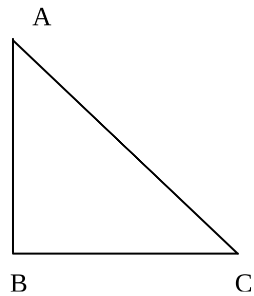
1. In the following triangles please mark a segment which can be a median to side AB.



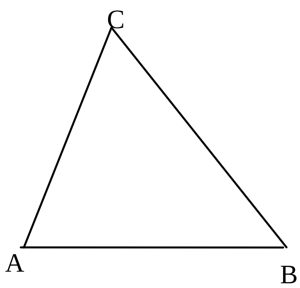


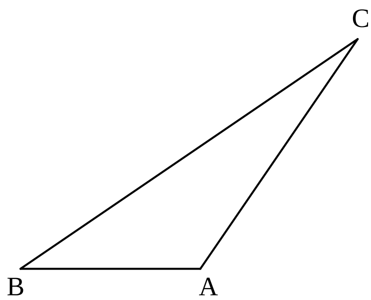
2. In the following triangles please mark a segment which can be an angle-bisector of $\angle B$.





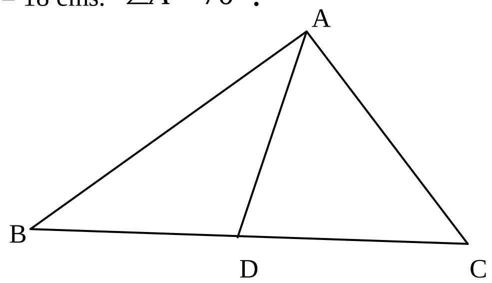
3. In the following triangles please mark a segment which can be the altitude to side AB.





4. Please look at triangle ABC. AD is an angle-bisector.

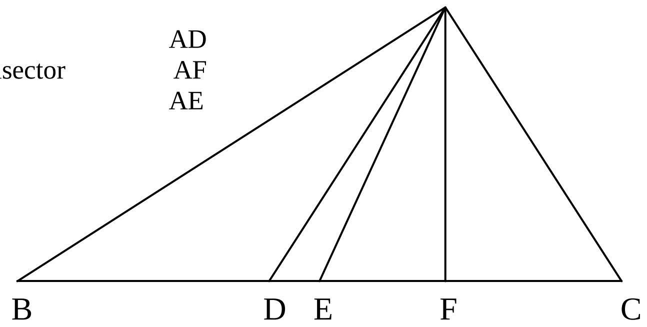
Given: BC = 18 cms. $\angle A = 70^0$.



- Can you calculate the length of BD?
  If yes, please calculate: ___________________________
  If not, please explain why not: ____________________
- Can you calculate the $\angle BAD$ angle?
  If yes, please calculate: ___________________________
  If not, please explain why not: _____________________

5. Please look at triangle ABC and try to match the segments with their names:

Altitude	AD
Angle-bisector	AF
Median	AE



6. Please define in words:
   A. Median in a triangle: ________________________
   B. Angle-bisector in a triangle: __________________
   C. Altitude in a triangle: _______________________

Appendix B – Interview Transcription

1. **Interviewer:** Dear students. I would like you to tell me a little about the special geometry classes which you had last week.
2. Student A: it was fun. We want more classes like that (2).
3. Student B: I wish all the classes were like that (3).
4. Student D: It was really fun and I also understood much more (4).
5. Student F: It was more fun than a regular lesson and I too understood (4).
6. Student K: I have always hated mathematics classes but this was a fun lesson.
7. Student A: Right, I always sleep during the lesson…
8. Student D: I never participate in mathematics classes but this was different.
9. All the others concur, nodding their heads in agreement.
10. Student H: It's fun when the lessons are diversified and not so boring.
11. Student F: I would like to see more demonstrations in geometry by means of paper folding.
12. **Interviewer:** You have all used the word "fun" many times. Have the lessons given you something other than "fun"?
13. Student D: Yes, I really understood the material well. When the teacher explains on the board I understand nothing.
14. Student J: Right, when we folded I actually understood.
15. Student D: I am the most stupid student in class and even I managed to fold the paper.
16. Student H: Now I find this subject very easy.
17. **Interviewer:** Can you try explaining why it is more comprehensible than a regular lesson?
18. Student D: Because it was tangible.
19. Student H: we actually saw these segments and all their properties.
20. Student A: And the teacher also helped each of us to fold the paper correctly.
21. Student B: I feel that now I understand the material because I actually saw it.
22. **Interviewer:** Do you think you could do better in a test on this subject if it comprised regular questions with regular drawings?
23. Student D: I am sure I will do better.
24. Student H: Me too. It is really easy now.
25. Student A: Perhaps it's just a sense of comprehension. I don't know if I will pass the test because drawings always confuse me.
26. **Interviewer:** Do you believe all geometry classes can be learnt by means of paper folding?
27. Student B: I don't think it can work with the material learnt in high school.
28. Student A: I wish it were possible but I don't think so.
29. Student H: It's good because it makes feel we understand but surely it's impossible to learn the entire subject of geometry in this way.

30. Student F: But if we have already done an angle-bisector, median and altitude by means of paper folding, so what can't we do with it? I believe everything can be done in this way.
31. Student D: I don't think so. High school geometry is much more difficult.
32. Student A: True. It's real nice and it's fun but this is not actually geometry.
33. Student J: I don't believe our teacher will consent to teach everything in this way. She always demands accurate writing with "givens" and "we should demonstrate that…"
34. Student H: I talked to my mother who is a middle school teacher. She thought it was great that we were given such a lesson and that it was a good idea but she believed this were not real geometry studies.
35. Student F: Why not? If we understand in this way, I think they should continue teaching us like that. Perhaps it is more suitable for medium tracking level. It's a fact that we understood better by using this method.
36. Student D: Do you think that the teacher will agree? Never!
37. **Interviewer:** I understand that, after the lesson, you felt you understood the material. I am glad you feel that way and after a certain period of time we will give you another test to check if you have really understood.
38. Student F: Don't be so sure that we will succeed. We are not that smart.
39. Student D: I believe I will do better.
40. Student J: Me too.
41. Student K: Well, teacher, you know, we are medium tracking level, so don't have high expectations…
42. **Interviewer:** Thank you all.