The Mathematics Educator
2008, Vol. 11, No.1/2, 79-88

Teachers' Mathematical Values for Developing Mathematical Thinking in Classrooms: Theory, Research and Policy

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Abstract: The paper discusses assisting teachers to nurture mathematical thinking in their students by using findings from research on mathematical values. The author begins by sharing three theoretical perspectives on how mathematical thinking develops in a student from Lancy (1989), Billett (1998), and Bishop (1988). Using White's (1959) three component analysis of culture, the author presents 6 mathematical values which are important to the development of Mathematics, and thus underpinning the development of mathematical thinking in the classroom. An exploratory Values and Mathematics Project (VAMP) shows that teachers found it difficult to discuss values they held about Mathematics education in relation to Mathematics. The introduction of some of the theoretical terminology helped teachers to discuss their teaching. In conclusion, the author proposes some implications for practice and policy.

Key words: Developing mathematical values; Mathematical thinking; Mathematics learning and teaching

Introduction

Mathematical thinking sounds like an essentially psychological topic. It seems to be just another branch of thinking, and therefore part of the psychological field of knowledge. However, we can never observe mathematical thinking—we can only observe what we assume to be its products, namely mathematical ideas and processes. But we can observe what conditions and contexts might have been responsible for the products of mathematical thinking, which brings us rather closer to the social context.

Mathematical Thinking from a Sociocultural Perspective

So what is the problem we are trying to consider here? In one sentence the major problem seems to be: "How can teachers help mathematical thinking to develop in their students?" A subsidiary problem is "How can research on values help with this?" and this is the problem I will focus on here. Also because of my research work in the field of mathematics education, I prefer to consider values and mathematical thinking not from a psychological perspective but from a socio-cultural perspective.

This perspective also makes me be clear that in considering cultural values, I should not claim these values to be 'pan-cultural' – indeed values are clearly specific to cultures. In my earlier work I also separated mathematics (with a small 'm' as the general pan-cultural area of knowledge, which all cultures develop) from Mathematics (the specific academic subject area which many societies now subscribe to) which is sometimes referred to as Western mathematics. Thus I will discuss in this paper the values related to Mathematics, and Mathematical thinking, and not claim any general connection with pan-cultural mathematical activity.

Three Theoretical Ideas

In trying to make research progress in solving the problem of helping Mathematical thinking develop, I believe we need to consider carefully any theoretical perspectives which might assist us. I will present here three theoretical ideas which I have found helpful in my research and which I believe can shape our understanding of the problem and lead to potential pedagogical solutions.

Lancy's (1983) developmental theory of cognition

David Lancy (1983) is a cultural psychologist who, in his major cross-cultural study in Papua New Guinea, developed a new stage theory of cognition. It was based on Piaget's theories but he developed them from a socio-cultural perspective. He was doing his research in Papua New Guinea, a very different culture from his own, and through investigating cognition with students in Papua New Guinea, he found that the theoretical developmental sequence of Piaget's stages were similar to, but not identical with, those which Piaget found in his European-based research.

Lancy found that Stage 1 was very similar to Piaget's sensori-motor and early concrete operational stages. He argued that this stage is where genetic programming has its major influence, and where socialisation is the key focus of communication. Many activities involving the child are completely similar across cultures.

He then argued that Stage 2, a later concrete operational stage, is where enculturation takes over from socialisation. As he says: "Stage 2 has much to do with culture and environment and less to do with genetics", and he demonstrated that this is the stage where different cultures will emphasise different knowledge and ideas. Even in relation to mathematics (which is where ethnomathematics develops) this is the case.

The big development in Lancy's theory from Piaget's is seen in Stage 3 which concerns the meta-cognitive level. Lancy says: "In addition to developing cognitive and linguistic strategies, individuals acquire 'theories' of language and cognition." Different cultural groups emphasise different 'theories of knowledge' and Piaget's 'formal operational' stage is one such theory of knowledge emphasised in Western culture. Confucian Heritage Cultures emphasise other theories of knowledge, for example. These theories of knowledge represent the ideals and values lying behind the actual language or symbols developed by a cultural group.

Thus it is in Stages 2 and 3 that values are inculcated in the individual learners. In a classic work by Kroebner and Kluckholm (1952) they strongly support this idea: "Values provide the only basis for the fully intelligible comprehension of culture, because the actual organisation of all cultures is primarily in terms of their values" (p. 340).

Thus for our problem, the idea of Mathematical thinking as a form of meta-cognition, affected by the cultural norms and values of the learner's society, is helpful.

Billett's (1998) analysis of the social genesis of knowledge

But where do these norms, values and knowledge come from, and how can we think about them from a more educational perspective? Stephen Billett's (1998) sociological work analyses and locates what he calls "the social genesis of knowledge" in 5 inter-relating levels:

Socio-historic knowledge. Socio-historic knowledge factors affect the values underpinning decisions made by both institutions and teachers. It is knowledge coming from the history and culture of the society, and is therefore value-laden knowledge.

Socio-cultural practice. Socio-cultural practice is defined by Billett as historically derived knowledge transformed by cultural needs, together with goals, techniques, and norms to guide practice. At the institutional level these are manifested by curricular decisions influenced by such factors as: (a) current institutional management philosophy with respect to educational and social values (*in loco parentis*); (b) State or national curricular frameworks and (c) the ethos of the mathematics faculty or teacher's peer group.

The community of practice in the classroom. The community of practice in the classroom is identified by Billett as particular socio-cultural practices shaped by a complex of circumstantial social factors (activity systems), and the norms and values which embody them. This community is influenced by (a) the teachers' goals with respect to, and portrayal of, pedagogical values, (b) students' goals and their portrayal of learning values, and personal values.

Microgenetic development. Microgenetic development is interpreted by Billett as individuals' (teachers' and students') moment-by-moment construction of socially derived knowledge, derived through routine and non-routine problem solving. The nature of teaching as a profession is reflected in the *relative* autonomy assumed within the walls of the classroom, where teachers' decisions are constantly being made or revised on the basis of a continuous flow of new information. The instantaneous nature of many decisions is likely to be influenced to a greater or lesser extent by the teacher's internalised sets of values.

Ontogenetic development. Ontogenetic development includes individuals' personal life histories, socially determined, which furnish the knowledge with which to interpret stimuli; this development includes participation in multiple overlapping communities.

This analysis points to the different sources of influence on Mathematics teachers' values. Billett's categorised knowledge is a powerful indicator of how different knowledge at these five levels can impinge on and influence teachers' values in the classroom.

Bishop's (1988) socio-cultural dimension and its levels

As stated above, my research context has been in the field of culture, and especially with considering mathematics as a form of cultural knowledge, with Mathematics as its 'Westernised' and academic version.. When we are considering how to develop values in relation to Mathematical thinking, I also believe we need to keep in mind the socio-cultural dimension of mathematics education. This dimension influences the values of mathematical thinking at five levels, which are similar but different to Billett's levels.

Cultural level. The overarching culture of the people, their language, their mathematics, their core values. In Billett's levels he combined together the cultural and the societal, which I believe in the case of mathematics education is not helpful. Evidence from research at the cultural level shows how different ethnomathematical ideas are not necessarily related to similar societal structures. Ethnomathematics points to specific cultural influences on mathematical thinking.

Societal level. The social institutions of the society, their goals, and their values regarding mathematics education. In most societies mathematics education is a contested field with many proponents of different educational 'solutions' vying for publicity and academic advantage. They inevitably affect what is considered to be important mathematical thinking, and who is capable of doing it.

Institutional level. The educational institutions' values and the place of mathematics within them. At this level we can see the ways institutional values influence the curriculum, the timetable and even the allocation of teaching space to each subject. These values also affect the development of mathematical thinking in different groups of students.

Pedagogical level. The teachers' values and decisions, the classroom culture of mathematical thinking. This is the same level as Billett's 'community of practice', and I confess that I prefer Billett's description of this level, as it emphasises the contribution of both teacher and students to the classroom knowledge culture.

Individual level. Individual learners' values and goals regarding mathematics, and mathematical thinking, which can differ markedly, and which do not necessarily follow the teachers' values and goals.

I will draw on these three perspectives in the rest of this paper, and I will start by assuming that my ideas about values regarding Mathematical thinking are:

1. Concerned with developing meta-cognition;
2. Located within the socio-cultural dimension;
3. Focused on the community of practice in the classroom.

Components of Mathematical Values

Now we turn to the values problem stated in Section 1 above. Building on the above analysis, I realised firstly that it was necessary to distinguish between three kinds of values:

- Mathematical values: values which have developed as the knowledge of Mathematics has developed within 'Westernised' cultures.
- General educational values: values associated with the norms of the particular society, and of the particular educational institution.
- Mathematics educational values: values embedded in the particular curriculum, textbooks, classroom practices, etc. as a result of the other sets of values.

My research approach to values and Mathematical thinking has to date been to focus on **Mathematical** values, and on the actions and choices concerning them (see Bishop, 1988, 1991, 1999). In my work I have used White's (1959) three component analysis of culture:

- Ideological component: composed of ideologies, dependent on symbols, philosophies,
- Sentimental (attitudinal) component: attitudes, feelings concerning people, behaviour,
- Sociological component: the customs, institutions, rules and patterns of interpersonal behaviour.

So how are these components interpretable in terms of Mathematical values and Mathematical thinking? I will describe the six values in more detail here, and also add several questions which readers can ask of themselves, as teachers.

The Ideological component of Mathematical values

In regard to this component of the Mathematical culture, I argued (Bishop, 1988, 1991) that the critical values concern *Rationalism* and *Objectism*.

Valuing **Rationalism** means emphasising argument, reasoning, logical analysis, and explanations, arguably the most relevant value in Mathematics education. To test this for yourself, ask yourself (as a teacher):

Do you encourage your students to argue in your classes?
Do you have debates?
Do you emphasise Mathematical proving?
Do you show the students examples of proofs from Mathematics history (for example, different proofs of Pythagoras' theorem)?

If you do, I would say that you are encouraging the value of Rationalism.

Valuing **Objectism** means emphasising objectifying, concretising, symbolising, and applying the ideas of Mathematics. Ask yourself:

Do you encourage your students to invent their own symbols and terminology before showing them the 'official' ones?
Do you use geometric diagrams to illustrate algebraic relationships?
Do you show them different numerals used by different cultural groups in history?
Do you discuss the need for simplicity and conciseness in choosing symbols?

If you do, you are encouraging the value of Objectism.

The Sentimental (Attitudinal) component of Mathematical values
In regard to this component, the important values are, I argued, *Control* and *Progress.*

Valuing **Control** means emphasising the power of Mathematical knowledge through the mastery of rules, facts, procedures and established criteria. Ask yourself as a teacher:

> *Do you emphasise not just 'right' answers, but also the checking of answers, and the reasons for other answers not being 'right'?*
> *Do you encourage the analysis and understanding of why routine calculations and algorithms 'work'?*
> *Do you always show examples of how the Mathematical ideas you are teaching are used in society?*

If you do, you are encouraging the value of Control.

Valuing **Progress** means emphasising the ways that Mathematical ideas grow and develop, through alternative theories, development of new methods and the questioning of existing ideas. Ask yourself:

> *Do you emphasise alternative, and non-routine, solution strategies together with their reasons?*
> *Do you encourage students to extend and generalise ideas from particular examples? Do you stimulate them with stories of Mathematical developments in history?*

If you do, you are encouraging the value of Progress.

The Sociological component of Mathematical values
In regard to this component, the important values are *Openness* and *Mystery.*

Valuing **Openness** means emphasising the democratisation of knowledge, through demonstrations, proofs and individual explanations. Ask yourself:

> *Do you encourage your students to defend and justify their answers publicly to the class?*
> *Do you encourage the creation of posters so that the students can display their ideas?*

Do you help them create student Mathematical newsletters, or web-pages, where they can present their ideas?

If you do, you are encouraging the value of Openness.

Valuing **Mystery** means emphasising the wonder, fascination, and mystique of Mathematical ideas. Ask yourself:

Do you tell them any stories about Mathematical puzzles in the past, about for example the 'search' for negative numbers, or for zero?
Do you stimulate their Mathematical imagination with pictures, artworks, images of infinity etc.?

If you do you are encouraging the value of Mystery.

These then are what I believe to be the values lying behind the development of Mathematics, and thus they are the crucial values underpinning the development of Mathematical thinking in the classroom. I think we will make good progress in solving our educational problems if more research is devoted to investigating ways of developing these values in our teachers, so that they can develop them in their students.

Values and Mathematical Thinking

These ideas form the background to our empirical work on values in Mathematics education, and here I present some ideas coming from our initial research with teachers. Researching values development is no easy matter, but in our exploratory Values and Mathematics Project (VAMP) we used the following approach with our teachers:

1. We interviewed the teachers before the lessons and asked them what values they thought they were going to develop in those lessons.
2. We then observed and video-recorded the lessons
3. We interviewed the teachers after the lessons to have them explain what they thought they had achieved in terms of values development.

The results showed that the teachers certainly held values about Mathematics and about Mathematics education. The teachers also had many goals in planning for lessons, often revealing their values, which included:

- the development of individual children

- helping them acquire Mathematics knowledge and gain confidence with doing Mathematics
- showing how Mathematics can be relevant to them personally as well as to society.

However the teachers found it difficult to discuss values in relation to Mathematics, but the introduction of some of the theoretical terminology above helped the teachers to discuss their teaching. They also chose to make explicit certain Mathematics or Mathematics education values or they 'showed' them implicitly. Also it was easier for the teachers to think about and recognise the values they were teaching, rather than to implement new values. This latter finding has many implications for the implementation of new policies in Mathematics education. More details of this research, together with downloadable papers, can be found at this address:

http://www.education.monash.edu.au/research/groups/smte/projects/vamp/vamppublications.html

Conclusions and Implications for Practice and Policy

1. With any educational policy developments it is essential to have good research as the foundation, as well as good theories to support and structure that research work. Researching new approaches is necessary but if we are to make substantial progress in developing Mathematics teaching then we also need solid theoretical foundations to those developments.

2. Mathematical thinking has been studied in many ways, but in relation to values it is useful to consider it as an aspect of meta-cognition and affectivity. It is not just doing Mathematical thinking which is important, but it is thinking about Mathematical thinking, and exploring the value preferences associated with that thinking.

3. The context for the developments should be the classroom, as it is there that the community of practice significantly influences the values of Mathematical thinking. This means that policy developments should be drawn up in such a way that they frame new practices rather than control and specify them. Our values research showed that the new theoretical ideas helped to frame teachers' ideas and activities, and gave them a language to discuss their practices, but did not control them.

4. Equally important to consider in this kind of research is the students' socio-cultural context, as any educational values are embedded in the culture of the particular society. Once again the implication for policy and practice development is

to frame teaching practice and not control it, so that teachers can respond appropriately to the needs and values of their particular students coming from their particular societal contexts.

5. Finally the teachers need special support in this research and development activity, as values teaching involves the teacher's own personal pedagogical identity, which must be respected (Chin, Leu & Lin, 2001).

References

Billett, S. (1998). Transfer and social practice. *Australian and New Zealand Journal of Vocational Education Research, 6*(1), 1-25.

Bishop, A. J. (1988). *Mathematical enculturation: a cultural perspective on mathematics education.* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Bishop, A. J. (1991). Mathematical values in the teaching process. In A. J. Bishop, S. Mellin-Olsen, & J. van Dormolen (Eds.), *Mathemaical knowledge: Its growth through teaching* (pp. 195-214). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Bishop, A. J. (1999). Mathematics teaching and values education—An intersection in need of research. *Zentralblatt für Didaktik der Mathematik, 31*(1), 1-4.

Chin, C., Leu, Y. C., & Lin, F. L. (2001). Pedagogical values, mathematics teaching, and teacher education: Case studies of two experienced teachers. In F. L., Lin, & T. J. Cooney (Eds.), *Making sense of mathematics teacher education* (pp. 247-269). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Kroebner, A. L., & Kluckholm, D. (1952). *Culture; a critical review of concepts and definitions*. New York: Vintage Books.

Lancy, D. F. (1983). *Cross-cultural studies in cognition and mathematics.* New York: Academic Press.

White, L. A. (1959). *The evolution of culture.* New York: McGraw-Hill.

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