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Using Case Discussion Materials to Improve Mathematics Teaching Practice

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Abstract: This paper discusses the development and implementation of a particular tool─cases and case discussions─for the professional development of teachers of mathematics at the secondary level. Case development and the use of cases to foster in-depth discussions about teaching and content knowledge offer a promising approach to help develop effective mathematics teaching practice in middle and secondary classrooms. The appendix to this article includes a case that illustrates how this tool describes, as completely as possible, practice in a mathematics classroom. The case presents multiple perspectives, including the views of the teacher, the students and other adults in the school. The case offers a depiction of a common teaching dilemma and represents a powerful tool for professional development.

Key words: Mathematics teachers' development; Mathematics teaching practices; Case discussions

> If we want schools to produce more powerful learning on the part of students, we have to offer more powerful learning opportunities to teachers at each stage of their careers. (Feiman-Nemser, 2001, pp. 1013-1014)

Introduction

The performance of many youngsters in mathematics classrooms in the United States is disappointing at best, dismal at worst. Despite the development of sophisticated curricula, expansive professional development, clearly defined standards and high-stakes performance assessments, the problem of enabling more youngsters to learn more mathematics seems resistant to solution. This issue of US student performance in mathematics is a topic of concern in cloakrooms of the United States Congress, editorial rooms of major newspapers, boardrooms of School Committees, teachers' rooms in schools and living rooms of parents. And while these concerns cross race and class lines, state borders and regional differences, nowhere is the issue of greater concern than in the nation's urban schools. For example, in the City of Boston, Massachusetts, home to many institutions of higher education, the performance of 15 year olds on the State's Comprehensive Mathematics exam was such that 45 % of the students either failed or were deemed in need of improvement (Massachusetts Department of Education, 2007).

Day after day, year after year, more and more urban youngsters are leaving classrooms with fragile and inadequate knowledge of mathematics that will likely inhibit their potential to live productive fulfilled lives. However, enabling urban youngsters to learn mathematics at higher levels is not a simple matter; it is a multi-faceted, complex and complicated problem that demands an equally complex solution. The challenge is real and increasing in severity every day. The time for action is now and the individuals to take action are leaders in the mathematics professional education community.

Several possible strategies and approaches exist to begin to address this need. Some suggest the use of data analysis to define instructional goals and the careful development of formative and summative assessments of students. Others call for higher standards for entering teachers, the education of teachers in mathematics content knowledge and an increased focus by principals and other district administrators on mathematics teaching practice. Finally, many urban districts have created school-based coaching positions to provide sustained and intensive support to groups of teachers (Charles A. Dana Center, 2004, p. 2).

This article specifically addresses the use of yet another strategy—the development and implementation of teacher written cases describing challenging topics in the teaching of mathematics. While cases have been used successfully to stimulate discussions in different K-12 subjects (c.f. Silverman, Welty, & Lyon, 1992; Shulman, 1992; Wasserman, 1994) and in mathematics at the elementary level (Barnett-Clarke & Ramirez, 2003; Goldenstein, Barnett-Clarke & Jackson, 2004), this article concentrates on the middle and high school levels and in the field of mathematics (Merseth, 2003). Case development and the use of cases to foster in-depth discussions about teaching and content knowledge offer a promising approach to help develop effective mathematics teaching practice in urban middle and secondary classrooms.

The use of these materials, as reported here, builds on the belief that teachers need powerful ways to engage in sustained, long-term investigations about their practice and the practice of their colleagues. Case use and development can help address the concerns of leading education scholars who have called for improved professional development opportunities for teachers. Substance and content should form the basis for these opportunities and they need to carry relevance to teachers' everyday practices. One scholar, Feiman-Nemser, describes an ideal approach to professional development for teachers in this way:

> In place of superficial, episodic sessions, teachers need sustained and substantive learning opportunities instead of discrete, external events provided of teaching…Although teachers need access to knowledgeable sources outside their immediate circle, professional development should also tap local expertise and the collective wisdom that thoughtful teachers can generate by working together. (Feiman-Nemser, 2001, p. 1042)

Case development and case discussions can squarely address these concerns and serve as opportunities for teachers to work together to explore effective practices.

What are Cases and How are They Used?

Cases are not a new pedagogical tool. They are used to train professionals in many fields such as business, law, medicine and social work (Merseth, 1991, 1996). In the view of several researchers, cases and case-based instruction can help users develop content knowledge and pedagogical skills (e.g. Carter, 1989; Shulman, 1992; Levin, 1993; Merseth, 2003). Case readers are able to study content in depth, learn to diagnose problems, recognize multiple influences and perspectives on student learning, and engage in the exercise of developing strong content and pedagogical knowledge through the exploration of the topic and the discussion and analysis of possible solutions and courses of action

Though there are many different definitions of a case, in this particular context cases are best understood as narratives that attempt to describe, as completely as possible, practice in mathematics classrooms. The cases present multiple perspectives, including the views of the teacher, the students and even other adults in the room. They are real; they are not made-up. Convincing cases bring a "chunk of practice or reality" into the teacher professional development setting for examination and explorations. In this way, they stimulate the opening of a window on practice by case discussants and facilitators. Cases used in this way are <u>not</u> case studies as in the research sense (Yin, 2003). Case studies in this genre often present both the event as well as an analysis of the event. In contrast, the cases presented in this article only offer a description of a teaching event in a classroom and do not offer any form of analysis of the situation. Instead, the analysis is the responsibility of the discussants.

Used as prompts to stimulate discussion, then, cases can help teachers engage in discussions of both teaching practice and mathematical content. They also offer an opportunity for teachers to reflect on their own classrooms and practices. Typically, the discussion of cases presents a situation to engage teachers in a discussion about teaching. While an individual may facilitate the discussion of the case, this is not always necessary; leader-less groups can be equally effective. Carefully developed cases frequently have discussion notes that help guide the participants with questions for their consideration. Though cases and case discussion methods may serve different purposes (see Merseth, 1996 for a more complete review), the cases here are simply a full account of classroom practice.

What are the Key Elements of Math Cases?

Cases for use in professional development settings for mathematics teachers typically contain a careful description of a classroom and a concept that a teacher is teaching. The mathematical topic often is one that traditionally has been identified as challenging and troublesome for many learners. Common examples might involve rational numbers, integers, independence in probability or limits in upper level mathematics. Successful cases for mathematics teachers have four identifiable elements: math content; clear dilemmas and questions about pedagogical approaches; representations of student understanding of the topic whether through the presentation of student work or student dialogue; and the reality of an educational setting including the demographics of the classroom or school, the grade level and the teacher's background.

The cases frequently encompass a dilemma--an action or decision forcing situation--in which reasonable readers may disagree about what the teacher has said or should do, or even about the nature of the student confusion. The cases are compelling, usually asking the reader to make a recommendation about what the teacher should do next. The cases are rich with student dialogue and things that present potential confusions of the students. The classroom description is an important component of these cases because they must seem real to the reader. Thus, characteristics about the learners, the teacher and any other important details such as the level of the class, the lesson objective, the time of the year, and the backgrounds of the students help add realism to the case.

What Knowledge and Skills are Important for Math Teachers to Know?

Several strands of knowledge seem important for teachers and hence become logical for inclusion and emphasis in the cases. These are:

- knowledge of mathematical content,
- knowledge of effective classroom practices, and
- the ability to apply this knowledge in varied learning situations for a wide array of teachers.

The following sections briefly explore each theme.

Mathematical content knowledge

While experts may argue about specific content, a publication, *Helping Children Learn Mathematics* from the National Research Council (2001) states the obvious:

> Teachers need a special kind of knowledge to teach mathematics well. They must themselves be proficient in mathematics, at a much deeper level than their students. They must understand how students develop mathematical proficiency and they must have a repertoire of teaching practices that can promote proficiency. (Kirkpatrick, Swafford, & Findell, p. 31)

Effective teaching requires this special knowledge. At the middle grade level, for example, such knowledge must include, but not be limited to, a complete and thorough understanding of the nature of number and number systems, a comprehensive understanding of the elements of algebra and algebraic thinking, a fundamental understanding of the ideas of calculus and an appreciation of geometric figures and their properties and the ability to discern viable proofs and logic. Further, for each of these topics, teachers will benefit enormously by understanding them through numerical, graphical and algebraic means. Finally, every teacher should have a firm grasp on appropriate applications for the concepts.

Cases written about these topics afford teachers an opportunity to examine closely the mathematical content, to discuss the content with peers and to explore possible mathematical extensions of the material. Cases are about the content as well as the pedagogy making a discussion of both topics likely in a professional development setting.

Knowledge of effective classroom practices in mathematics

Cognitive research about the difference between expert and novice teachers suggests a set of skills that are helpful for effective classroom practice. Experts and novices do not differ in general ability, intelligence, memory agility, or in use of general strategies. Where they do differ, however, is in what they notice about a given situation and how they organize, represent and interpret information.

Experts notice features and patterns and have content knowledge that is organized in ways that are fluidly accessed. Their content knowledge evidences facile connections between and among seemingly disparate concepts and illuminates deep understanding. Further, experts have knowledge that is “conditionalized” for the particular set of circumstances in which it occurs (Bransford, Brown, & Cocking, 2003, p. 37).

In the ideal, mathematics reform seeks to place an expert teacher in every classroom. This would be a teacher whose pedagogical actions were based on “seeing the differences” among learners. Instead of teaching the way they were taught, these well informed teachers would “conditionalize” or tailor their approach to instruction as a result of seeing these differences among the learners in front of them. An effective teacher is one who can notice patterns of student thinking, anticipate approaches commonly taken by students, and have a firm grasp of where the content will lead and where it has come from. Such a teacher also has a repertoire of pedagogical moves available to match the various possible understandings and interpretations of the concepts.

Cases can help teachers see differences—to understand what is happening when students are and are not learning—and to practice offering alternative explanations. Also the discussion of a case with other teachers offers an opportunity to hear how others might approach the topic and what pedagogical moves or actions they might take if faced with this teaching situation.

The ability to apply knowledge in varied contexts

Teachers need practice in identifying pedagogical approaches and techniques in classrooms. Sometimes this practice can occur in a teacher’s classroom but often this is tempered by curricular demands, student reality and school contexts. Learning about teaching, while teaching, is hard and takes particularly skillful and reflective teachers to recall details and generate possible alternative approaches.

Cases are vicarious enactments of classrooms and so have the advantage of being real depictions of classrooms but not necessarily classrooms that are familiar to the reader. Thus, teachers may have exposure to many different classrooms, grade levels, curricular conundrums, and various learning styles of children without ever leaving their classroom.

Cases can also show teachers what changed practice might look like. They can offer alternative, novel approaches and encourage the reader to analyze whether this approach would work in the individual teacher’s setting. Several well-meaning teachers may *think* they are teaching in reform relevant ways, when they are not. Cohen wrote a classic description of this phenomenon in his article about the California Math teacher, he cleverly named Mrs Oublier, who blithely taught a mathematics reform curriculum in old unproductive ways (Cohen, 1990).

Outcomes from Case Use

It is challenging to define a research study that unequivocally would establish the effect of case use in the professional development of teachers. Confounding factors of content and delivery as well as teachers' particular contexts and backgrounds make generalizations difficult at best. Thus, it is not the purpose of this article to present empirical research findings. Instead, the purpose is to describe the use of cases and to include an example case in order to broaden international knowledge of and familiarity with this tool.

Since 2004, the Massachusetts Math/Science Partnership at the Harvard Graduate School of Education (MMSP) has developed and used case discussion materials with practicing middle school mathematics teachers. From 2004 to 2007, nearly 100 middle school math teachers attended seminars using cases (when space was available, elementary and/or high school teachers were also included.) in order to enhance their mathematical and pedagogical knowledge. The seminars were 45 hours in length and the objectives of the seminars were:

- to increase teachers' content knowledge in mathematics,
- to improve teachers' ability to understand student and teacher misconceptions through case-based discussions of mathematics classrooms, and
- to prepare teachers to respond to student misconceptions.

During the seminars, educators discussed anywhere from 6-8 cases and also participated in other activities including graphing calculator work, techniques to review student work, classroom warm-up exercises which required teacher collaboration within the context of the seminar. Teachers used the cases to discuss the classroom practices of the protagonist in the case as well as extending their discussions and reflections to their own classrooms and practices. All teacher participants were required to read the cases assigned for each class session, and to offer comments and reactions to several questions posed by the discussion facilitator. All of the cases illuminated particular student and teacher misconceptions. Through the readings as well as discussions in class, teachers were encouraged to think through a variety of scenarios that helped the teachers understand how the teachers and students were thinking in the case. By doing this, they were better able to prepare themselves for a variety of misconceptions related to mathematical content.

Because the cases often stress student misconceptions, teachers often discussed their own hypotheses about what the students in the case were thinking and based on this hypothesis, they then developed specific pedagogical approaches to clarify the student confusions. The teachers practiced asking questions of each other with one teacher taking the position of the confused student and the teacher attempting to uncover the confusion. Because the cases were not specifically about their individual classrooms and their own students (but rather some other teacher and student not known to them), the teachers reported feeling safe in the discussions and were willing to take risks in proposing alternative approaches and assessing the classroom environments. By focusing on the teacher in the case as the discussion point, the teachers discussing the case did not need to expose their own uncertainties and confusions. This characteristic helps increase productive participation and explorations, resulting in conversations and discussions that were active, fun-filled and robust.

Qualitatively, teachers reported enjoying the case based courses and said they felt more prepared to respond to student misconceptions as a result of the seminars. These self-reported data are one indication that teachers' confidence increased with regard to meeting student needs.

In addition to improving the quality of math instruction through seminars or school site discussions, case studies can leave a legacy of support and procedures for future professional development work in schools. The cases can be used frequently, both with the same or different group of teachers because there are always new insights or different ideas to be explored. In this way, cases can be extremely helpful for schools wishing to build capacity for internal professional development activities in mathematics. Cases are excellent tools for the examination of mathematics instruction and provide a model of a professional development process for other disciplines as well.

The appendix of this article offers an example of a case titled "Lost in Translation" (Merseth, 2003). The case depicts an Elementary Algebra class at the beginning of the school year. Several students in the class have specific learning challenges and tend to be reluctant participants. The teacher must decide whether to encourage participation by recording of answers on the board, even though the answer may be wrong and invite embarrassment and ridicule. The mathematics in the case concerns the challenge of translating the meaning of a sentence into an algebraic expression. Students struggle with the notion of "less than" and the symbol $<$, as well as how to write "five less than two times a number."

While there are multiple approaches to open a case discussion, a case discussion leader might begin the discussion by asking a general question such as "What difficulties occur for students in translating from words to mathematical symbols or from symbols to words? Are these different?" More specific questions are possible as well. For example, "What do you think of the teacher's approach to have each student explain his or her thinking to the class?" or "If a student records a wrong answer on the board, what action should the teacher take?" And an even more specific discussion can ensure about how to help a student understand that "five less than two times a number" is not written as 5-2*n*, a common mistake in algebra classes all over the world.

The reader is encouraged to explore the case in the appendix and to examine it as a potential example of a powerful tool to engage mathematics teachers in discussions about teaching.

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Appendix

Lost in Translation

"I got $2n - 5 = 12 + n$. Is that the same as Donna's answer, $5 - 2n = 12 + n$?" called out Trish, waiting for an answer.

"What's going on with these kids?" Mrs. Harper remembered thinking to herself, as she got really worried. "Didn't they learn *anything* last year?"

It was Friday evening and Michelle Harper should have been enjoying a pleasant dinner with friends. Instead, she found herself still trying to make sense of what had happened in her Intermediate Algebra class earlier in the day. It had been the second class meeting of the year and Mrs. Harper had given her students three problems to work on at their seats while she processed the book sign-up sheets. The problems mirrored the first section of the text they were using, which focused on translating English sentences into algebraic expressions. Even though this topic was a review from Algebra I, she wanted to be sure to go over a few problems before sending the students home to try them on their own. Because there was an assembly scheduled, she only had about 20 minutes with this group instead of the usual 49.

The Class

Intermediate Algebra was the lowest level course available for students who had completed Algebra I in this highly academic, suburban school. Over 90% of the students went on to attend four-year colleges and universities. This particular class, however, consisted mostly of students who would not be among that ninety percent. Of the fifteen students, there was one ninth-grader and one senior. The rest were evenly split between tenth- and eleventh-graders. Nine of these students had learning disabilities. Of these, three had severe language processing difficulties and were mainstreamed for mathematics only. In fact, there were so many special education students in the class that the school had offered to provide Mrs. Harper with an aide. She refused, afraid, as had happened in years past, that the aide would not be trained in mathematics. Mrs. Harper hoped that having only one teacher explaining things would make it less confusing for her students. The special education department also offered to make someone available on a more temporary basis to help during special projects or with units when a second pair of adult hands would be beneficial. Mrs. Harper was grateful for this offer.

She was not quite sure what to expect from this group of students. She knew that many of them still had difficulties adding signed numbers. And most of them were dependent on their calculators for even the most basic computations. She often had to defend keeping students in this course rather than putting them in a "Basic Math"

class. "You can teach students basic skills using algebra as a vehicle instead of holding them back doing arithmetic over and over again. They feel better about the process because they think they're doing high school math instead of baby stuff."
After the class had some time to work, she asked for a volunteer to give an answer to the following problem:

> *Translate into an algebraic expression or sentence:*
> *5 less than 2 times a number is 12 more than the number.*

Kenny

Kenny, a large 10th-grader who was also a member of the football team, was the only student to raise his hand. But he didn't just raise his hand; he shook it wildly back and forth while yelling out, "I know the answer!" Kenny had been a member of Mrs. Harper's 9th-grade homeroom the previous year, where he had some serious behavior problems. She knew he had been in a fair amount of trouble in other classes as well from the many administrative detention notices which she had to deliver to him throughout the year. Apparently, he did not deal well with conflict, usually responding by shoving or punching those with whom he disagreed. Kenny and another boy, Alan, engaged in shoving and pushing episodes on a daily basis. Kenny almost always was the instigator. When she asked Kenny why he couldn't just ignore the other boy, he responded, "Because I hate him. Looking at him makes me want to bash his face in." After trying, unsuccessfully, to reach his parents a number of times, Mrs. Harper asked the assistant principal, Mrs. Kalkstein, for help. "You won't be able to reach them," she had told her. "Kenny is basically raising himself. If you saw his home environment, you'd just want to cry. He's a real survivor. Let me talk to him. Maybe the best thing to do is to move him to another homeroom."

After much consideration of Kenny's difficulty dealing with change, Mrs. Kalkstein decided to move Alan to another homeroom instead. With Alan gone, Kenny became much easier to deal with. Mrs. Kalkstein also suggested that Mrs. Harper speak to the school psychologist about Kenny. When she did, he told her a bit about Kenny's very difficult home life. He explained that Kenny's reactions to stressful situations were often exaggerated. He gave her some suggestions about strategies for dealing effectively with Kenny. Now, it seemed her efforts had paid off; she was pleased to see Kenny so animated and willing to participate. When she called on him, he blurted out,

> "Five less than two *n* equals twelve greater than *n*."

She asked him to come to the board and write it out. He lumbered out of his seat, took the marker from her hand, and wrote on the board in large characters:

$$5 < 2n = 12 > n$$

Then he returned to his desk.

Mrs. Harper stared at the board for a moment. When she turned toward the class she saw Kenny sitting at his seat, proudly grinning at her. "Thank you, Kenny" she said out loud while grimacing internally. Then she paused as she looked around the room waiting for a reaction. When nothing happened, she looked around the room a second time, more desperately. "Does anyone have another answer?"

After a moment or two, Damon responded to the invitation with, "I got the same thing." "Anyone else?" she asked again.

Trish and Donna

Donna volunteered her answer and then Mrs. Harper asked her to put her solution on the board under Kenny's.

$$5 - 2n = 12 + n$$

Now there were two answers on the board to consider. She asked again, "Did anyone get anything other than these two answers?"

It was then that Trish had asked the question that Mrs. Harper was reviewing in her mind later that evening. Was her answer of $2n - 5 = 12 + n$ the same as Donna's answer? Mrs. Harper tried not to show how worried she was becoming. "Trish, that's a good question. Could you hang onto it for a second until everyone has had a chance to tell us what he or she got as an answer? Then we can discuss it." Mrs. Harper asked Trish to put her solution on the board under Kenny's and Donna's.

$$2n - 5n = 12 + n$$

Trish and Donna had worked together on this problem. At the beginning of class they had approached Mrs. Harper with a request. "If we have to work in groups this year, can Trish and I work together?" Donna asked. "We've known each other since kindergarten and we work really well together."

Trish added, "You can ask Mr. Sullivan. We had him for math last year and he let us work together all the time and we did really well!"

Mrs. Harper had recently read Donna's cume (cumulative record file), which revealed some of her history. In sixth grade, Donna had had problems reading. School special educators realized late in the year that she was dyslexic, but not before Donna had fallen far behind her classmates. As a result, she was forced to repeat sixth grade. Since then, Donna had made great strides in conquering her disability. When she and Trish were reunited in high school, she began to flourish. It was no surprise, then, that Trish echoed Donna's wish to work together.

Mrs. Harper responded positively to their request to sit next to each other. "I'll let you try it for a while unless I find that you don't concentrate well together. But I may mix the groups up in a few weeks so everyone gets a chance to meet other people in the class.” They seemed content, and found two seats next to each other.

Two was Mrs. Harper’s preferred ‘group’ size. In her experience, larger groups tended to spend more time gossiping and fooling around than working. In those instances, it seemed one or two students would end up doing most of the work for the whole group. With two, there was more pressure on both group members to focus on the task at hand. Because she knew there were wide-spread misconceptions, she hoped that by asking students to work together, they would be forced to discuss differing responses. She also hoped that students would help each other so that fewer students would give up without a good try.

No one else volunteered another answer, but Colin quietly added, "I got the same thing as Kenny and Damon."

Joe

Then Joe, an energetic young man, noticed one of the answers on the board was similar to his own. "I got the same thing on the right," he exclaimed. He almost tipped over as he leaned forward over the arm of the chair and pointed to the board where the 12+ n was written. "But, I got two minus five n on the left."

"Come on up, Joe, and add your answer to the list," Mrs. Harper invited. He jumped out of his seat and went to the board. After inspecting the colored markers lying in the tray, he selected a red one and wrote:

$$2 - 5n = 12 + n$$

Then he signed his name in decorative letters next to what he had written. Trish giggled as he ended his last name with a big flourish. "Give it up, Joe," Kenny yelled as Joe returned to his seat and the class dissolved into quiet conversation for a moment.

"Folks," Mrs. Harper called out. She paused as the class quieted down again. "Are there any other solutions out there?" No one else volunteered another answer, but Amy quietly added her vote for Kenny's solution.

Explaining the Solutions

After she was sure no one else had an answer to share, Mrs. Harper continued, "Kenny, can you explain how you got your answer?"

"Well, I just wrote down what the words said. I know "is" always means "equals" and the rest is obvious." A few kids nodded in agreement after Kenny completed his explanation. Kenny continued to grin while glancing around the classroom with pride, still confident that his answer was correct.

Mrs. Harper turned to Donna, "Can you explain how you did this problem, Donna?"

"I think that five less than two times a number means you should subtract. And 12 more than a number means you should add. So that's what I did." Many of the students looked puzzled.

"So, you used addition and subtraction instead of less than and greater than signs, is that right?"

"I don't think you can have all those signs in the same answer," Donna added, pointing to Kenny's solution.

Next, Mrs. Harper directed her attention to Joe. "Joe, why did you choose to multiply the 5 by the *n*?"

"I didn't!" he yelled in surprise. "You have to subtract the 2 - 5 first. *Then* multiply by *n*. It said five less than two, so you have to subtract those two numbers!"

"Oh," Mrs. Harper replied. She paused. "Do you remember PEMDAS?" Colin yelled out, "Please Excuse My Dear Aunt Sally! We did that last year!" "Please excuse who?" Kenny asked dramatically.

"What's PEMDAS for, Colin?" Mrs. Harper pursued.

"I think I remember. Parenthesis, Exponents, Multiplication, Division, Addition, and Subtraction. It's the order you have to do things in." he replied.

Colin

Colin was typical of many learning disabled students. He was very intelligent, with an excellent memory and creative ideas. But he had difficulty putting his thoughts on paper. After reading his file, Mrs. Harper discovered that his 9th grade history teacher had allowed him to record his thoughts on audio-tape. Then an aide in the special education department transcribed his words. It had made all the difference for Colin who was finally able to demonstrate his understanding of historical events. The special education contract which the school had written with Colin and his parents required "alternative assessment" in situations where Colin felt he was unable to demonstrate his understanding on paper. Mrs. Harper considered how she might modify tests for Colin if necessary, but was not sure how she could use the tape recorder approach. Although she agreed that it was most important to find out if Colin understood the mathematical concepts, she also felt it was important that she be able to see his work. In math, details mattered. She made a note to talk to the special education director about effective assessment techniques for use in math classrooms.

"Thank you, Colin." Mrs. Harper responded. "You have a good memory." Colin grinned.

She continued, "Now let's look at Joe's answer. The way it's written, you should multiply the five and the *n* first and then subtract the product from the two. How could we rewrite it so that it says what Joe means?"

No one said a word. The silence was deafening. "No one has an idea?" Still no answer. Damon turned towards the window and watched some students playing frisbee out on the lawn.

"What about parentheses?" Mrs. Harper injected into the silence. "You could put them around the parts you want done first since they come first in the order of operations, right?'

Joe responded, "OK, put the 2 - 5 in parenthesis. Then it will work."

Mrs. Harper drew in the parenthesis around the 2 - 5.

$$(2 - 5)n = 12 + n$$

Then she turned to the class and asked, "What do you think about this now?"

Donna said, "I still don't think it's right. I think the n goes with the 2, not the 5. Wouldn't it have to say ‘five less than two *all* times n' or something to get what Joe wrote?"

"Good observation, Donna. Actually, if I wrote '5 less than two *comma* (she drew a comma in the air with her index finger as she talked) times a number,' then Joe's answer would be correct. That little comma would make a big difference!" Mrs. Harper directed the final comment to Joe, who nodded that he understood.

Finally, Mrs. Harper turned back to Trish, who was doodling on her paper. "Now, Trish, you had something similar to Donna, but you had a question about it. Could you repeat the question you asked before so everyone can remember it?"

Trish looked up and impatiently responded. "I just wanted to know if it matters if you write $2n$ - 5 or 5 - $2n$."

Mrs. Harper was halfway through her sentence, "Well, what do you think about. . . ." when the bell rang. As most of the class bolted from the room, Kenny rose slowly, having lost all of his earlier enthusiasm. Mrs. Harper called quickly after the students, "We'll continue with this problem on Monday," while wondering what she could have done differently.

Thank God It's Friday!

After school, Mrs. Harper walked upstairs to talk with Mrs. Salisbury, a colleague who was teaching the same course. Between the two of them, they taught all sections of Intermediate Algebra. Mrs. Salisbury had been teaching math for over 25 years, and had been teaching Intermediate Algebra most of that time. When Mrs. Harper first started teaching 8 years earlier, Mrs. Salisbury had become a mentor teacher to her. Because they worked well together, the department head had assigned them to many of the same courses over the years. Mrs. Harper had only taught Intermediate Algebra once before and had lobbied strongly to update the course, which had been taught using the same materials for two decades. During the previous year, the two teachers wrote up a proposal for a four-day workshop to be held over the summer for curriculum development. Although they only received funding for one day, they were able to revise the course outline and select new books. They planned to share ideas and materials throughout the year. When Mrs. Harper walked into Mrs. Salisbury's classroom, she was working at her desk.

"How was your 5th period class? My 3rd period was a disaster!" Mrs. Harper exclaimed as she settled into one of the student chairs. Mrs. Salisbury had official memos about opening-week procedures scattered about her desk, mostly on top of books. She had cleared a spot in the middle where she was alphabetizing book receipts.

"What a mess!" she replied. "I don't know if we can assume these kids remember *anything* from last year." Mrs. Harper was amazed to find that Mrs. Salisbury had had a similar experience in her class. Mrs. Salisbury continued, "These kids come from so many different places. Some are 9th graders who took Algebra I at the junior high. Some are taking this course for the second time. We have kids at all different levels who learn in all different ways." She threw up her hands and, shaking her head, looked to the ceiling for guidance. "Maybe we need to start from scratch."

Equally exasperated, Mrs. Harper started to pick up her things. "I need to think about how I'm going to deal with this. If you have any brainstorms, give me a call. I've got to run." Halfway out the door she called over her shoulder "Now I know what that expression--TGIF--really means!"

Case Questions

1. Was this class a "disaster", as Mrs. Harper proclaimed? Why or why not?
2. What should Mrs. Harper do on Monday, given everything that has happened?
3. How else could Mrs. Harper have responded when Kenny gave his answer? How should you – as a teacher – deal with a student's disappointment when they learn that the answer they were so proud of is not the "right answer"?
4. What are the mathematical problems here? How might you address them?
5. How were the special needs of students with learning disabilities addressed in Mrs. Harper's class? What would you do differently?

Lost in Translation
Pre-Case Discussion Worksheet

Given the following situation:

Keisha has just come down with a bad cold. Suppose f(t) is the function which represents her body temperature, measured in degrees, Fahrenheit, where t is the number of hours since 6 AM today.

Translate:

Express the following information symbolically.

a. At 9 this morning, 20 minutes after taking some aspirin, Keisha's temperature was measured to be 101.2. At this time, her temperature began to decrease.

b. By 10 AM, her temperature was still decreasing, but not as quickly as it was at 9 o'clock.

c. By 11:30, Keisha's temperature "bottomed out" at 100 degrees.

d. At 3 PM, after taking some more aspirin, Keisha's temperature was 100.5. Until 3:45, her temperature continued to increase at a constant rate of 1 degree per hour.

e. By 8 PM, her temperature had been "normal" for 2 hours, so she concluded that her cold had run its course.

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