The Mathematician Educator
2020, Vol. 1, No.1, 25-42

Fostering Critical Thinking in Singapore Primary Mathematics Classrooms

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In order to prepare students for the 21st century, it is important to foster critical thinking in students. This paper explores the teaching of critical thinking during mathematics lessons at the primary level in Singapore. It suggests lesson features of primary mathematics lessons that could further encourage students to think critically, which include the kind of tasks used, the use of Socratic questioning by the teacher, supportive behavior of teachers as well as communicating assessment criteria to students. The identified lesson features have been enacted in a Primary 4 mathematics classroom. In this paper, students' work showing elements of critical thinking in a lesson on the topic of decimals are presented and discussed.

Keywords: critical thinking • 21st century competencies • primary mathematics

Introduction

Students from Singapore have done comparatively well at international assessments in mathematics and science. For instance, Singapore ranked first in mathematics and science at the 2015 Trends in Mathematics and Science Study (TIMSS) involving 10-year-olds and 14-year-olds (Teng, 2016). In a report that combined data from the 2011 TIMSS for 14-year-olds and the 2012 Programme for International Student Assessment (PISA) for 15-year-olds, Singapore also ranked first in mathematics and science (Ng, 2015).

Singapore's performance at these international assessments may suggest that students in Singapore have good grounding in mathematics and science. However, the director of the Organization of Economic Cooperation and Development (OECD) commented that Singapore could place greater emphasis on skills such as creativity critical thinking and collaborative skills (Ng, 2015). As early as 2009, the Ministry of Education (MOE) in Singapore has also recognized the importance of imbuing such skills in students and has taken steps to incorporate these skills into the curriculum. That year, MOE launched the Framework for 21st Century Competencies and Student Outcomes. The latest version of the framework is shown in Figure 1. At the centre of the framework lies the core values of responsibility, respect, resilience, integrity, care and harmony. The social and emotional competencies are listed in the inner ring while the outer ring shows the emerging 21st century competencies. The four desired outcomes of education in Singapore are shown at the four corners (Poon et al., 2017). Critical and inventive thinking is listed as one of the 21st century competencies. It is explained in an annex accompanying the framework that critical and inventive thinking is important as our young need to be able to think critically, assess choices and make considered decisions in order to be ready for the future (MOE, 2018).

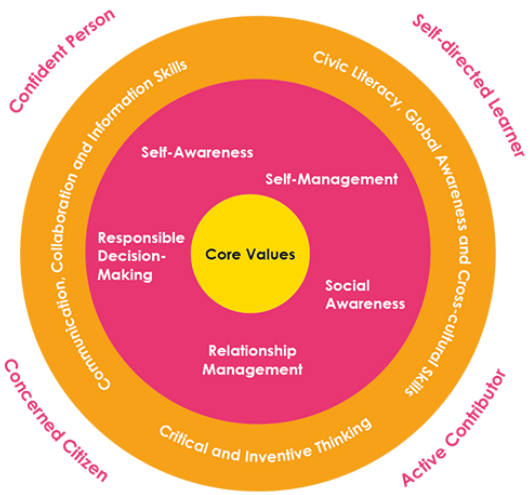

Figure 1. Framework for 21st Century Competencies and Student Outcomes
© Ministry of Education, Singapore (MOE, 2018).

Tan, Choo, Kang and Liem (2017) listed the major policy initiatives for educating for the 21st century and to groom future-ready learners. These included the Thinking Schools, Learning Nation (TSLN) initiative in 1997 and the formation of the Primary Education Review and Implementation (PERI) committee in 2010. One of TSLN's focus was on developing creative and critical thinking skills. Curriculum content was reduced and there was greater emphasis on the process of learning (Poon et al., 2017). PERI had a similar aim of fostering in young students a set of 21st century competencies and dispositions (Tan et al., 2017). One recommendation of PERI was that teachers adopt more creative learning strategies (MOE, 2019).

Responding to the need to foster 21st century competencies in students, a number of researchers have implemented evidence-based practices that promote such competencies (Toh & Kaur, 2016). For example, Toh, Cheng, Jiang and Lim (2016) explored using comics to enhance students' learning as well as to develop 21st century competencies in the mathematics classroom. In a more detailed paper, Toh, Cheng, Ho, Jiang and Lim (2017) reported the experiences of a teacher who made use of a comics package to elicit 21st century competencies. One of the comics in the package showed a man with half his hair cut when he visited a salon offering a 50% discount. Students had the opportunity to discuss what a 50% discount meant and whether he hair salon had provided misleading information. This task required students to discern the legitimacy of given information, which was regarded as a component of critical thinking in their study. The paper did not report student responses as the focus was on the teacher. Nonetheless, it is heartening to note that there are already practices which aim to foster 21st century competencies in the classrooms.

However, more could be done to explicitly equip students with 21st century competencies. Tan et al. (2017) stated a perennial question related to the teaching of 21st century competencies that had engaged international education communities: How can 21st century competencies be more effectively taught, learnt, and assessed in formal and informal educational contexts? The scope of the question posed in Tan et al. (2017) is vast and this paper seeks to partially address

it. This paper focuses on how critical thinking could be effectively taught or learnt in the Singapore primary mathematics classroom. Particularly, the key purpose of this paper is to provide an example of a context in which critical thinking might be developed in a Primary Four mathematics classroom in Singapore and to report the extent to which the students responded to the stimuli provided.

Literature Review

With the intention of identifying the most feasible approach of teaching critical thinking in Singapore primary schools, literature on approaches in teaching critical thinking was reviewed. In addition, several definitions of critical thinking were studied and crucial characteristics of critical thinking were selected for use in the working definition of critical thinking in this study. Before designing lessons which incorporated critical thinking, literature on what had already been done to foster critical thinking in Singapore schools was explored. Such literature provided insights on what had worked and what more needed to be done. Finally, literature on critical thinking in mathematics education was reviewed to find out what factors of a mathematics lesson were conducive in fostering critical thinking. This section summarizes finding from various literature and explains how the findings have guided the study.

Approaches to teaching critical thinking

According to Ennis (1989), critical thinking can be taught through four approaches, namely the general approach, the infusion approach, the immersion approach and the mixed approach). In the general approach, general critical thinking skills are taught independently of any subject matter during lessons specially set aside for critical thinking. In the infusion approach, general critical thinking skills are made explicit during regular lessons and applied to subject-specific content. In the immersion approach, a subject-specific definition of critical thinking is adopted and subject-specific critical thinking skills are implicitly embedded in subject-specific content. Lastly, in the mixed approach, general critical thinking skills are taught during lessons specially set aside for critical thinking and applied to both subject-specific and general content.

The general and mixed approaches are time-consuming and are unlikely to be accepted by teachers in Singapore as extra curriculum time needs to be set aside for the teaching of general critical thinking skills. Indeed, Tan (2017) found that while the teachers in her study believed it was important to fulfil the aim of imparting critical thinking skills, they still desired to have sufficient time for examination preparation. If additional time was to be set aside for the teaching of general critical thinking skills, the teachers' desire for enough time for examination preparation might not be fulfilled. Moreover, opponents of teaching of critical thinking as generic skills independent of subject content suggested that people who learn general principles in non-specific situations are usually not able to apply these principles in new situations (e.g. Willingham, 2019).

In the immersion approach, critical thinking is restricted to the subject which has been identified for the infusion of critical thinking. Abrami et al. (2008) showed that the immersion approach was the least effective in enhancing critical thinking among the four approaches. Angeli and Valanides (2009) found that students taught with the infusion approach demonstrated better understanding of critical thinking than students in the immersion group. These findings suggest that the immersion approach may not be as effective in developing critical thinking as other approaches.

The infusion approach addresses the problems faced by the other approaches. In the infusion approach, general critical thinking principles are made explicit during mathematics lessons and are applied to mathematics content (Ennis, 1989). In this study, authentic tasks with mathematics elements would be used so that the critical thinking skills that students learn can be applied in daily living. Moreover, the general critical thinking skills are incorporated in mathematics lessons instead of being addressed in specially-designed critical thinking lessons, so teachers do not have to specially allocate time for the teaching of critical thinking. Thus, the infusion approach is used in this study. As such, the definition of critical thinking used in this study will be a general, rather than a subject-specific one.

Operationalization of critical thinking

Common themes recur in the definitions of general critical thinking. Recognizing and examining assumptions was listed as a characteristic of critical thinking by Socrates (Paul, Elder & Bartell, 1997) and Watson and Glaser (2002). Gathering and assessing information was referred to by Socrates (Paul et al., 1997) and Paul and Elder (2006a). Reflective thought or open-mindedness to alternative views was included in the definitions of critical thinking by Dewey (1933), Ennis (1987), Paul and Elder (2006a) and Rahim (2013). In addition, Paul and Elder (2006a), Watson and Glaser (2002), Reynolds (2011) and Rahim (2013) mentioned that critical thinking involved well-reasoned conclusions.

For the purpose of this paper, critical thinking will be operationalized by four characteristics, namely: the ability to gather and assess relevant information; recognizing and examining assumptions; maintaining an open mind to alternative viewpoints as well as being able to justify decisions with supporting evidence and reasons. As mentioned, these characteristics have been found to recur in several definitions of critical thinking. They are also relevant to mathematics.

Critical thinking in the primary mathematics curriculum

With critical thinking operationalized, this section discusses how the curriculum has already incorporated such thinking skills. The Primary Mathematics Syllabus (MOE, 2012) highlights the need to design learning so that students have the opportunity to think critically about mathematical solutions. However, the syllabus does not provide examples on how critical thinking can be elicited in the classroom. Neither does it give the definitions of critical thinking.

Of the three commonly used primary mathematics textbooks in Singapore, examples of how critical thinking can be encouraged in the classroom are explicitly included in the teacher's guides of two titles, *Shaping Maths* and *My Pals are Here*. As this study is focussed on the Primary Four level, examples found in Koay and Lee (2016) as well as Fong and Gan (2016) will be discussed. These are the teachers' guides for the Primary Four *Shaping Maths* and *My Pals are Here* titles respectively.

Koay and Lee (2016)gave three suggestions of how critical thinking could be addressed. In one of the scenarios, students use wedges to measure angles. Students could "infer the relative sizes of the angles from the number of wedges used to measure angles a and c" (p. 4.5). It was also suggested that students be "encouraged to reflect on their experiences with using the wedges to measure angles, review their limitations and more importantly, consider alternatives to overcome their limitations" (p. 4.5).

In the topic of Geometrical Figures, Koay and Lee (2016) suggested getting students to come up with various ways of cutting a square hole and a rectangular hole at the centre of a grid

paper. The third example of a critical thinking task in Koay and Lee (2016) was in the topic of Symmetric Figures. It was suggested that students could compare the similarities and differences of the figures on the posters shown in a figure. Koay and Lee (2016) stated that there could be different answers and students should be open to alternative opinions. All three tasks from Koay and Lee (2016) had the potential to elicit critical thinking as the tasks were open-ended and not restricted to one fixed response. There was also guidance to teachers on how to do so. However, this was the only title among the three commonly used textbooks that provided explicit instruction on eliciting critical thinking. There could perhaps also have been a definition of critical thinking and more focus on students identifying assumptions and justifying their responses with reasons and evidence.

In each unit of Fong and Gan (2016), there is a Put on Your Thinking Cap section, where problems that are supposed to foster critical thinking are posed. While the problems are interesting and challenging, most of the problems do not seem to have the capability of eliciting critical thinking, as defined in this paper. The problems had one fixed solution and did not require students to identify relevant information or make assumptions. They also did not require students to be open to alternative viewpoints or justify their responses. A case in point is the following example: "How many times does the digit 5 appear from 10 000 to 11 000?" (p. 90). This problem, although challenging, has a fixed answer. Students may solve the problem by listing or by looking for patterns. Students are not required to gather or assess information. Neither do they need to identify assumptions. There are not many variations in how students can arrive at an answer.

From the Primary Mathematics Syllabus (MOE, 2012) and the textbooks cited, it can be seen that there have been attempts to incorporate critical thinking into the mathematics curriculum in Singapore. However, as pointed out, there are few examples of how critical thinking could be developed in the classrooms. In addition, more support could be given to teachers to ensure that they knew what critical thinking was and how they could incorporate critical thinking in their lessons. In this light, it may be helpful to find out what features of a mathematics lesson can foster the development of critical thinking so that more resources can be developed to support teachers.

Research on critical thinking in mathematics education

In order to find out what features of a mathematics lesson can address critical thinking, research studies relating critical thinking and mathematics education are examined. In the literature on critical thinking and mathematics education, there is consensus that the content used has to have the capacity to elicit critical thinking. For example, Broadbear (2012) used ill-structured problems (problems that are encountered in daily life and have unclear goals or incomplete information) to elicit critical thinking while Glazer (2001) suggested that problems used to elicit critical thinking should allow for multiple solutions or representations and require reasoning. Based on the consensus in literature that the content used had to have the potential to encourage critical thinking, this study would make use of authentic tasks, which are set in realistic contexts, in the lessons on critical thinking. Such tasks can be designed to allow students to select relevant information and make assumptions. They can also be designed to allow for different opinions and require students to justify their decisions. This is consistent with the characteristics of critical thinking identified in this study.

Using an authentic task alone will not be sufficient to elicit critical thinking as a teacher could carry out the task in a teacher-centred manner and provide answers to the students instead of

letting them come up with the answers on their own. Peter (2012) and Glazer (2001) suggested that for critical thinking to be elicited, teacher questioning is important. Also in relation to teacher questioning, Paul and Elder (2006b) suggested using Socratic questions to elicit critical thinking. Socratic questions are derived from the ideas of the renowned philosopher Socrates, who emphasized the challenging of assumptions and constantly reflecting on the accuracy of information. Such questions are often associated with critical thinking as the questions seem to draw out the characteristics of critical thinking. Paul and Elder (2006b) suggested a number of categories of Socratic questions and gave examples for each category. There can be questions that clarify; probe assumptions; probe information, reasons, evidence and causes; probe different viewpoints and perspectives; consider implications and consequences and question the question. Most of the categories of Socratic questions suggested by Paul and Elder (2006b) are closely related to this study's operationalization of critical thinking. Hence, Socratic questioning has been included as a feature of a lesson that seeks to draw out critical thinking in this study. The relationship between the categories of Socratic questions and the characteristics of critical thinking identified by this study is summarized in Table 1.

Table 1.
Relationship between categories of Socratic questions and the characteristics of critical thinking

Category of Socratic question (example)	**Characteristic of critical thinking**
Clarifying (What do you mean by …?)	Assessing information
Probing assumptions (Is it always the case? Why do you think the assumption holds here?)	Recognizing and examining assumptions
Probing information, reasons, evidence and causes (What are your reasons for saying that?)	Being able to justify decision with reasons and evidence
Probing different viewpoints and perspectives (How would other groups of people respond? Why?)	Being open-minded to alternative viewpoints and perspectives
Considering implications and consequences (What effect would that have?)	Being able to justify decision with reasons and evidence
Questioning the question (Can we break down this question?)	Gathering and assessing information

Besides asking Socratic questions, teachers could encourage students to learn independently and support them in their development of reasoning skills (Glazer, 2001). Teachers could also help students discern between facts and opinions, identify relationships, evaluate sources of information and encourage students to generate multiple ideas (Chye & Ee, 2009). Hence, teacher behaviour that supports independent learning, reasoning and coming up with multiple ideas is important and has been included as a feature of a lesson that elicits critical thinking.

Lastly, Broadbear (2012) suggested that students need to know the criteria of assessing thinking. One way of doing this is to show students model examples of work that demonstrate critical thinking. There are other ways to transmit the criteria of assessment, like explicitly telling students what the criteria are or training students to use the rubric for assessment to grade their work. However, the latter could be time consuming. Thus, for this study, model examples of work that demonstrate critical thinking were shown to students.

To summarize, four features of a lesson that could elicit critical thinking have been identified. They are: the use of authentic tasks, the use of Socratic questions, teacher behaviour that supports critical thinking and student awareness of the criteria of assessing thinking.

The Study

The case study approach has been adopted in this study. Through observing students' behavior and their work, it is hoped that manifestations of students' critical thinking of the students can be described clearly and sufficiently. Also, Yin (2012) commented that the case study method is appropriate when data collection is done in natural settings. In this study, the class setting is kept natural with the regular teacher teaching the class.

Subjects

A Primary Four class was chosen as the unit of analysis since the researcher had previously worked with Primary Five students and found that Primary Five students may already exhibit characteristics of critical thinking. The school had grouped the Primary Four students into three bands - high, middle and low - according to their Primary Three mathematics achievement scores. The class in this study was placed in the middle band. There were 36 students in the class and the students were divided into eight groups. Four groups had four students and four groups had five students.

Procedures

Together with other Primary Four mathematics teachers in her school, the teacher of the selected Primary Four class attended a workshop on critical thinking conducted by the researcher. During the workshop, the importance of critical thinking and the characteristics of critical thinking used in the study were discussed. Thereafter, the researcher presented a task that was previously conducted in a Primary 5 class and demonstrated how the task could be introduced to students. The workshop participants assumed the role of students and worked on the task in pairs. The researcher observed the participants working and asked Socratic questions such as "Could there be other views?" (probing different viewpoints) and "Why do you say so?" (probing reasons and evidence). There was also a demonstration on how to provide a safe environment for differing views, as well as for encouraging independence by not providing answers. Three participants with different answers were called to write their responses on the whiteboard and the different points of view were discussed. After the discussion, the researcher explicitly mentioned how critical thinking could be elicited and how the various parts of the activity were related to critical thinking. The workshop participants were also shown responses from students obtained from a pilot study conducted earlier and given suggestions on how they could respond to each student.

Subsequently, the participants of the workshop were given an authentic task that involved the four operations of decimals, which was the topic that the Primary Four teachers were about to teach. The task, which can be found in Appendix 1 and which will be henceforth referred to as the Stationery Task, required the participants to choose what stationery to buy, on the pretext that they had to make thank-you cards for ten teachers. There were three kinds of paper, three kinds of writing materials and three kinds of stickers in a given catalogue. This authentic task allowed participants to select and assess relevant information, as well as to compare materials using different information. The solvers had to write down considerations and assumptions made and explain how they arrived at their decision on what to buy. This is aligned with the study's operationalization of critical thinking. This time, the participants were supposed to

discuss possible student responses to the task and come up with a model solution that would demonstrate critical thinking. After discussing the participants' solutions, the researcher presented a sample model solution, emphasizing that there could be various solutions. The sample model solution is found in Appendix 2. It was stressed that students not only had to make a choice, but they also had to justify their decisions.

After the workshop, the teacher of the selected Primary Four class had a separate discussion with the researcher. The researcher went through Socratic questions with the teacher and gave examples of Socratic questions that could be asked as students performed the task. The teacher was assured that she could administer the task in any way she liked so long as the four characteristics of critical thinking were elicited. She was given the presentation slides on the importance and definition of critical thinking that were used during the workshop and was told that she could use or adapt the slides.

The teacher conducted the Stationery Task in her class slightly more than three weeks after the workshop. During the lesson, which lasted 1.5 hours, the teacher introduced critical thinking to her students and explained the importance of critical thinking using the presentation slides from the workshop. She also explained what considerations and assumptions were and gave examples that were relevant to them. Then, she went through the task and mentioned that students could consider the costs, or in the case of the paper, the quality of the paper. The students were reminded not to make a decision immediately, but to compare the different items and to justify their decision. They were then given time to discuss which kind of paper to purchase. After the discussion, some students were selected to present their decisions as well as the justification behind their decisions. The teacher asked questions as they were presenting and acknowledged interesting ideas. She also showed a sample of a model solution for the choice of paper so that the students knew what they were expected to include in their answers.

Next, the students were given more time to discuss which kind of writing materials and stickers they would purchase. The teacher went around to facilitate the discussion and asked Socratic questions such as "Is that the most expensive? What makes you say so?" (probing reasons and evidence) and "So you are choosing Type 2 because it is waterproof?" (clarifying). She did not provide answers but encouraged students to think deeper about their claims. After the discussion, selected groups were asked to present their decisions and to justify them. Once again, the teacher asked Socratic questions like "What is his reason in choosing Type 2?" (probing reasons and evidence) and "So if you choose a thicker card, the chances of it smudging will be lesser. Will it smudge? I don't know. So this is an ……?" (probing assumptions) to elicit critical thinking.

Hence, the four identified features that could promote the development of critical thinking – the use of authentic tasks, the use of Socratic questions by the teacher, teacher behaviour that supports critical thinking and the communication of the criteria of assessing thinking through showing a sample model answer – were included in this lesson. Students' responses produced during the lesson were collected for analysis. The next section briefly explains how the data was analysed.

Data Analysis

The lesson conducted by the teacher was observed and video-recorded by the first author. The first author transcribed the video recording of the lesson. As the authors were interested to find out the extent to which students displayed the four characteristics of critical thinking during the lesson, four codes referring to the four characteristics of critical thinking were used in coding the video transcript. The codes were information (for gathering and assessing information); assumptions (for recognising and examining assumptions); alternative (for being open to alternative viewpoints) and justifications (for justifying decisions with reasons and evidences). Instances of the four chacteristics of critical thinking were also identified in student work. There were also codes for teacher actions, but those will not be discussed in this paper as the focus of this paper is on student responses. In the next section, student responses during the lesson and in their written work will be discussed.

Findings and Discussion

As iterated before, the purpose of this paper is to provide a context in which critical thinking might be developed in a Primary Four mathematics classroom in Singapore and to report how the students responded to the stimuli provided. In this section, responses from students during the lesson and in their work are analyzed and categorized into the four characteristics of critical thinking.

(1) Gathering and assessing relevant information

After the Stationery Task was explained to the students, they got into groups and discussed which type of paper to buy. For the choice of paper, students could consider (a) the number of pieces of paper needed, (b) the quality of the paper, (c1) the cost of each sheet of paper, (c2) the total cost of the paper and the budget (d). An analysis of the students' written responses showed that all the groups were able to identify at least two categories of the relevant information. Four representative examples are shown in Table 2.

Table 2.
Examples of students' written responses and the information considered

Group	Response	Information category
1	We choose type 2 because if we tore or draw wrongly, we still have 10 extra to make a new one but type 1 has 20 sheets but a lower quality than type 2 and type 3. Type 3 is 35 cents per 1 sheet of paper but we only need 10.	a, b, c1
2	We choose type 3 because it is much more cheaper and thick. When we write black marker on the paper, it will not damage the back and in total of ten papers, it will cost $3.50.	b, c2
3	We bought Type 3 colour papers because its cheaper than type 2 and the quality is quite good. If we buy 10 colour cards we would spend $3.50 and we can spend $16.50 on other decorations on the cards.	b, c2, d
8	We choose Type 2 because it is cheap and have 20 sheets. And it is thick so it will not tear easily. And it is only $3.95 for 20 sheets.	a, b, c2

Group 1 managed to identify that the number of pieces of paper, the quality of paper and the cost of each sheet of paper were relevant information. They noticed that type 1 and type 2 paper had the same number of sheets, but the quality of type 1 was inferior to that of type 2. They

also assessed that type 2 would yield ten extra sheets, which could be useful as spare copies should they destroy any of the sheets. They mentioned that type 3 cost 35 cents per sheet and that only ten sheets were needed, probably implying that the total cost of $3.50 for ten sheets of type 3 paper was cheaper than the total cost of $3.95 for 20 sheets of type 2 paper. They could have further mentioned how this information influenced their decision.

Group 3 considered the total cost, the quality of the paper and the budget. They were able to assess that the quality of type 3 paper was good and that ten sheets of type 3 paper would cost $3.50. They also found out how much they had left from the budget to buy other items. However, Group 3 claimed that type 3 was cheaper than type 2 without explaining how they reached that conclusion.

Group 8 considered the cost, the number of sheets and the quality of the paper. They mentioned that type 2 paper cost $3.95 for 20 sheets but did not elaborate on how this information was used to compare the cost of the different types of paper.

Group 2 managed to identify that the cost of each sheet of paper and the quality of the paper were relevant, but they did not mention what they were comparing type 3 to. Like Group 3, they managed to calculate that the cost of ten pieces of type 3 paper was $3.50. The members of this group were asked to present their responses verbally during the lesson and the teacher asked them how they compared the cost since they claimed that type 3 was cheaper. The group mentioned that type 2 cost $3.95 and type 3 cost $3.50 for ten sheets, thus type 3 was cheaper. The teacher further prompted the class to think if type 3 was really cheaper. In response, one student mentioned that type 3 was cheaper since only 10 sheets were needed and it was not necessary to compare the same number of sheets. Another student mentioned that they did not need extra sheets because if they made mistakes, they could erase the mistakes instead of using a fresh sheet of paper. Yet another student mentioned that the savings of 45 cents could be used for their future. Some groups worked out that 20 sheets of type 3 paper would cost $7 and so type 2 was cheaper. These verbal responses from students showed that they were assessing the information, albeit in different ways.

Thus, although the task was different from the usual mathematics word problems where all the information had to be used to arrive at one fixed answer, the students showed that they were capable of gathering at least two categories of relevant information. Moreover, although none of the groups identified all the relevant information in their written work, the students' verbal responses to their teacher's prompts showed that they could assess the information in varied ways. Certain ideas that were not expressed in the written form were verbalized during the whole class discussion. The students' language ability might have impeded how they expressed their ideas in the written form, but when given the opportunity to speak up, they were able to express themselves more effectively.

(2) Recognizing and examining assumptions

In introducing the concept of critical thinking at the beginning of the lesson, the teacher had explained the word "assumptions" and given examples of "assumptions". However, during the whole class discussion, there were only two instances when assumptions were mentioned.

In one of the instances, the teacher referred to some students' claim that certain types of markers would smudge while discussing the choice of writing materials. She elicited from

students that since they did not know for sure that the markers would smudge, they were making an assumption.

The second instance occurred during the discussion. Group 6 mentioned their assumption that their teacher liked food in this statement "I am assuming that she likes food and all the time she ran to the canteen." This was the only example where the students brought up the word "assuming" on their own. The teacher responded by mentioning that another group had asked her if she liked Mickey Mouse (which was one of the types of stickers featured in the catalogue). For them, since she had told them her preference, they were not making an assumption.

In some groups' written responses, assumptions were made, but they were not identified to be assumptions. It was not clear from the responses if the students knew they were making assumptions. Two examples are as follows:

Group 5: "There might be the teacher's favourite colour in the pack."

Group 5: "Everyone likes food."

Hence, if recognizing and examining assumptions is an important characteristic of critical thinking, more emphasis may need to be placed on it. Perhaps, more examples of assumptions could be given to students. Alternatively, the context in this study might have been limiting in terms of the assumptions that could be made. Some possible assumptions that students could make were the number of pieces of paper they needed to buy, the quality of the markers, the preferences of the teachers and whether other materials were already available. For future studies, the context could be better considered to provide more opportunities for assumptions to be made.

(3) Maintaining an open mind to alternative viewpoints

In the first part of the lesson, when introducing critical thinking and the importance of critical thinking to her students, the teacher showed them a piece of fake news in the form of a picture of young children lying down on the floor with the caption "At Sam's Early Learning Centre, children are made to sleep on the floor without mattresses". Students were asked for their reactions to the picture. The initial responses from the students were those of sympathy for the children like "I pity the kids" or negativity towards the centre like "The teacher doesn't care about the children". When the teacher prompted the students to think of alternative views with questions such as "Does anyone else think differently?" and "Do you think there is a reason for them to do this?", the following responses were suggested:

Student 1: "They could be playing a game."

Student 2: "It was too hot to sleep on a mattress."

Student 3: "Someone wanted the centre to close down."

In reality, the picture was circulated by an unhappy former employee who had since returned to her country of origin. The centre issued a statement that the children were doing exercises on the floor and were not sleeping. From this example, it appeared that the students' initial views were clustered around the same issues. However, after the teacher actively asked Socratic

questions to encourage different views, the students' responses showed that they were open to alternative viewpoints.

Although it was mentioned earlier that students were able to gather and assess information when they made a decision on which type of paper to buy, only one group (Group 1) explicitly mentioned the comparison of the three types of paper. Since being open to alternative viewpoints was a characteristic of critical thinking in this study, it was important that students should consider all possible alternatives, then compare and contrast the various options before coming to a decision. Therefore, after the discussion on the choice of paper, the teacher showed the students how she expected them to communicate their decision. This included an explicit comparison of the three types. She also reassured students that there was no right or wrong answer. What she was looking for was that they were able to justify their decisions.

For the choice of writing materials and stickers that followed, more student responses displayed a comparison of all three types of items. Examples of responses that included a comparison of all three types of items are included in Table 3. These responses showed that the students did not immediately make a decision on which items to buy, but they explored and considered the pros and cons of each type and were able to articulate the various viewpoints. For instance, on Group 1's choice of writing materials, they had worked out the unit cost of each type and found that the unit cost of type 3 was the highest. Yet, they chose type 3 and justified their decision by saying that type 3 had more colours and they could make their cards more attractive with the two additional colours.

Table 3.
Examples of students' written responses that included a comparison of all three types of items

Stationery	**Group**	**Response**
Writing materials	1	We choose type 3 because it is 8 for $12.85. We can make it more colourful. But type 2 is 6 for $6.90 and type 1 is 5 for $6. We would rather type 3 than the rest. Type 3 has 8 and we can make the cards more colourful and beautiful. Type 3 1.60 per marker, type 2 1.15 per marker and type 1 1.20 per marker.
Writing materials	4	We choose type 1 because 1 marker only cost $1.15 and there are more markers than type 2 and 3. Type 2 cost more than type 3 and 1. Type 3 is very expensive and it will leave not that much money.
Writing materials	7	Type 2. It has more colours and it is cheap. It is cheap because type 1 is $6.00 but type 2 is $6.90 and got more colours but type 3 has 2 more but the 2 more cost $5.95.
Stickers	4	We choose type 1 because it is cheaper than type 3 and 2 and we can save more money. And there are more stickers than type 3 and 1.
Stickers	8	We choose type 1 because it has more stickers and cost $1.90 and type 1 has more stickers than type 2 and type 3.

Apart from the written response, a student from Group 7 was asked to explain his view on the cost of the writing materials verbally. The student shared that his group chose type 2 even though it cost 90 cents more than type 1 because 90 cents would give them 1 additional colour. Type 3 would also give them more markers, but the additional price of $5.95 compared to type 2 was much too high. This response showed that the student was not only comparing the three types of writing materials, but he was also justifying his decision. Thus, he demonstrated both

the third and fourth characteristics of critical thinking in his response. The teacher affirmed the student for good thinking. She also encouraged the class to think how else they could compare the cost. Another student suggested that they could find the cost of one marker for each type, following which the teacher asked the class to perform the calculation. Through teacher questioning, the teacher challenged the students to consider different ways of deciding which kind of writing materials was more value for money and students were able to come up with different responses.

(4) Justifying decisions with supporting evidence and reasons

From Group 1's justification for their choice of paper (cited in point 1), it can be seen that the students were able to discuss different considerations. They mentioned that type 2 will yield ten extra pieces of paper and that type 1's quality was lower than that of type 2 and type 3. They communicated their decision clearly and supported the decision with justifications.

All the responses cited in point 3 also showed evidence that students were able to justify their decisions. In addition to the responses cited in point 3, Table 4 shows more examples of students' ability to justify their decisions with supporting evidence and reasons.

Table 4.
Examples of students' written responses that show justification of decisions

Stationery	Group	Response
Writing materials	3	We choose type 3 markers because there are more colours and we can decorate more and if there are other markers that run out of ink, we can still use it. And there are two sides that can be a different shape. If we buy the type 3 marker, we will have 8 colour markers so we can decorate more. Type 1 or type 2, we will have less colour and only 1 sided shape.
Stickers	1	We chose food item stickers as it is cheap and there's a lot of stickers on it. The food item sticker and the Mickey sticker price is the same. Mickey Mouse sticker has lesser stickers than cat stickers, but cat stickers is 3.50 and it has lesser sticker than the food item sticker and it is over budget."

Group 3 stated their choice of writing materials clearly and justified their decision using the information on the number of colours and the double tips of type 3 markers. They also pointed out the disadvantages of the alternatives (type 1 and type 2) and justified why those were not chosen. They could have improved in their justification by also considering the cost of the writing materials as cost was an important factor in buying items. The teacher reminded them to do so.

Group 1 found that the prices of the type 1 and type 2 stickers were the same, so they compared their number of stickers and concluded that type 1 stickers were a better buy. They also justified why they did not choose type 3 stickers. Type 3 stickers would cause them not only to overspend the given budget but also to receive fewer stickers than type 1 stickers. The reasons given by the group well-supported their decision.

As a number of groups had indicated that the food stickers were the cheapest during the whole class discussion, the teacher asked the students verbally if there was any evidence that type 1 stickers were the cheapest. A student responded by saying they could not divide the total cost by the number of stickers for type 1 stickers to find the unit cost as there were too many stickers

to be counted, but they concluded that visually, it was obvious that type 1 had the most number of stickers. This response was not found in students' written work, so the teacher might have to prompt such responses during class discussion.

Implications

Some examples of students' work or their presentations have been cited and discussed in this paper. The examples are positive in showing that students are capable of gathering and assessing relevant information, considering alternative viewpoints and justifying their decisions with reasons. These are characteristics of critical thinking as operationalized in this study. It is hoped that teachers would be encouraged by the findings and hence be more willing to actively incorporate critical thinking into their lessons. However, not many students referred to or recognized assumptions, so if that is considered to be an important characteristic of critical thinking, more would have to be done to address it.

Another implication from this study was that the four features identified for inclusion in a lesson that aimed to foster critical thinking were all crucial in eliciting all the components of critical thinking. When the students began to discuss which type of paper to purchase, the teacher had not asked Socratic questions pertaining to that task nor had she shown the students how they should present their responses. Although all the groups were able to gather at least two categories of relevant information, most of the groups did not explicitly compare the different types of paper at first. Their justification of their choices of paper was mainly based on the advantages of that choice. The teacher had to ask Socratic questions to probe alternative views or to probe reasons and evidence. She also asked questions to help students recognize assumptions and to clarify their answers. In addition, the teacher did not provide immediate answers, but encouraged students to think from different perspectives. She also asked the class to applaud interesting ideas and was accepting to all answers provided, even when one student mentioned that he chose the food stickers because he was hungry. She emphasized that as long as students considered all possibilities and could justify their decisions, that was what she was looking for. Lastly, she showed a sample of what was expected in students' answers after they had presented on their choice of paper. Students' justifications for the choice of writing materials and stickers showed more instances of comparison of types and consideration of more factors. Thus, this study provided more information on the features of a lesson that could promote the development of critical thinking.

Conclusion

In closing, this study has fulfilled its purpose of providing an example of a context in which critical thinking might be developed in a Primary Four mathematics classroom in Singapore. It has also reported the extent to which Primary Four students demonstrate critical thinking when four identified features are present in the lesson. This study also contributes theoretically by proposing that critical thinking can be operationalized by four characteristics. Future studies could explore using tasks of other topics, or students of other ages to confirm if the lesson features are generalizable to other topics and age groups.

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Appendix 1

Primary 4 decimals task

You intend to make 10 thank-you cards for your teachers. For this task, you need paper, writing materials and stickers. The catalogue below shows the costs of the items you need.

Stationery type	**Type 1**	**Type 2**	**Type 3**
Paper	 A4 coloured cards 20 sheets (160gsm) $3.50	 A4 coloured cards 20 sheets (200gsm) $3.95	 Individual sheets of A4 coloured cards (200gsm) $0.35 per sheet
Writing materials	 Type 1 markers (5 colours) $6.00	 Type 2 gel pens (6 colours) $6.90	 Type 3 markers (8 colours) $12.85
Stickers	 Food item stickers $1.90	 Mickey Mouse stickers $1.90	 Cat stickers $3.50

With a $20 budget, which materials in the catalogue will you buy?
Explain how you arrive at your decision.
Please state any assumptions you have made and write down all your considerations.

Appendix 2

Sample model solution to decimals task

Paper
Decision: Type 2 cards (200 gsm)
Reasons: Type 2 costs $0.45 more than type 1 cards, but each piece is thicker and probably more durable. Type 3 sheets are also 200 gsm, but 10 sheets cost $3.50 (= 20 sheets cost $7) while 20 sheets of Type 2 cost $3.95. Type 2 is cheaper per sheet.
(Considers cost, quality of paper, number of sheets needed and makes comparisons)

Writing materials
Decision: Type 2
Reasons: Lowest unit cost, fine tip writes better, least likely to soak through paper
Type 1 has fewer colours and slightly higher unit cost
Type 3 costs $5.95 more than type 2.

Supports with calculations
Type 1: 1 marker → $1.20
Type 2: 1 pen → $1.15
Type 3: 1 marker → $1.61 (approximately)
(Considers unit cost, writing comfort, ink suitability)

Stickers
Decision: Type 1 stickers
Reasons: It is the cheapest and contains the most number of stickers. Everybody likes food (assumption).
Type 2 has the same cost as type 1, but much fewer stickers.
Type 3 is more expensive than type 1 and has fewer stickers than type 1.
(Considers cost, number of stickers, what teachers like)

Budget: $20
Total cost: $3.95 + $6.90 + $1.90 = $12.45
(Checks that amount is within budget)