ASSOCIATION OF MATHEMATICS EDUCATORS

Website: http://math.nie.edu.sg/ame/

Maths Buzz

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President's Message

Dear AME Members,

On behalf of the Association of Mathematics Educators, I wish you all a very Happy New Year. We, the exco of AME, hope 2010 will bring you good health and rich experiences as you meticulously make the learning of mathematics both meaningful and fun for your charges. Mdm Foo Kum Fong and A/P Ng Swee Fong were the editors of the Maths Buzz from October 2003 till May 2009. It was under your able leadership that 12 issues of the newsletter, i.e. 6 volumes with 2 issues per volume, were produced

The editors of the Maths Buzz, with effect from July 2009 are Ms Chua Kwee Gek and Asst/P Lee Ngan Hoe. This issue of the Maths Buzz is their first and they have decided to give the newsletter a new look! It certainly has expanded in scope and the editors will explain their rationale.

Berinderjeet Kaur
President,
AME (2008 – 2010)

Contents

Editors' message

Dear Readers of Maths Buzz,

Welcome to a new issue of Maths Buzz, and a big thank for your support of Maths Buzz over the years. With the new year ahead of us, we offer you a new format to Maths Buzz – a way to continuously upgrade ourselves to make this newsletter into a platform that is more suitable for all mathematics educators and researchers to buzz around the mathematics education scene keeping us all connected and informed.

From this issue onwards, we are not only going to include contributions from researchers, but also practicing teachers. We want to set up a channel for excited teachers to share some of their creative teaching ideas for the mathematics classrooms. You are welcome to email us your ideas at mathsbuzz.ame@gmail.com. In fact, in our initial call for such ideas to AME members at the end of last year, we have received many contributions from enthusiastic teachers. We welcome your continuous support. However, due to the limited space, we would like to appeal to potential contributors to submit their write-ups in no more than two pages. Some teachers in fact, also emailed and shared with us the related worksheets to their lesson ideas. To allow readers access to such rich materials for published ideas digitally, we have created a website; http://math.nie.edu.sg/ame where the e-copy of Maths Buzz and links to related resources are available electronically.

As a way to serve our readers better, we have also created another channel for discussion and clarification of issues related to teaching and learning of mathematics. You may email your questions or views to askdrmathsteaching.sg@gmail.com – the Singapore version of "Ask Dr Maths Teaching". We will be directing such contributions to the relevant authority or expertise for comments and we hope to publish some of these that may be of interest to most of the readers.

We sincerely hope you will be able to better relate with the new format for Maths Buzz as we continue to seek to improve this channel of communication with our readers. We also welcome comments and suggestions to better position Maths Buzz. You may contact us at mathsbuzz.ame@gmail.com.

Happy Mathematicsteaching.

Editors
Maths Buzz

Promote Student Questioning in Mathematics Lessons

Wong Khoon Yoong & Quek Khiok Seng
National Institute of Education

Two scenarios

Question: Do your students normally ask questions in class when they do not follow your explanations?

Teacher 1: Yes, I have created a cordial learning environment and my students are not afraid to ask questions. However, some of them keep asking me to explain everything from the beginning. This can be rather frustrating.

Teacher 2: No, most of them are very shy. Some of them are afraid to ask questions in case their classmates think they are "stupid".

The willingness of students to ask questions when they do not understand is a metacognitive attribute that Teacher 1 has managed to promote in her class, whereas the students under Teacher 2 need some encouragement. Students in both classes, however, can benefit from a stronger metacognitive ability to ask appropriate questions related to the math. This ability will help students under Teacher 1 to ask focused rather than very broad questions and students under Teacher 2 to reduce the anxiety of being seen as asking "stupid" questions. We propose here one technique to help students learn to ask appropriate math questions through initial scaffolding using a set of Student Question Cards (SQC).

Four types of math-related questions

Most math lessons are about one or more of these four aspects: Meaning, Method, Reasoning, and Applications. A variety of questions can be asked about each of these aspects. Suppose the teacher has just spent about 15 minutes explaining congruency between triangles ABC and XYZ. The students may not have understood certain parts of the explanation and want to ask some focussed questions. Below are some possible questions.

Meaning: How is this symbol "≡" different from the equal sign?
Method: Do we have to strictly keep to the order of pairing A with X, B with Y, and C with Z?
Reasoning: Why do congruent triangles have the same area?
Application: When do people use congruent triangles in real life?

These specific questions about a short segment of a lesson can be generalised into standard question forms. These forms (the set of SQC) are shown below. They were arrived at with consultation with several primary and secondary mathematics teachers from a much longer list of questions. The students can fill in the ellipses (…) with words related to specifics of the lessons. The last option for each aspect is (Your own question), allowing students to frame their own questions. These questions are printed on laminated cards with one aspect per card; the label of each aspect on one side and the sample questions on the reverse side of the same card.

Meaning	***Method***
M1: What do you mean by …. M2: What is the difference between …. and …. M3: Can you use a diagram to show …. M4: (Your own question)	Md1: Can you show us how to do this problem in another way? Md2: Can you explain/show us this step (….) again? Md3: What will you do next? Md4: (Your own question)
Reasoning	***Application***
R1: Why do you do that ….? R2: What happens if you change …. to ….? R3: (Your own question)	A1: Why do we study this topic (….)? A2: How do we use this (….) in everyday life? A3: (Your own question)

Using the SQC

Teachers can set aside time during their math lessons to allow students to ask questions. For example, after about 15 minutes of explanation or activity, the teacher stops the class and requires students to ask questions. These pauses in the lesson are called Question Times (QT). During these QTs, the students are encouraged to refer to the SQC to find a question to ask about that part of the lesson. This is the scaffolding part, which is especially important for students who do not know how to phrase appropriate math questions. The teacher may proceed differently from this stage, including the following:

Ask every student to choose a question from SQC and call on a few of them to ask their questions. This forces every student to think about their learning.

- Ask students who really have doubts about that part of the lesson to ask a question from SQC.
- Focus on a particular aspect, for example, ask students to choose only Reasoning questions. Change the aspect to focus on during the lesson and across several lessons. This method will help students to become familiar with all the four ways of thinking about math.
- Call on only specific students to ask questions from SQC. This will encourage participation from as many students as possible, including those who normally keep quiet in class.
- Organize students into groups, allow them to discuss the questions they are going to ask, and pick a few groups to ask their questions.

Answering student questions is a challenge

Once a question has been asked, the teacher has to answer it as carefully as possible. Teachers are used to asking students questions and they know what answers are correct or wrong. The SQC technique reverses the roles of the teacher and students with respect to questions and answers. Teachers may find some student questions easy to answer while other questions, especially the Application ones, quite challenging. They have to answer these questions without prior preparation and this can be a real test of their mathematical and context knowledge. Trying to give convincing answers to challenging student questions is the beginning of a self-learning journey for the teachers!

In one particular lesson, the teacher had explained how to factorise expressions such as 9 – 25x2. At the immediate QT, there were 8 questions about what is the meaning of factorisation and 8 questions asking the teacher to explain the technique again (plus a few other questions). The teacher had no difficulty answering these questions. She was also aware of the students' difficulty; she said:

> I think they are also confused by the x there. You see, given 25 alone and 9, they're alright. But given an x in the expression, I think they are confused. They cannot handle variables and numbers together, they cannot see it, they get confused. Numbers alone, alright. But mixed, I don't think they can do it.

Application questions are quite popular because as one student in our study explained, it can be asked for all topics. In another lesson, the teacher taught the conversion of fractions into decimals, resulting in decimals that terminate, recurring, and not recurring. At the QT, 19 students chose Application 1: Why do we study this topic? The teacher explained:

> We need several skills in numeracy. Example, you go supermarket

and bought something for $1.58. You bought 18 of them. You work in McDonalds' you're paid $2.50 per hour and you worked 7 hours a week. Doesn't mean (only) whole numbers exist.

This explained why we need decimals (some educators do not think that money involves decimals, as some people read $2.50 as two fifty!) but not why we need to know the conversion or the three types of decimals. It is not easy to give convincing answers to Application questions targeted at specific skills. Nevertheless, one of the aims of the Singapore Mathematics Curriculum is for students to "acquire and apply skills and knowledge relating to number, measure and space in mathematical situations that they will meet in life".

This SQC technique includes training the students to pay attention to the questions asked by their classmates as well as the answers given by the teacher. This kind of attention does not happen by itself!

Concluding remarks

The SQC technique is flexible and easy enough to use so that the teachers can adapt it to suit their teaching style and target students. It can break the monotony of the lessons and stimulate the quiet or shy students to become more participative. When the students are competent at asking their own questions and eager to think about the answers from the teachers or own mental struggle, eventually they will become self-regulated learners, making it easier for teachers to "cover" the syllabus. Certain aspects of this SQC technique can be a meaningful area for teacher's next Action Research (AR) project.

Read more about this project: Wong, K. Y., & Quek, K. S. (2009). Enhancing mathematics performance of mathematically weak pupils: An exploratory study.

(http://www.crpp.nie.edu.sg/~pubs/CRP47_03WKY_FinalResRpt.pdf)

Use of Blogs To Engage Students in the Learning of Mathematics

Kris, Ng Tjin-Hwai
Tanjong Katong Girls' School

Blogging has gained popularity among young digital natives, our students. How could we tap this "hip" platform for teaching?

I got started after I attended a course in TRAISI; on the Use of Blogs on the Teaching and Learning of Mathematics by ETD. Despite dismal responses from the students for the first blog attempt I did (http://www.simultaneouslinearequations.blogspot.com), I persevered. The excited reactions from the students convinced me that this was going to work.

For the March holidays, I did up http://www.factorise.blogspot.com. By then, it was obvious to me that the students liked the idea of learning of Mathematics via blog. Yet something was poignantly missing – participation from the weaker students. None of them posted any response on the blog. What were the factors deterring them from doing so? Was it a lack of confidence?

My previous blogs had always been optional. To involve the group of persistent non-participators, I created http://www.quadgraphs2e8.blogspot.com, I concurrently did up a set of accompanying worksheets to complement the learning. Having a whole class logging on to the blog simultaneously proved challenging. Ensuring that they do not end up surfing the net but follow the activities set was yet another challenge. Which part of a teacher's job isn't challenging anyway?

With the new lessons' structure, I observed how the weak students took a liking to comfortable scaffolding of the tasks; the relevant links made to the textbook and the worksheet as well as the indirect "guidance" they got by reading the responses of their peers prior to inputting their own.

Many would agree with me that engagement is key to learning. Now that I have engaged the weak students, I was further motivated to continue this modern form of teaching for my other teaching subject; English, http://www.setlanguage2e2.blogspot.com.

I noticed the confidence in the students growing, particularly among the weaker students. They were no longer daydreaming. Nor were they dumbfounded when you call upon them to answer questions. They were sitting up straight, listening for instructions, switching almost immediately to face the computer once instruction was given to attempt the task on the blog. Once they had posted their responses, they took the initiative to read others' responses to see a wider range of acceptable answers and thus understand the concept better.

From perennially failing Mathematics, these students, who could never catch up in class, had problems answering questions when asked and not to mention had trouble with homework, were now transformed. With this engagement came newfound confidence. With confidence came the zest to work harder.

Of course, it is also important to look at changes in their grades. Beacuse of their weak foundation from the start, the grades were only inching up slightly. But to me, it was heartening enough knowing how their enthusiasm for learning Mathematics had been ignited and will continue to burn from that point on. With this positive attitude towards learning, they will continue to ascend the curve for learning in the future. I'm glad I had a part to play in moulding them.

Mathematical Investigation from Convex to Concave Polygons

Fung Hwee Hua & Rosalind Soh
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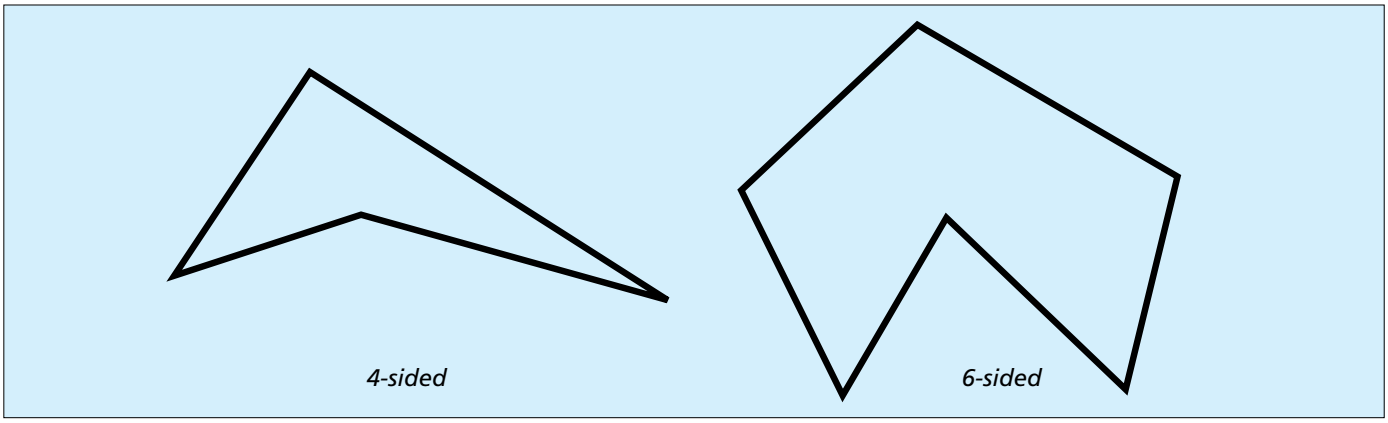

1. On each of the above concave polygon, mark, with the letter R, the vertex that has a reflex interior angle. Measure and write down the size of the reflex angle.
2. Draw two more concave polygons.
3. Does the formula of the sum of interior angles apply to concave polygons? Let's investigate.

Pointers:

- Use the given and drawn polygons to find out more
- Explain in words, providing the necessary mathematics to support your reasoning
- Organise and tabulate your results
- Draw your conclusions based on your observation and findings from the polygons.

Questions:

- Does the formula for the sum of interior angles of convex polygon (i.e, (n-2) x 180°) also apply to concave polygons?

Answer : Yes

- How does one define an exterior angle of a concave polygon at the vertex that carries the reflex angle?

Investigation:

Let's take a walk around a triangle.
Let the traingle be ABC and ABC are in the counter-clockwise direction.

Start at point A facing B.
Raise your right arm to a horizontal position and pointing at B.
Walk to B.
While at B, turn to face C then walk to C.
While at C, turn to face A.
Walk to A.
While at A, turn to B.

By doing so, one would have walked around the triangle and you get back to the same position. In short, one would have turned 360 degrees. An arm has swept across the three exterior angles. That is to say: the sum of the exterior angles is 360 degrees.
One can also walk a convex quadrilateral, or any convex polygon.
One will have always turned 360 degrees.

Walk around a convex polygon.
Walk around again a concave polygon.
One will notice that for a convex polygon, the turning is always counter-clockwise.
For a concave polygon, the turning is also in the counter-clockwise direction except at the point when the polygon is concave.

Here is the formula: the sum of the counter-clockwise angles minus the sum of all clockwise angles is always 360 degrees.

Walk a few times and one will see how it works and why it works.
In fact, the formula has nothing to do with polygons.
Can you generalize the above formula?

- Is the sum of exterior angles (ie 360°) also true for concave polygons?

Answer : Yes

Further Exploration for Teachers:

- How could you extend this concept to higher level such as the topic on vectors?

Think of n vectors v_1, v_2, ..., v_n.
Connect them consecutively and v_n ends up at the starting point of v_1.

In short, this is a connected chain of vectors.
The vectors are allowed to cross each others.
The same formula holds, namely, the sum of all angles is a multiple of 360 degrees.

Polygon, convex or concave, is a special case.

Acknowledgement: A/P Lee Peng Yee for his advice and guidance. The Singapore Ministry of Education (MOE) has made extensive changes in its curriculum focus in the last ten years. This change was brought about by the recognition of the global social changes and the need for Singapore in producing "the creative, autonomous and flexible work force required to compete in value-added market" (Sharpe L & Gopinathan S, 2002).

Problem- Based Learning- Implementation and Assessment

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The assessment in Singapore has also changed in tandem with the new curriculum focus, which is to possess "high level of communication skills, interpersonal skills and proactive mindsets and the competencies to seek, evaluate, process and apply new knowledge and information." (Sellen, R . et al., 2006). In response to the Singapore changing educational landscape, our department embarked on Problem-Based Learning (PBL) in mathematics.

Problem-Based Learning is a student-centered instructional approach that anchors the curriculum in ill-structured, authentic problems (Hmelo-Silver, 2004; Savery, 2006). The use of PBL is not new in secondary schools and we have adapted it into our mathematics curriculum to suit our pupils. Although it has been mentioned frequently from literature reviews that PBL has numerous advantages such as having positive impact on motivation (Pedersen, 2003) and problem-solving performance and collaboration skills (Achilles & Hoover, 1996), the planning, implementation and assessment stage are often tedious and riddled with intricate details. These three aspects needed to be looked into critically in order to reap the benefits of PBL.

Before the implementation of PBL, our teachers understudied educators in Republic Polytechnic to gain an understanding of how PBL lessons are conducted. We then decided to pilot PBL lessons to the first class of Secondary 1 (2009 batch) that has an average PSLE T-score of 220. We had one PBL lesson fortnightly. This is to create the opportunity for students to hone their problem solving skills regularly without unduly pressuring them.

Lessons were carried out in our school's Celebratory Learning Centre (CLC) which is equipped with technological resources to support learning. Students had the luxury of using wireless computers and seating configuration which facilitates better discussion and interaction among the students on the assigned problem.

The teachers met fortnightly to craft the problem statements for the PBL lesson. Although the problem statement should be an ill-structured statement, it is by no means easy to craft as we had to take into account the entry level of the students in terms of their content knowledge, thinking skills and problem-solving skills. The final problem statement crafted should be appropriate and had the potential of getting our students learn the intended content and skills stated in the syllabus.

During the fortnightly PBL lessons, the teacher adopted the facilitator's role and provided guidance to students' queries. At the end of each PBL lesson, two groups were selected to present their solutions to the class. The teacher gave comments to the group for future improvement and presented the solution with emphasis on the objective of the day's lesson.

Two surveys were administered half-yearly. The students responded that they had benefited and enjoyed the PBL lessons. From the many reflections by students, one stated, "I really enjoy PBL lessons because it allows me to learn things outside of the textbook".

Initially, the students were graded according to their teamwork and accuracy of solutions. However, as we reflected on our initial objectives of embarking the PBL process, we found that there are many other skills that we should be measuring. The difficulty lies in the aspect of grading thinking skills like creativity and critical thinking, proactive mindsets, the competencies to seek and apply new knowledge and information.

As such, our department engaged the help of the participants from Management and Leadership in Schools (MLS) to assist us in crafting the rubrics to measure the intended skills. We are hopeful that we will be more structured in our assessment of skills through PBL next year and are looking forward to sharing the rubrics and our experience in our new approach of assessment.

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Sample of a Problem - Based Learning Task

You are organizing your own birthday party. You are to prepare goody bags for your guests. Your mother has brought you to Cold Storage. She has instructed you to purchase 4 types snacks — M&M SMALL BAG CHOCOLATE , MARS FUNSIZE CHOCOLATE, SNICKERS FUNSIZE, TOBLERONE MILK CHOCOLATE MINIS BAG. All these items come in share packs.

You then need to divide the items to obtain as many equivalent goody bags as possible from what you have bought, so that you can invite the most number of people to the party. You may not have any left over snacks or sweets.

Resources:
1) Textbook
2) http://www.coldstorage.com.sg/onlineshopping/default.aspx

Setter: Mr Mazri Bin Misawar

Explaining why 0! is defined to be 1

Tay Eng Guan
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The teacher could begin writing on the board 2×1, 3×2×1, 4×3×2×1 and state that such products are quite common in counting problems. Again, the teacher may then ask the students if they have encountered a way of representing these products in a more succinct manner. Once, students realize what a hassle it would be to write 100! as a product of 100 terms, either by discussion or previous knowledge, it is easy to tell students that the notation n! is a good way of representing 'a product of n consecutive integers beginning with 1'. Thus 4! = 4 · 3 · 2 · 1 = 24. The notation 'n!' is read 'n factorial'. The teacher explains this as a case where something is defined for ease of notation.

At this point, the teacher may ask the students, "For what values of n is the notation meaningful?" Since n! has now been defined to be 'a product of n consecutive integers beginning with 1', it is only logical that n must be a positive integer. Now, the teacher may write $n! = n(n-1)(n-2)\ldots3\cdot2\cdot1$, where n is a positive integer.

The teacher goes on to show that this notation is useful in simplifying expressions in certain counting formulae. Using the 'factorial' notation, we now have the following:

$$P_r^n = n(n-1)\ldots(n-r+1) = \frac{n(n-1)\ldots(n-r+1)(n-r)(n-r-1)\ldots2\cdot1}{(n-r)(n-r-1)\ldots2\cdot1} = \frac{n!}{(n-r)!}.$$

Next the teacher may ask,

"For what values of n and r is the formula $P_r^n = \frac{n!}{(n-r)!}$?"

The answer (hopefully from the students) is that since m! is defined only for positive integers m, the numbers n and $n-r$ must both be positive integers. Thus, the formula is valid for positive integers r, n such that $0 < r < n$. Now the teacher writes Theorem 3 completely as follows: for positive $P_r^n = \frac{n!}{(n-r)!}$ integers n and r such that $r < n$.

Consider the extreme case when r = n. From definition, we have that is the number of ways of arranging n distinct objects from n distinct P_r^n objects. Thus, by counting the number of choices for each position and using the Multiplication Principle,

we have $P_n^n = n(n-1)(n-2)\cdot\ldots\cdot3\cdot2\cdot1 = n!$.

On the other hand, from Theorem 3, we have $P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$.

By now, teachers and students would have experience enough to remark, "How nice it would be if 0! = 1. Then the theorem would work for $r = n$ as well." And so we define 0! = 1 so as to extend Theorem 3 to include P_r^n

TIMSS 2007 - Performance in Mathematics of Primary Four Pupils from Singapore

Koay Phong Lee
National Institute of Education

TIMSS (Trends in International Mathematics and Science Study) 2007 is the fourth cycle of the international comparative study on the mathematics and science performance of fourth grade (Primary 4 in Singapore) and eight grade (Secondary 2 in Singapore) pupils. The first study was held in 1995. Fourth graders, including their teachers and principals, from 37 countries participated in the 2007 study. The findings can be found in the TIMSS 2007 International Mathematics Report (Mullis, Martin and Foy, 2008). This article attempts to summarize the performance of Primary 4 participants from Singapore with respect to the international benchmarks.

The subjects

5041 Primary 4 pupils in Singapore were selected according to TIMSS guidelines on all phases of the sampling procedures. They sat for the mathematics test at the end of the school year. Their average age at the point of testing was 10.4 years.

The test

The test consisted of 179 items testing three mathematics content areas in two test formats, multiple-choice and constructed-response. Three cognitive domains were tested, namely, knowing, applying and reasoning. Table 1 shows the distribution of these test items.

Table 1. Distribution of Items by Content Domain and Cognitive Domain

Content Domain	MCQ items	Constructed response items	Total number of items	Total number of score points
Number	50	43	93	98 (51%)
Geometric shapes and measures	32	28	60	65 (34%)
Data Display	14	12	26	29 (15%)
Total	96	83	179	192 (100%)
Cognitive domain				
Knowing	45	24	69	73 (38%)
Applying	37	33	70	75 (39%)
Reasoning	14	26	40	44 (23%)
Total	96	83	179	192 (100%)

The test tested on 19 topics on number, 11 topics on geometric shapes and measures, and 5 topics on data display. About a quarter of the topics tested are not be found in Singapore Primary 4 mathematics syllabus as shown Table 2. However, pupils may have been exposed to some of these topics in enrichment classes.

Table 2. TIMSS Assessment Topics not found in Singapore Primary 4 Mathematics Syllabus

TIMSS mathematics topics	Total number of topics	Topics not taught at Primary 4 in Singapore
Number	19	• model simple situations involving unknowns with expressions or number sentences • describing relationships between adjacent terms in a sequence • generate pairs of numbers following a given rule • finding a rule for a relationship given some pairs of numbers
Geometric shapes and measures	11	• recognizing relationships between three-dimensional shapes and their two-dimensional representations • use informal coordinate systems to locate point in a plane • reflection and rotation
Data display	5	• comparing and matching different representations of the same data
Total	35 (100%)	8 (23%)

Overall Performance of Primary 4 pupils

The top four performing countries in the world (see Table 3) are from four East and South East Asian countries. The overall performance of Grade 4 pupils from Hong Kong SAR and Singapore was well above the international average. The difference in achievement between Hong Kong SAR and Singapore is relatively small. This is commendable as about a quarter of the topics tested are not in Singapore Primary 4 mathematics syllabus.

Table 3. Average Mathematics Achievement of the Top Four Performing Countries

Country*	Rank	Average Scale Score	Human Development Index#
Hong Kong SAR	1	607 (3.6)^	0.937
Singapore	2	599 (3.7)	0.922
Chinese Taipei	3	576 (1.7)	0.932
Japan	4	568 (2.1)	0.953
International average	500		

* Only Grade 4 pupils from these four East and South East Asian countries participated in TIMSS 2007.
^ Standard errors
\# The human development index (HDI) measures a country's human development. The index measures the country's average achievements in three basic areas of human development; health, knowledge, and a decent standard of living.

Table 4 shows the average percentage of pupils responding correctly to the items in each domain. Grade 4 pupils in Singapore performed extremely well in both the mathematics content domain and the cognitive domain as compared to the Grade 4 pupils in the rest of world.

Table 4. Average Percent Correct in the Mathematics Content Domain and Cognitive Domain

Domains		Singapore average	International average	Ranking
Mathematics content	Number	75 (0.9)	46 (0.1)	Sharing 1st
	Geometric shapes and measures	70 (0.8)	47 (0.1)	2nd
	Data display	82 (0.7)	54 (0.1)	2nd
Mathematics cognitive	Knowing	80 (0.7)	51 (0.1)	2nd
	Applying	76 (0.8)	49 (0.1)	2nd
	Reasoning	63 (1.1)	38 (0.1)	2nd

TIMSS uses a four-point scale to establish the international benchmarks. The benchmarks indicate what pupils typically know and can do in mathematics. Table 5 gives a brief description of each benchmark and the percentage attained by Singapore Primary 4 pupils.

Table 5. Description of International Benchmarks for Grade 4

International Benchmark	Score	Description of mathematical knowledge and ability	Percentage of pupils reaching the benchmark	
			Singapore	International median
Advanced	625	Pupils can apply their understanding and knowledge in a variety of relatively complex situations, including justification.	41 (2.1)	5
High	550	Pupils can apply their knowledge and understanding to solve problems.	74 (1.7)	26
Intermediate	475	Pupils can apply basic mathematical knowledge in straightforward situations.	92 (0.9)	67
Low	400	Pupils have some basic mathematical knowledge.	98 (0.3)	90

According to the TIMSS results, 98% of the Primary 4 pupils in Singapore had some basic mathematical knowledge and slightly half of these pupils were able to apply their knowledge in relatively complex situations and justify their solutions. Only Hong Kong SAR (100%) and Chinese Taipei (99%) attained a higher percentage than Singapore for the TIMSS 2007 Low International Benchmark of mathematics performance. Gender wise, the Primary 4 girls in Singapore seem to perform slightly better than the boys in both domains (see Table 6).

Table 6. Average Achievement in the Math Content and Cognitive Domains

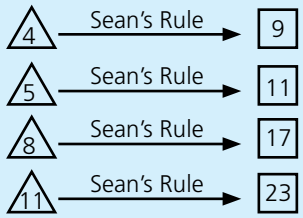
Domains		Total	girls (49%)	boys (51%)
Content:	Number	611 (4.3)	611 (4.4)	610 (4.8)
	Geometric shapes & measures	570 (3.6)	574 (3.6)*	567 (4.1)
	Data display	583 (3.2)	589 (3.6)*	578 (4.0)
Cognitive:	Knowing	590 (3.7)	593 (3.8)*	586 (4.1)
	Applying	620 (4.0)	622 (4.5)	619 (4.5)
	Reasoning	578 (3.8)	581 (3.9)*	575 (4.1)
Overall		599 (3.7)	603 (3.8)	596 (4.1)

*significantly higher than the boys

Performance in some specific test items

Table 7a to Table 10 show the performance of Singapore pupils in some items at each of the benchmarks; Advanced, High, Intermediate and Low.

Content domain: Number



Sean used the same rule to get the number in the □ from the number in the △.

What was the rule?

Top five performing countries:
(Percent full credit)

Hong Kong SAR	39 (2.7)
Japan	38 (2.1)
Singapore	36 (2.1)
Armenia	35 (2.9)
Chinese Taipei	33 (2.4)

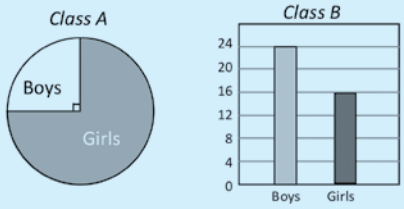
International average: 15 (0.3)

Singapore Primary 4 pupils' performance at the High International Benchmark seems high compared to their counterparts from the other countries. The emphasis put on the thinking skills in the Singapore mathematics curriculum seems to help pupils develop their ability to apply their knowledge in unfamiliar problem situations. For example, while bar graph is introduced in Primary 3, pie chart is introduced only in Primary 6. Yet 63% of the Primary 4 pupils at the High International Benchmark were able to answer the item on Data Display correctly (see Table 7b above). These pupils were able to make the connection between fraction and data display. They probably applied their knowledge of fraction to determine the number of girls in Class A and proceeded to compare with 16 which they could read from the bar graph for Class B.

Table 7b. Items Likely to be Answered Correctly by the Pupils Performing at the Advanced International Benchmark

Content domain: Data Display

Class A and B each have 40 pupils.



There are more girls in Class A than in Class B.
How many more?

(A) 14 (C) 24
(B) 16 (D) 30

Top five performing countries:
(Percent full credit)

Singapore	63 (2.3)
Hong Kong SAR	63 (2.3)
Kazakhstan	51 (3.7)
Chinese Taipei	47 (2.5)
Lithuania	46 (2.1)

International average:
32 (0.4)

Table 8a. Items Likely to be Answered Correctly by Pupils Performing at the High International Benchmark

Content domain: Number

$$\begin{array}{r} 942 \\ -\ 5\square 7 \\ \hline 415 \end{array}$$

Mano did the subtraction problem above for homework but spilled some of his drink on it. One digit could not be read. His answer of 415 was correct. What is the missing digit?

Top five performing countries:
(Percent full credit)

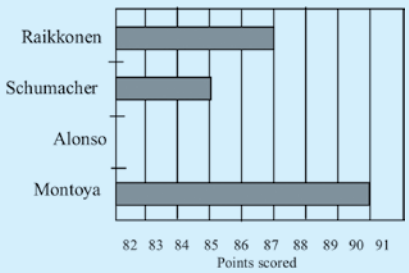
Chinese Taipei	88 (1.6)
Hong Kong SAR	85 (1.9)
Singapore	**85 (1.4)**
Russian Federation	84 (1.8)
Kazakhstan	83 (3.1)

International average:
42 (0.4)

Table 8b. Items Likely to be Answered Correctly by Pupils Performing at the High International Benchmark

Content domain: Data Display

This graph shows the point obtained by 4 drivers in the car racing championship. Montoya is in the first place, Alonso is in the third place. Draw a bar which shows how many points Alonso has scored.



Top five performing countries:
(Percent full credit)

Hong Kong SAR	77(1.9)
Chinese Taipei	72 (1.8)
Japan	71 (2.0)
Singapore	**70 (2.1)**
Kazakhstan	63 (3.7)

International average:
38 (0.4)

For the two items in Table 8a & 8b, the performance of Singapore Primary 4 pupils at the High Benchmark was not far behind the corresponding pupils from the top performing countries, Chinese Taipei and Hong Kong SAR, and was well above the international average. However Singapore pupils did not perform as well on Geometry Shapes and Measures item at the Intermediate International Benchmark (see Table 9).

The results reveal one of the areas of concern among the pupils. Primary 4 pupils in Singapore are taught the concepts of parallel lines and perpendicular lines, including how to construct these lines. They are taught the properties of rectangles as well. Yet the pupils in the study were not able to draw the rectangles with the help of the given grid lines. Many Primary 4 pupils seem not to have mastered the skills to visualize the rectangle and mentally manipulate it in different orientations. For the item on Number, Singapore pupils did very well, their performance ranks second, just behind Chinese Taipei.

Table 9. Items Likely to be Answered Correctly by Pupils Performing at the Intermediate International Benchmark

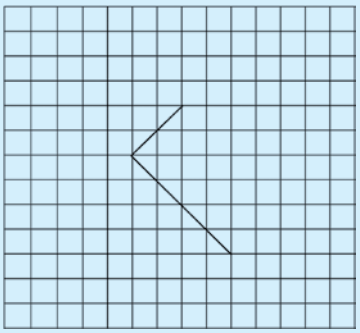
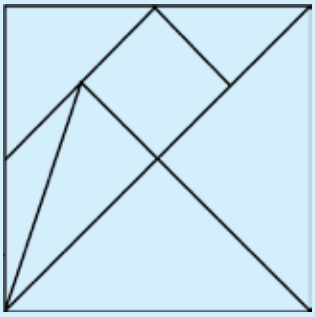
Content domain: Geometric Shapes & Measures Here are two sides of a rectangle. Draw the other two sides. 	**Top five performing countries:** (Percent full credit) Hong Kong SAR 90 (1.4) Japan 78 (1.8) Chinese Taipei 77 (1.9) Russian Federation 75 (2.8) Czech Republic 72 (2.2) **Singapore 69 (2.3) 7th** International average: 54 (0.4)
Content domain: Number Al wanted to find how much his cat weighed. He weighed himself and noted that the scale read 57 kg. He then stepped on the scale holding his cat and found that it read 62 kg. What was the weight of the cat in kilograms?	**Top five performing countries:** (Percent full credit) Chinese Taipei 95 (1.2) Singapore 87 (1.3) Russian Federation 86 (1.8) Hong Kong SAR 86 (1.7) Kazakhstan 85 (2.6) International average: 60 (0.3)

Table 10 shows the performance of pupils from top five performing nations on this item likely to be answered correctly by pupils performing at the Low International Benchmark. The performance of Singapore pupils is only a few percentage points below the top performing nation, Hong Kong SAR.

Table 10. Items Likely to be Answered Correctly by Pupils Performing at the Low International Benchmark

Content domain: Geometric Shapes & Measures The square is cut into 7 pieces. Put an X on each of the 2 triangles that are the same size and shape. 	**Top five performing countries:** (Percent full credit) Hong Kong SAR 91(1.2) Slovenia 91(1.3) Lithuania 89 (1.3) Denmark 88 (1.8) Scotland 88 (1.4) **Singapore 88 (1.4)** International average: 72 (0.3)

Conclusion

The performance of Primary 4 pupils in Singapore in TIMSS 2007 is commendable. Even though many (74%) pupils were found to reach the High International Benchmark, we should follow up with the group of pupils who failed to achieve the Low International Benchmark. This group of pupils, albeit small (2%), needs special assistance so that they can at least attain the most basic standard in mathematics. Singapore, a resource scarce nation, cannot afford to let these pupils fall through the gap.

The TIMSS results also identify the strengths and areas of concern of the Primary 4 pupils in Singapore. Apparently, many pupils (41%) were able to think at the higher order level. They were able to apply their understanding and knowledge of mathematics in various unfamiliar and complex situations. Singapore pupils also performed relatively better in items testing Number and Data Display than in items testing Geometric Shapes and Measures. Perhaps, teachers could take a cue to two from the study. They may wish to reexamine their instructional approaches on the topics in Geometry and Measurement so as to improve pupils' conceptual understanding in these areas.

Definitions of x^n where n is a positive integer, zero or a negative integer

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The teacher could begin, "It is good to reduce the number of variables when introducing new definitions at the secondary school level. Thus, one can initially replace x with 10."

Instead of just stating some numerical examples such as $10^2 = 10 \times 10$ and $10^3 = 10 \times 10 \times 10$, and then summarising this feature algebraically as $10^n = \overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}}$, where n is a positive integer, the teacher could write on the board 10×10, 10×10×10, 10×10×10×10 and then ask the students if they have encountered a way of representing these products in a more succinct manner. Once, students realize what a hassle it would be to write 10^{100} as a product of 100 terms, either by discussion or previous knowledge, it is easy to tell students that the *notation* 10^n is a good way of representing 'a product of n terms of 10'. The teacher can explain this as a case where something is defined for ease of notation.

At this point, the teacher may ask the students, "For what values of n is the notation meaningful?" Since 10^n has now been defined to be 'a product of n terms of 10', it is only logical that n must be a positive integer, thus leading to the teacher writing on the board $10^n = \overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}}$, where n is a positive integer.

The teacher goes on to show, first through some concrete examples and then generally, that this notation is useful in simplifying computations involving powers of 10 as follows:

$$10^m \times 10^n = \overbrace{10 \times 10 \ldots \times 10}^{m \text{ terms}} \times \overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}} = \overbrace{10 \times 10 \ldots \times 10}^{m+n \text{ terms}} = 10^{m+n}$$

Next, the teacher may ask, "For what values of m and n is the theorem $10^m \times 10^n = 10^{m+n}$ valid?" The answer is easy enough (m and n are positive integers) but sets the stage for a similar question in the division case. So the teacher writes Theorem 1 completely as follows:

$10^m \times 10^n = 10^{m+n}$ for positive integers m and n.

What about division? How would we simplify $10^{51} \div 10^{32}$? Again, the teacher goes on to show, first through some concrete examples and then generally, that:

$$10^m \div 10^n = \overbrace{(10 \times 10 \ldots \times 10)}^{m \text{ terms}} \div \overbrace{(10 \times 10 \ldots \times 10)}^{n \text{ terms}} = \frac{\overbrace{10 \times 10 \ldots \times 10}^{m \text{ terms}}}{\overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}}}$$

$$= \frac{\overbrace{10 \times 10 \ldots \times 10}^{m \text{ terms}} \times \overbrace{10 \times 10 \ldots \times 10}^{(m-n) \text{ terms}}}{\overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}}} = \frac{\overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}}}{\overbrace{10 \times 10 \ldots \times 10}^{n \text{ terms}}} \times \overbrace{10 \times 10 \ldots \times 10}^{m-n \text{ terms}} = \overbrace{10 \times 10 \ldots \times 10}^{m-n \text{ terms}} = 10^{m-n}$$

And again, the teacher may ask, "For what values of m and n is the theorem $10^m \div 10^n = 10^{m-n}$ valid?" Since the definition is restricted to positive integral values of the exponent, we must have that m, n and $m-n$ are all positive integers. Thus, m must also be strictly more than n, with the result Theorem 2 as follows: $10^m \div 10^n = 10^{m-n}$ for positive integers m and n such that $m > n$.

How then do we deal with $10^5 \div 10^8$? Can we use the result above to obtain $10^5 \div 10^8 = 10^{5-8} = 10^{-3}$? If so, what does 10-3 mean? Even, before all these, what does 100 mean? (How good would it be if these questions came from the students themselves!)

The teacher may then expand on what we implied earlier in this paper: In mathematics, we like to extend known definitions to cover as much area as possible, partly because this will mean that we do not need totally new definitions or theorems – a common principle of parsimony in science. Now, to extend the definitions of powers of 10 to include 'negative' powers, we need the extended definition to be consistent with the two theorems (formulae) we obtained earlier, i.e., $10^m \times 10^n = 10^{m+n}$ and $10^m \div 10^n = 10^{m-n}$.

The teacher now works with the students to obtain well-definitions of 10^n where n is 0 or a negative integer. The first definition is suggested by the statement $1 = 10^n \div 10^n = 10^{n-n} = 10^0$. Now, 10^0 has not been defined. But how nice it would be if $10^0 = 1$. So, let's define 10^0 to be 1! Then our Theorem 2 can now be extended to include the case when $m = n$ and also when m or n is 0. Now, the teacher may define $10^0 = 1$; and extend Theorem 2 to $10^m \div 10^n = 10^{m-n}$ for nonnegative integers m and n such that $m \geq n$.

The next definition is then suggested by the statement $\frac{1}{10^n} = 1 \div 10^n = 10^0 \div 10^n = 10^{0-n} = 10^{-n}$. Since n is a positive integer, 10^{-n} has not been defined. But how nice it would be if $10^{-n} = \frac{1}{10^n}$. So, let's define 10^{-n} to be $\frac{1}{10^n}$. Then our Theorem 2 can now be even further extended to include the case when m or n are negative integers. So the teacher may define $10^{-n} = \frac{1}{10^n}$, where n is an integer; and further extend Theorem 2 to $10^m \div 10^n = 10^{m-n}$ for integers m and n.

Thus, we have two (new) definitions that arise from an extension of a previously defined notation (10^n) to include more values of n. Note here how different the motivation for the definitions is when compared with the first 'intuitive' definition of 10^n as 'the product of n terms of 10'. In fact, in our new definitions, the motivation is everything – the final definitions are stated without explanation of what they could represent, as really they cannot represent anything concrete! It is often overlooked but we think that it is good for students to be directed to this nuance in notation. Otherwise, some students would think that 10^{-3} is 'the product of -3 terms of 10' and then have the impression that Mathematics is ridiculous, while some others may, because of this impossibility, go further to 'forget' that the definition makes a lot of sense for positive integral values of n. In 'forgetting', the latter group of students loses the logical foundation for understanding the algebra of indices, i.e., Theorems 1 and 2.

Ask Dr Maths Teaching

1. A number, when divided by 3, 5 or 7 leaves a remainder of 2. What is the least positive number? Is the answer 2 ? or 107 ?

Response: The answer is obviously meant to be 105; 3 X 5 X 7 = 105 or =2. On another note: If question for the primary level, then it should be "What is the number? If question for the secondary level, then it should be "Find two such numbers?

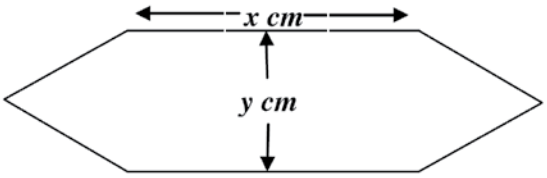
2. Must "length" always be the longer side of a rectangle?

Response: Length, breadth, and height are concepts that are associated with 3-dimensional objects. From the 3-dimensional perspective, one could quickly appreciate that these terms, when applied to an object, is dependent on how the object is placed in relation to the external environment, such as the observer. For a given cupboard, for example, there would be unlikely an issue for an observer in identifying the height.

However, when such concepts are applied to 2-dimentionsl objects placed in a 3-dimensional environment, issues often arose out of difficulty in relating the 2-dimnesional object to the 3-dimensional environment, such as the observer.

In such a case, convention may help in facilitating the communication of the related ideas to the young and mathematically inexperienced learners. While, some may adopt the convention that the "length" of a rectangle is the longer side of the shape, others may determine the "length" based on the orientation of the rectangle with respect to the observer. Whatever the convention is adopted, it would help to establish some form of consistency within a given system so as to minimize confusion.

3.



A piece of wire, 26 cm, is bent form the shape as shown in the diagram. This shape encloses a plane region, of area A cm2, consisting of a rectangle, x cm by , with an equilateral triangle on each side of length x cm. Show that area A has a maximum value when x = 4 + $\sqrt{3}$

Response:

Method I: Solving

a) Find the perimeter, b) find the area c) differentiate $\frac{dA}{dx}$ *d) differentiate* $\frac{d^2A}{dx^2}$

Method II: Guess and Check

a)) differentiate $\frac{dA}{dx}$ *b) substitute* $x = 4 + \sqrt{3}$ to show $\frac{dA}{dx} = 0$ *c) differentiate* $\frac{d^2A}{dx^2}$

It is alright to use Method II, however, students ***must*** learn the algebraic method.

Upcoming AME Activity 2010

No	Talk/Workshop	Speaker	Date & Time	Venue	Joint Organiser	Audience
1	Assessment in Secondary Mathematics Classroom for Teachers	Dr Hang Kim Woo and Prof Magdalena Mok	6 Sep 2010 Monday 9am – 12 pm	SP Auditorium	SP	Secondary teachers

Contributions Invited

Snail mail submission – You may send your contributions via mail to the following:

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c/o Mathematics & Mathematics Education
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- mathsbuzz.ame@gmail.com – For sharing of teaching ideas or research findings
- askdrmathsteaching.sg@gmail.com – For discussion and clarification of issues related to teaching and learning of mathematics.