Learning mathematical induction through experiencing authentic problem solving

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The anomaly of MI in the JC syllabus

- Not within any field of mathematics but rather it is a technique.
- This 'abnormality' would perhaps suggest a different way of teaching.
- This workshop sets the pedagogy of the technique of mathematical induction within its natural environment of problem solving where
  - a problem is explored,
  - a conjecture is made,
  - and an attempt to prove the conjecture using some techniques is made on the basis of the earlier exploration.

ex·pe·ri·ence [ik-speer-ee-uh ns] noun

- a particular instance of personally encountering or undergoing something: My encounter with the bear in the woods was a frightening experience.
- the process or fact of personally observing, encountering, or undergoing something: business experience.
- the observing, encountering, or undergoing of things generally as they occur in the course of time: to learn from experience; the range of human experience.
- knowledge or practical wisdom gained from what one has observed, encountered, or undergone: a man of experience.
- *Philosophy*. the totality of the cognitions given by perception; all that is perceived, understood, and remembered.

Learning Experience

- Must be personal

- Must be memorable

- Must be understood ∞

Problem 1

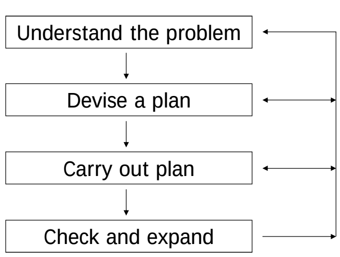
Find and prove some number patterns from the pyramid of consecutive odd numbers below where the *n*-th row contains *n* odd numbers.

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
…

Problem 1

- By exploring, students will make their own conjectures in an authentic manner.
- By seeing where the conjecture came from, ideas of how to prove it (may be by MI) are formed.

Polya's Problem Solving Strategy



Problem 2

Make a conjecture for a formula in closed form for the series

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n\times(n+1)}.$$

Use mathematical induction to prove your conjecture.

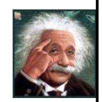
Problem 2

- Let students 'see' your problem solving framework by writing down the 'template' on a portion of the whiteboard.
- The overarching strategy will then be more clear and students can be scaffolded.
- Use UP, DP, CP, C/E.

Smart as Einstein

Here is a proof by mathematical induction that you are as smart as Einstein!

<u>Theorem</u>: I am as smart as Einstein.



<u>Proof</u>: Let P(n) be the statement: All n persons in a group containing n persons have the same IQ.

P(1) is obviously true.

Suppose P(k) is true for some positive integer k.

Take a group X of k+1 persons.

Remove a person A from the group, leaving behind a group X' of k persons. By the induction hypothesis, all the k persons in X' have the same IQ.

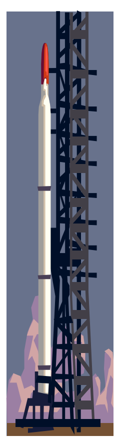
Remove a person B from X' and put back the first removed person A, thus forming a set X'' of k persons. By the induction hypothesis, all the k persons in X'' have the same IQ.

Finally, put back the second removed person B to re-form the group X with k+1 persons, all of whom have the same IQ. Thus if P(k) is true, then P(k+1) is also true.

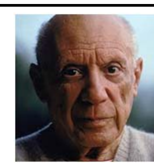
Since P(1) is true and P(k) $\Rightarrow$ P(k+1), by mathematical induction, P(n) is true for all positive integers n, i.e. all n persons in a group containing n persons have the same IQ. Finally, put me in a group with Einstein, and I will be as smart as he is!

Smart as Einstein

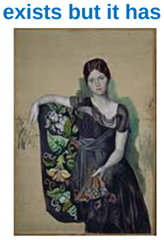
- "Thinking out of the box" is often misused when people do so without first "mastering the box"!
- The discipline of mathematics requires working within the s (or conditions).



Smart as Picasso



- Master the box **~ Inspiration exists but it has to find you working.**



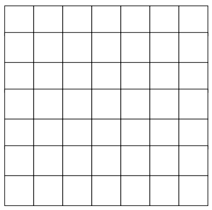
Portrait of wife Olga, 1917

Portrait of self, 1907

Portrait of mother, 1896

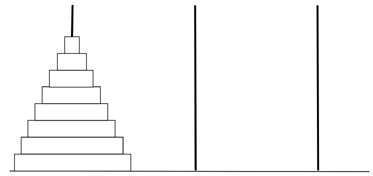
H3 Problems

Find the number of squares in an n by n grid consisting of 1 by 1 square cells. (The figure shows a 7 by 7 grid.)



H3 Problems

A tower of n circular discs of different diameters is stacked on one of the three vertical pegs as shown below.



The task is to transfer the entire tower to another peg by a number of moves subject to the following rules:

(i) each move carries exactly one disc; and

(ii) no disc can be placed on a smaller one.

What is the minimum number of moves required to accomplish the task?

Final Problem

Let $a^{(n)}$ denote $a^{a^{\cdots^{a}}}$, a tower of n a's.

Find the smallest integer m such that

$$9^{(n)} < 3^{(m)}.$$