

Learning experiences in mathematics through problem solving approach

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Some background information

Current objectives of mathematics for elementary school in Japan

Through mathematical activities students will

- (1) acquire basic and fundamental knowledge and skills about numbers, quantities and geometric figures,
- (2) cultivate their ability to consider phenomena from their daily lives with foresight to generate and organize logical thinking steps to follow through, and to represent those phenomena,
- (3) recognize the joy of mathematical activities and the merit of mathematical manipulation, and
- (4) foster a disposition to willingly make use of mathematics in daily life and studies.

(Ministry of Education, Culture, Sports, Science and Technology - Japan, 2008)

Japanese lesson pattern

(Stigler & Hiebert, 1999)

- Reviewing the previous lesson
- Presenting the problem for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the major points

Structured Problem Solving

“Number and Calculation” content domain

- In this domain, the objectives are to understand the meaning and representation of whole numbers, decimal numbers, and fractions, and to have a rich sense of numbers.
- Also, students will understand the meaning of calculations with whole numbers, decimal numbers, and fractions and learn to think about ways to calculate with them, acquire calculation skills and the ability to apply them.
- Additionally, it is also an important object in this domain for students to recognize the joys of mathematical activity and the merits of mathematical manipulation by acquiring the ability to think mathematically. (Ministry of Education, 2008)

Main content in each grade

Grade	Numbers	Calculations
Grade 1	<ul style="list-style-type: none"> • 2-digit numbers • Simple 3-digit numbers 	<ul style="list-style-type: none"> • Addition of 1-digit numbers and its inverse, subtraction • Addition/subtraction of simple 2-digit numbers.
Grade 2	<ul style="list-style-type: none"> • 4-digit numbers (numbers up to 1 <i>man</i> [10000]) • Base-ten notation system • Simple fractions 	<ul style="list-style-type: none"> • Addition and its inverse, subtraction of 2-digit numbers • Addition/subtraction of simple 3-digit numbers. • Multiplication table (<i>ku-ku</i>) • Multiplying simple 2-digit number by 1-digit number
Grade 3	<ul style="list-style-type: none"> • Units of <i>man</i> (numbers up to 1 <i>oku</i> [100,000,000]) • Decimal numbers (tenths) • Fractions 	<ul style="list-style-type: none"> • Addition/Subtraction of whole numbers (3-digit, 4-digit numbers) • Multiplication of whole numbers (2-digit numbers, 3-digit numbers) • Division of whole numbers (divisors and remainders are 1-digit numbers) • Simple division (remainders are 1-digit numbers and divisors are 2-digit numbers) • (mental calculation) • Calculation by using an abacus • Addition/subtraction of simple decimal numbers and fractions.

<p>Grade 4</p>	<ul style="list-style-type: none"> • Units of <i>oku</i> [100 million] and <i>chou</i> [1 trillion] • Approximate numbers • Decimal numbers • Fractions (proper fraction, improper fraction, mixed fraction) 	<ul style="list-style-type: none"> • Division of whole numbers (Divisors are 1-digit numbers or 2-digit numbers, and dividends are 2-digit or 3-digit numbers) • Estimating results of calculations (simple mental calculation) • Acquiring calculation skills of whole numbers • Calculation by using an abacus • Addition/subtraction of decimal numbers • Multiplication and division of decimal numbers in which multipliers and divisors are whole numbers • Addition/subtraction of fractions with like denominators.
<p>Grade 5</p>	<ul style="list-style-type: none"> • Even numbers, odd numbers • Divisors, multiples (greatest common divisor, least common multiple) • <i>Prime numbers</i> 	<ul style="list-style-type: none"> • Multiplication and division of decimal numbers in which multipliers and divisors are decimal numbers • Addition/subtraction of fractions with unlike denominators • Multiplication and division of fractions in which multipliers and divisors are whole numbers
<p>Grade 6</p>	<ul style="list-style-type: none"> • <i>Reciprocal</i> 	<ul style="list-style-type: none"> • Multiplication and division of fractions in which multipliers and divisors are fractions • Acquiring calculation skills of decimal numbers and fractions

Counting marbles

- Grade 2 -

Getting a feel of problem solving
approach

Important points of teaching

- Using open-ended problem
- Eliciting and organizing different ways of thinking
- Proposing and making sense of mathematically-significant focus
- Using symbolic representations (e.g., words, mathematical expression, children's self-developed symbols)

How many blocks in total?
- Grade 3 -

Organizing children's multiple
solutions

Polishing up children's solutions (Koto and his colleagues, 1992, 2010)

- Four principles of organizing discussion
 - Examination of the **validity** of each solution
 - Examination of the **relationship** among solutions
 - Comparison of different solutions from the point of view of **relevance, generality, or utility**
 - Looking back the solutions from **self-evaluation** by each child

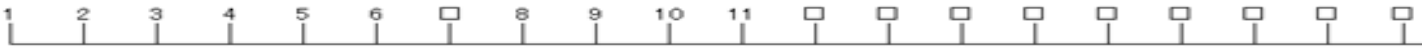

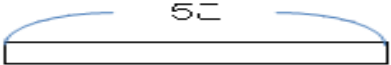
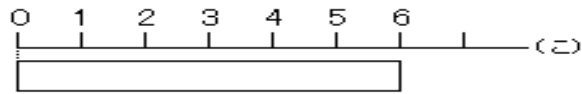
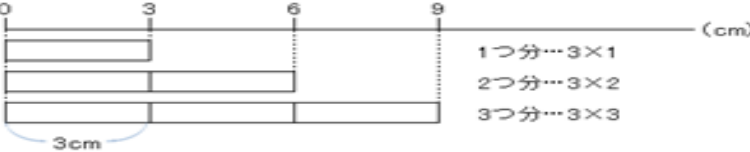
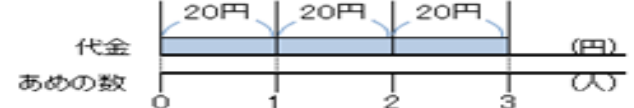

Polishing up children's solutions

- Four types of multiple solutions
 - *Independent type*: Each solution method has its own validity as a mathematical idea, but the relationship among the solutions is weak or non-existent. Each solution method is considered equally valid to the other methods.
 - *Ordered type*: Solution methods can be ordered from the perspective of mathematics or the objective of the lesson, such as, mathematical relevance, generality, or utility. For example, calculation method A may be judged better than calculation method B because A can apply to other numbers, while B can only apply to specific cases of numbers.
 - *Unified type*: By focusing on similarities, different solution methods can be unified under a common mathematical idea or principle.
 - *Connected type*: By focusing on the relationships among the solution methods, they can be synthesized into several interconnected groups.

Paint a board - Grade 6 -

Thinking the ways of division with
fractions

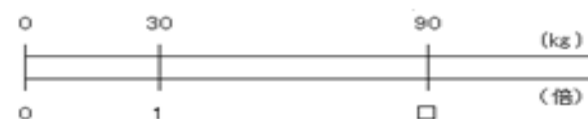
Use of number line across grades (Iwasawa & Hino, 2011)

子どもの中に積み上げられていく内容	扱う学年の目安
<p>・数の線及び数の系列を知る。</p> 	<p>↑ 1年 ↑ 2年 ↑ 3年 ↑ 4年 ↑ 5年</p>
<p>・ものの個数を正しく数える。</p> <p>① 具体物からブロック図で数量を表す。</p>	
	<p>↑ 2年</p>
<p>② テープ図に表し、テープの左端から右端までいくつという見方を知る。</p> 	
<p>③ テープ図に表した量を数直線上の点としてみる見方を知る。</p> 	
<p>④ 乗法の指導で、テープ1つ分、2つ分、...をそれぞれ分けてかき表し、1本の数直線に対応する点をかき入れる。</p> 	<p>↑ 2年</p>
<p>⑤ テープを1つ分、2つ分、...と分けずに1本に表し、どこまでが1つ分か、どこまでが2つ分かを、もう1本の数直線に表す。</p> 	<p>↑ 3年</p>
<p>⑥ 2本の数直線で数量の関係を表す。テープは徐々に取り除いていく。</p> 	

⑦ 問題文より、異種2量(ex.枚数と金額)を見つけ出し、1枚20円なら、5枚では□円という対応関係をつかむ。(答えが分かった後、□に数値をかき入れる)

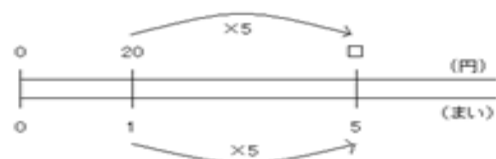


⑧ 問題文より異種2量(ex.重さと倍)を見つけ出し、30kgをもとにすると、90kgは30kgの□倍という対応関係をつかむ。(答えが分かった後、□に数値をかき入れる)



⑨ 1枚20円なら、5枚では100円という結果より、枚数が2倍、3倍になると金額も2倍、3倍になるという関係を見いだす。

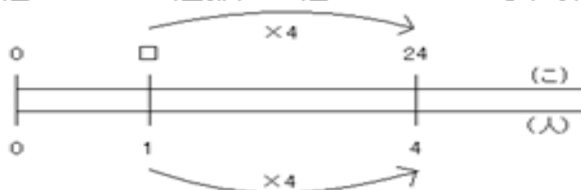
$$20 \times 5 = \square$$



⑩ 人数を表す数直線が4倍になると個数も4倍になるという関係を見いだす

$$\square \times 4 = 24$$

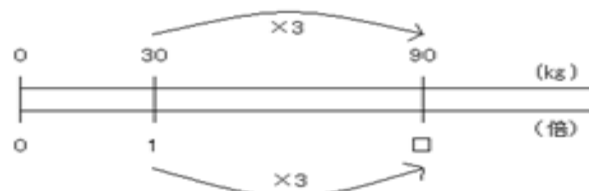
$$\square = 24 \div 4$$



⑪ 倍を表す数直線が□倍になると重さも□倍になるという関係を見いだす

$$30 \times \square = 90$$

$$\square = 90 \div 30$$



⑫ 演算決定の根拠を説明する際、数直線が有効であることに気づき、進んで用いる。

⑬ 数直線上の点の配置より、答えを求める前に結果の見通しをたてるのがたやすいことに気づいたり、求めた答えの確かめに有効であることがわかり、進んで活用する。

↓

4年

5年

6年

↓

↓

Analysis of solutions by other people

- (Given a mathematical expression by A), let's explain how A solved the problem.
- (Given solutions by B and by C), explain the idea of C to make the solution simpler than B.
- (Given an idea by D), do you think what D says is correct or not? Why?
- (Given a solution by E), E's solution is not correct. Let's find the mistake and correct it.

Final remark

The Heart of problem solving approach (Sugiyama & Ito, 1990)

- To have children experience problem solving means more than letting them to solve the problem at hand. It means to have them learn *to think, how to overcome difficulty*, and experience the desire, effort, struggle, joy, and so on in the process of solving the problem. In order to achieve this, it is important for teachers:
 - ✓ To have children experience the confidence and joy of being able to find a provisional solution to the problem by fully drawing on their own knowledge, and
 - ✓ To have children appreciate more fully elaborated solution methods, and experience the joy of continuously seeking for better ways. (p. 155; emphasis original)

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Thank you very much