## Fraction as part of a shape

## Divide the shape into quarters.

Hansen, A. (2011). Children's errors in mathematics: Understanding common misconceptions in primary school. Exeter: Learning Matters.

## Fraction as part of a set of objects



Hansen, A. (2011). Children's errors in mathematics: Understanding common misconceptions in primary school. Exeter: Learning Matters.

## Fractions as numbers on a number line



Hansen, A. (2011). Children's errors in mathematics: Understanding common misconceptions in primary school. Exeter: Learning Matters.

## Naming fractions



Hansen, A. (2011). Children's errors in mathematics: Understanding common misconceptions in primary school. Exeter: Learning Matters.

## Writing fractions



Hansen, A. (2011). Children's errors in mathematics: Understanding common misconceptions in primary school. Exeter: Learning Matters.

## History of Fractions

- Did you know that fractions as we use them today didn't exist in Europe until the 17th century?
- Who first used fractions?
- Were they always written in the same way?
- How did fractions reach us here?


## FRACTION

- comes from the Latin "fractio" which means to break.

From as early as 1800 BC, the Egyptians were writing fractions. Their number system was a base 10 idea (a little bit like ours now) so they had separate symbols for 1 , 10, $100,1000,10000,100000$ and 1000000.

## History of Fractions

| \# | fil | 9 | $\overbrace{4}$ | 1 | 44 | E |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 100 | 1000 | 10000 | 100000 | $10^{6}$ |
| Egyptian numeral hieroglyphs |  |  |  |  |  |  |

## Example:



Can you do this 1582 ?


## History of Fractions

The Egyptians wrote all their fractions using what we call unit fractions. A unit fraction has 1 as its numerator (top number). They put a mouth picture (which meant part) above a number to make it into a unit fraction. For example:


## History of Fractions

They expressed other fractions as the sum of unit fractions, but they weren't allowed to repeat a unit fraction in this addition. For example this is fine:

$$
\frac{3}{4}=\frac{1}{2}+\frac{1}{4}
$$

But this is not:

$$
\frac{2}{7}=\frac{1}{7}+\frac{1}{7}
$$

The huge disadvantage of the Egyptian system for representing fractions is that it is very difficult to do any calculations. To try to overcome this, the Egyptians made lots of tables so they could look up answers to problems.

## History of Fractions

## Ancient Rome

$\frac{1}{12}$ was called uncia
$\frac{6}{12}$ was called semis
$\frac{1}{24}$ was called semuncia
$\frac{1}{144}$ was called scripulum

As with the Egyptian system, the words made it very difficult to do calculations.

## History of Fractions

## Babylonians



Here are the numbers from 1 to 20 .

Can you see the symbol for 1 ?
What about the symbol for 10 ?

How would you write 47

## History of Fractions

From the table above, you can see that the two numbers are 12 and 15 . Now, this is where it becomes confusing. This could mean several different things:

## 《

| X60 | Units | Sixtieths | Number |
| :---: | :---: | :---: | :---: |
|  | 15 | 15 | $12+\frac{15}{60}=12 \frac{15}{60}$ |
| 12 | 15 |  | $720+15$ |

## History of Fractions

The success of their way of writing fractions is due to the number system they created which has three main ideas:
i. Each figure has a symbol which isn't like the value it represents
ii. The value of the figure depends on the position of it within the entire number
iii. A zero is needed to mean nothing and also to fill the place of units that are missing

History of Fractions

15. Cent.
16. Cent. (Dürer)

## History of Fractions

## Indian fractions



## History of Fractions

## Arabian

$$
\frac{3}{4}
$$

## What are fractions?

## Mathematically:

- part of the set of rational numbers that can be expressed in the form $\frac{a}{b}$ where $\boldsymbol{a}$ is any whole number and $\boldsymbol{b}$ is any non-zero whole number


## Symbolically:

- expressed as common fractions ( $\frac{1}{2}$ and $\frac{2}{3}$ ), as decimal fractions ( 0.5 and $0.6666 \ldots$..) and as percents ( $50 \%$ and $\left.66 \frac{2}{3} \%\right)$.


## Notations

Part of a complete unit or 'whole'




## Equivalent fractions



## Operations on Fractions

## Addition - Like Fractions



$$
\frac{1}{8}+\frac{2}{8}=\frac{3}{8}
$$

## Operations on Fractions

Addition - Related Fractions


$$
\frac{1}{2}+\frac{x_{4}}{x}=\frac{3}{8}+\frac{3}{8}=\frac{7}{8}
$$

## Operations on Fractions

Addition - Unrelated Fractions


$$
\frac{2}{3}{ }_{\times 4}+\frac{1}{4}{ }_{\times 3}^{=}=\frac{8}{12}+\frac{3}{12}=\frac{11}{12}
$$

## Operations on Fractions



## Operations on Fractions

## Multiplication - Fraction as part of a whole



$$
\begin{aligned}
\frac{1}{3} \times 2 & =1 \text { thirds } \times 2 \\
& =(1 \times 2) \text { thirds } \\
& =\frac{2}{3}
\end{aligned}
$$

## Operations on Fractions

## Division - Sharing

Method 1


Edge, D., and Yeap, B. H. (2012). Teaching of fractions: From research to practice, Singapore: Marshall Cevendish

## Operations on Fractions

## Division - Sharing

## Method 2

$$
\begin{aligned}
& \frac{2}{3} \div 2 \\
= & \frac{1}{2} \times \frac{2}{3} \\
= & \frac{1}{3}
\end{aligned}
$$



Edge, D., and Yeap, B. H. (2012). Teaching of fractions: From research to practice, Singapore: Marshall Cevendish

## Contact

## Angeline Nadia Lim

Nadia lim@moe.gov.sg

Yueh Yuan<br>Lee-goh kok hong@moe.gov.sg

