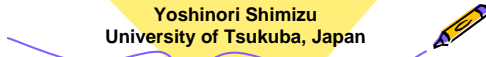


**Assessing Secondary School Students' Mathematical Literacy: A Focus on Mathematics as the Language of Science**

Yoshinori Shimizu  
University of Tsukuba, Japan

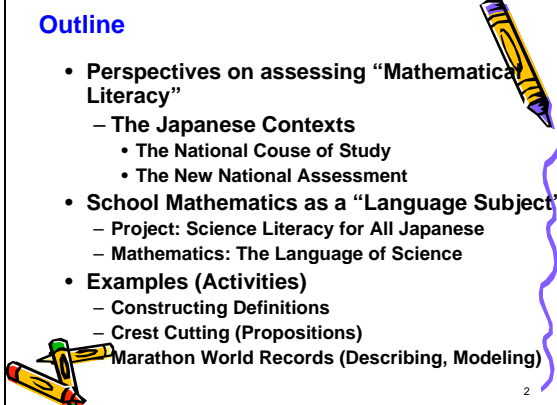
AME-SMS Conference 2014  
NUS High School of Mathematics & Science  
June 5th, 2014



1

**Outline**

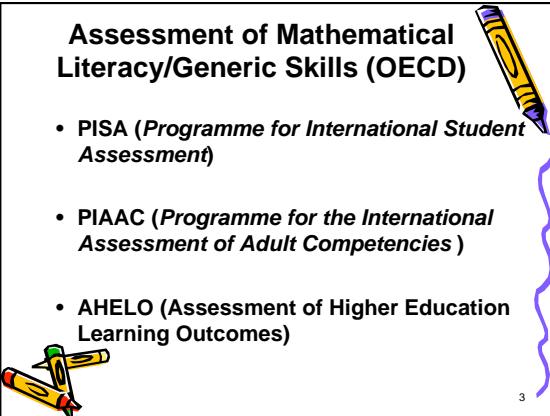
- Perspectives on assessing “Mathematical Literacy”
  - The Japanese Contexts
    - The National Course of Study
    - The New National Assessment
- School Mathematics as a “Language Subject”
  - Project: Science Literacy for All Japanese
  - Mathematics: The Language of Science
- Examples (Activities)
  - Constructing Definitions
  - Crest Cutting (Propositions)
  - Marathon World Records (Describing, Modeling)



2

**Assessment of Mathematical Literacy/Generic Skills (OECD)**

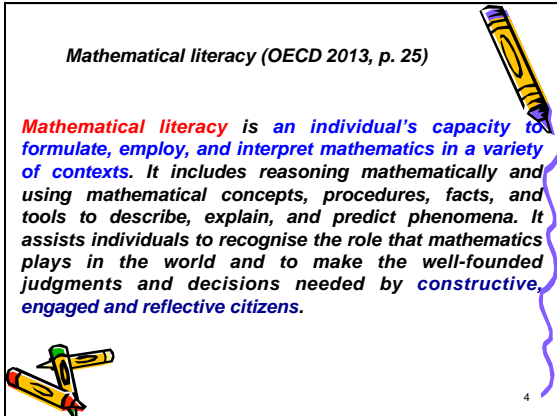
- PISA (*Programme for International Student Assessment*)
- PIAAC (*Programme for the International Assessment of Adult Competencies*)
- AHELO (*Assessment of Higher Education Learning Outcomes*)



3

*Mathematical literacy (OECD 2013, p. 25)*

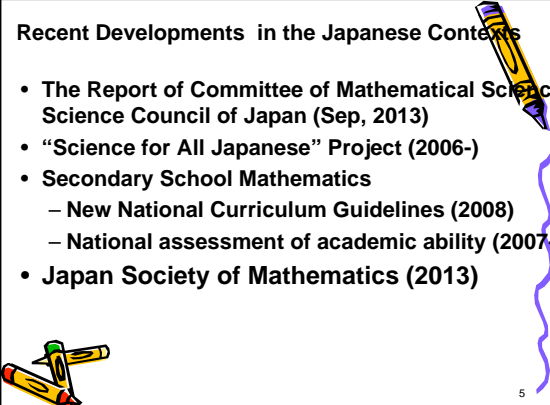
*Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.*



4

**Recent Developments in the Japanese Contexts**

- The Report of Committee of Mathematical Science Science Council of Japan (Sep, 2013)
- “Science for All Japanese” Project (2006-)
- Secondary School Mathematics
  - New National Curriculum Guidelines (2008)
  - National assessment of academic ability (2007-)
- Japan Society of Mathematics (2013)

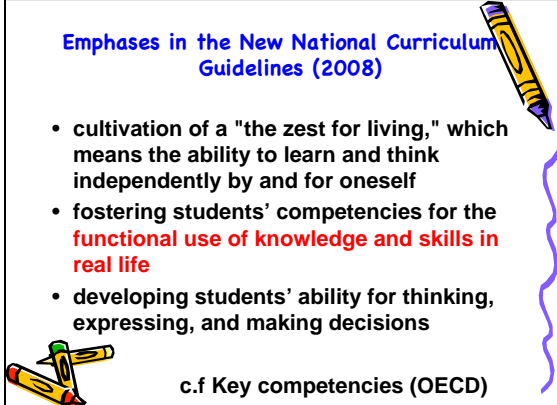


5

**Emphases in the New National Curriculum Guidelines (2008)**

- cultivation of a "the zest for living," which means the ability to learn and think independently by and for oneself
- fostering students' competencies for the **functional use of knowledge and skills in real life**
- developing students' ability for thinking, expressing, and making decisions

c.f Key competencies (OECD)



6

### Recent Large-Scale National Assessments in Japan

National Assessment of Academic Ability and Learning Environments	Assessment of Implementation of National Curriculum	Assessment of Specific Aspects in Students' Learning
To check and improve educational policy. To establish the PDCA (Plan-Do-Check-Action) cycle in educational policy. To improve classroom practices in each school.	To monitor the implementation of new national course of study.  To improve classroom practices in each school.	To investigate specific issues in teaching and learning which are not explored by the Assessment of Implementation of National Curriculum.
Grade 6 and 9	Grade 5 through 9	Grade 4 through 10
Complete (2007-2009, 2013-), Sampling (2010-2012)	Sampling	Sampling
Japanese and Mathematics, Science (every three years)	Japanese, Mathematics, Social Studies, Science, and English	All the school subjects, as well as "logical thinking"

### Two Strands in National Assessment of Academic Ability

- Knowledge and skills needed for further learning in schools and for applying in the real life situations (**Tasks A: "Knowledge"**)
- Competencies for applying knowledge and skills to the situations in real life, and for planning, implementing, reflecting, and improving the plan to solve problems. (**TASKS B: "Functional Use"**)

### The Processes focused in Task B

**α1: Mathematization**  
**α2: Use of information**  
**α3: Interpreting the mathematical results**

**β1: Planning for solving a problem to implementation the plan**  
**β2: Evaluating and improving the results**

**γ: Connecting and Integrating**

### NIER (2010) Task B (2008 Grade 9) : Mt. Fuji

Rina and her friends are planning to visit the Five Lakes of Mt. Fuji and climb up to the sixth stage of the mountain this August.

**Question 1.** You will take photos at two lakes among the five. How many different choices of two lakes do you have, if we ignore the order of the visits?

*A Map of Mt. Fuji Climbing and the Five Lakes of Mt. Fuji*

54.7%

### Task B (2008 Grade 9): Mt. Fuji (Continued)

**Question 2.** Rina and Ken-ichi are talking about the temperature of the sixth stage of Mt. Fuji.

**Rina:** *I have tried to investigate the temperature of the sixth stage, but I couldn't find it because there is no observatory on the stage.*

**Ken-ichi:** *It is known that the temperature falls at a constant rate as one climbs higher until an altitude of 10,000 meters.*

**Rina:** *We may use the fact to find the temperature of the sixth stage.*

If we hypothesize that the temperature falls at a constant rate as one climbs higher until an altitude of 10,000 meters, what is the relationship that holds anytime between altitude  $x$  meters and temperature  $y$  °C? Choose the correct one from the followings.

1. $y$ is proportional to $x$ .	30.9%
2. $y$ is an inverse proportion to $x$ .	26.1%
3. $y$ is a linear function of $x$ .	25.0%
4. Sum of $x$ and $y$ is a constant.	6.9%
5. Difference of $x$ and $y$ is a constant.	8.7%

### Task B (2008 Grade 9) : Mt. Fuji (Continued)

**Question 3.** Rina investigated the mean temperature in August on the top of the mountain and around Mt. Fuji. She completed table below and drew a graph, measuring altitude as  $x$  meters and temperature  $y$  °C.

Observation Points	Altitude (m)	Mean Temp (°C)	Observation Points	Altitude (m)	Mean Temp (°C)
A	273	27.7	D	860	23.3
B	394	26.7	E	992	21.7
C	552	24.9	F	3775	6.4

*Graph and table of the relation between Altitude and Temperature*


Rina understood that the temperature falls at a constant rate as one climbs higher. Then, she tried to estimate the temperature of the sixth stage of Mt. Fuji using data at the points D and F in the given table and graph. Explain your method of estimating the temperature at the sixth stage (2,500m). You do not need to actually find the temperature.

13.3%

## Emphasis on Explaining Mathematically

The open-construction tasks require the students to explain;


- The observed facts in a situation
- **Approaches and methods to solve problems**
- Reasons for the facts and properties



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## Explaining a method


- (a) Draw a line which go through D & F
- + (b) read the value  $y$  when  $x=2,500$
- (c) Find a expression of linear function
- + (d) calculate value  $y$  by substituting  $x=2,500$
- (e) Find a rate of change by using table
- + (f) Find the temperature at height of 2,500



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## Beyond the Existing Assessments



- More than “generic skills” for knowledge based society?
- Discipline-specific skills in mathematics in general, and in mathematical modeling, in particular?
- Entirely “intra-mathematical” items to be developed, as opposed to PISA items?




15

## Mathematics is the alphabet with which God has written the Universe.

Galileo Galilei (1564-1642)






16



Science Literacy for all Japanese?


<http://www.science-for-all.jp/>

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## Science Literacy for All Japanese

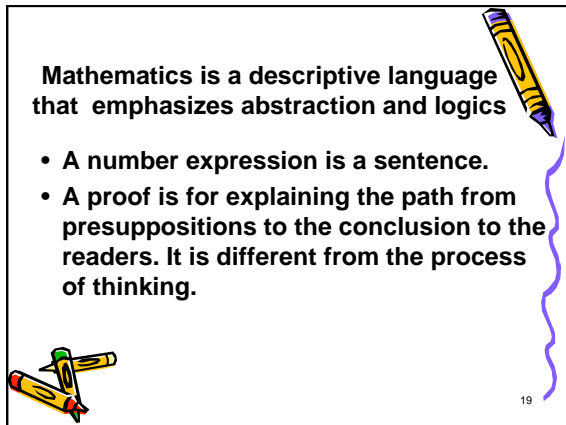
- Reshaping public image of science in society
- Reforming science education from a “literacy” perspective
- A group of roughly 150 members, including scientists, engineers, educators, science museum personnel, members of the media, and various NPO's worked together



18

Mathematics is a descriptive language that emphasizes abstraction and logics

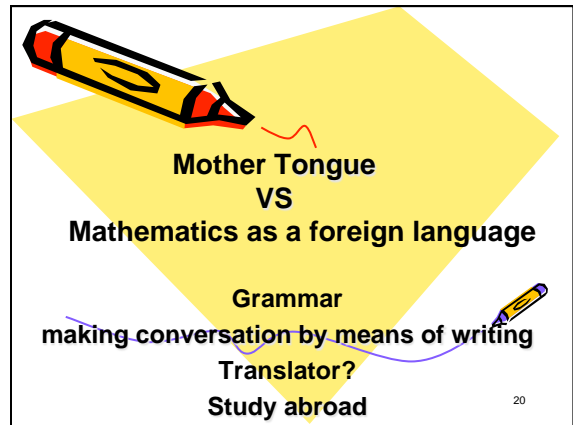
- A number expression is a sentence.
- A proof is for explaining the path from presuppositions to the conclusion to the readers. It is different from the process of thinking.



19

Mother Tongue  
VS  
Mathematics as a foreign language

Grammar  
making conversation by means of writing  
Translator?  
Study abroad

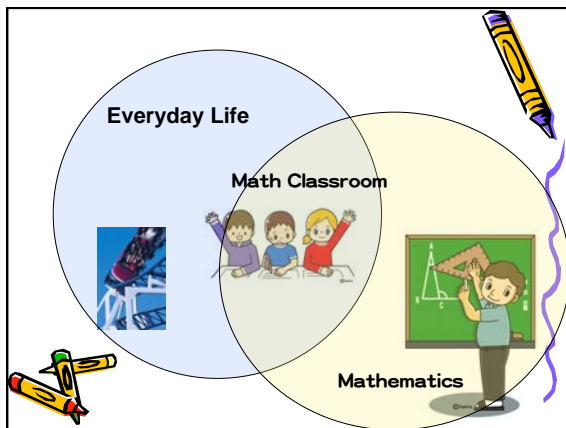


20

Everyday Life

Math Classroom

Mathematics



21

Use of Language

Everyday Life

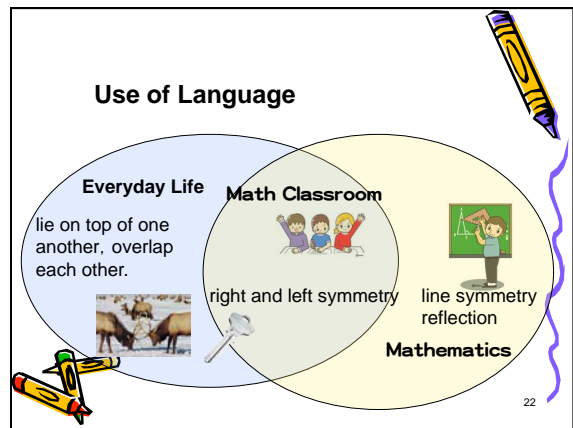
Math Classroom

Mathematics

lie on top of one another, overlap each other.

right and left symmetry

line symmetry reflection



22

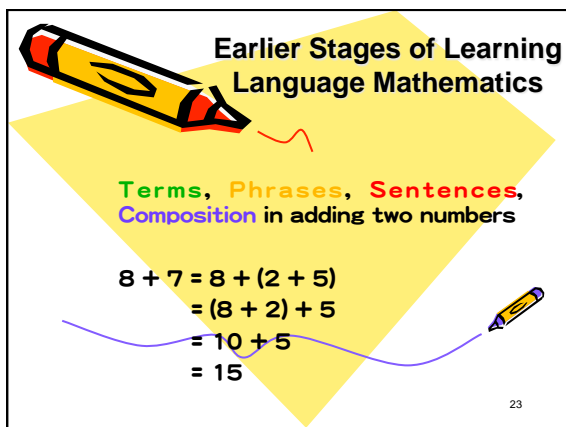
Earlier Stages of Learning Language Mathematics

Terms, Phrases, Sentences, Composition in adding two numbers

$$8 + 7 = 8 + (2 + 5)$$

$$= (8 + 2) + 5$$

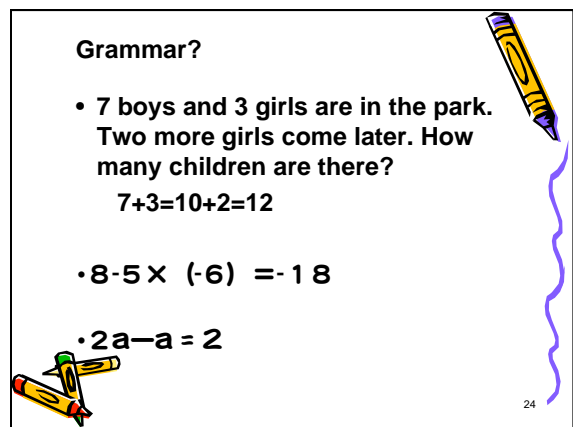
$$= 10 + 5$$

$$= 15$$


23

Grammar?

- 7 boys and 3 girls are in the park. Two more girls come later. How many children are there?  
 $7+3=10+2=12$
- $8-5 \times (-6) = -18$
- $2a-a = a$



24

## Mathematics is a descriptive language that emphasizes abstraction and logics

- **Term** "a round shape" or "a circle"  
Concept, Definition, Example
- **Sentence**  
Proposition, the subject and the predicate
- **Composition** Thinking, Expressing  
Explanation, Computation, Proof

25

## NIER (2010) (Grade 9)

When we express a two-digit number by using  $x$  for the tenth digit and  $y$  for the unit, which of the following expressions is correct one? Choose it.

1.  $xy$
2.  $x + y$
3.  $10xy$
4.  $10x + y$

Category	Choice	Response rate (%)
1	$xy$	11.5
2	$x + y$	11.0
3	$10xy$	8.9
4	$10x + y$	67.7
9	Others	0.2
0	No Answer	0.8

26

## Japan Society of Mathematics (2013)

- Proof of a proposition:  
*An addition of an even number to an odd number makes an odd number.*

Percentage of collect responses; 34.0%

- "Because,  $2+1=3$ , and  $4+1=5$ "
- "An odd number cannot be an even number without adding another odd."

University students, N=5946  
(from 48 universities and colleges)

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## Expression and Interpretation of Language Mathematics

$$n + (n+1) + (n+2) = 3n+3 = 3(n+1)$$

The sum of three consecutive natural numbers is  
a multiple of 3

Three times of the central number

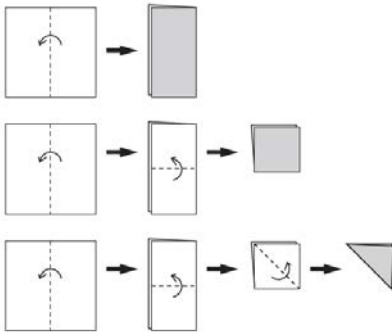
$$(n-1) + n + (n+1) = 3n$$

"Cancellation" of "Mean"

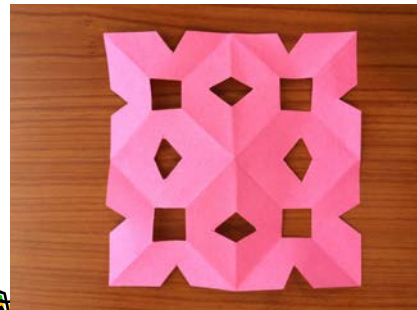
How about five consecutive numbers?

28

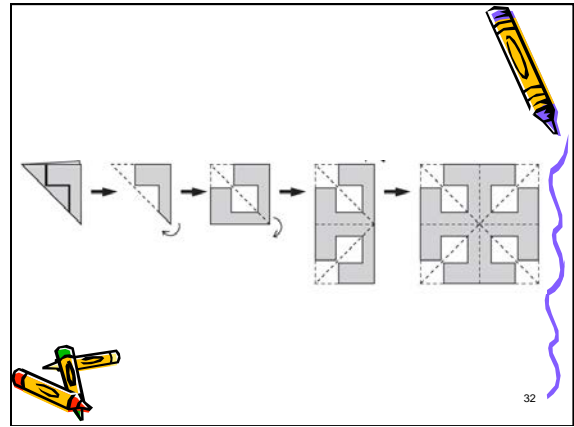
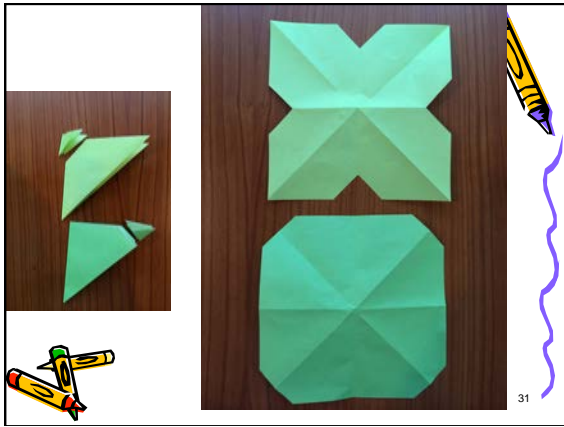
## Example1 Crest Cutting



29



30



1回折り  
折り目

2009, 1(1)

Which one of the following figures can be produced by cutting a paper that was hold once?

86.7%

33

2009, 1(2)

Which of two groups is the result of crest cutting? Chose it. Then describe the property of the figure in the group.

47.2%

34

**Emphasis on Explaining Mathematically**

The open-construction tasks require the students to explain;

- The observed facts in a situation
- Approaches and methods to solve problems
- Reasons for the facts and properties

35


**Mathematics is a descriptive language that emphasizes abstraction and logics**

**Sentence**  
Proposition, the subject and the predicate



Use of Mathematical terms  
Being aware of the subject and the predicate

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Which one of the following figures can be produced by cutting a paper that was hold three times?



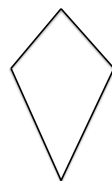


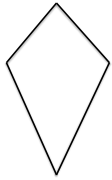
54.3%



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**Example 2**

Write a definition of a kite

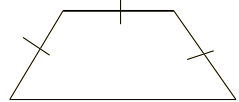







- Two adjacent sides are equal?
- Diagonals are perpendicular to each other?
- One pair of the opposite angles are equal?

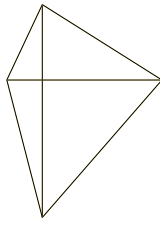


39

- Two equal adjacent sides?

40

Diagonals are perpendicular to each other

41

**Definitions of Kite**

A kite is...

K(1): a quadrilateral which has two distinct' pairs of adjacent sides of the same length.

K(2): a quadrilateral in which exactly one diagonal is a perpendicular bisector of the other.

K(3): a quadrilateral in which one diagonal is a perpendicular bisector of the other.



K(4): a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent.

K(5): a convex quadrilateral with two distinct pairs of adjacent sides of the same length.

K(6): a four-side with an axis of symmetry.

\* The words "disjoint" or "non overlapping" are often used in place of "distinct". These modifiers for the word "pair" exist to avoid the possibility that {a, b} and {a, c} are considered as two different pairs.

Usiskin, Z. & Griffin, J. (2008). *The classification of quadrilaterals: A study of definition*. Information Age Publishing.



### A Rhombus: Inclusive or Exclusive

- Rh (I1): a parallelogram with four equal sides (37)
- Rh (I2): a parallelogram in which at least two consecutive sides are congruent (20)
- Rh (I3): a parallelogram in which all four equal sides are equal in length/congruent (12)



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### A Rhombus: Inclusive or Exclusive, continued

- Rh (E1): a rhomboid having all sides equal (7)
- Rh (E2): a parallelogram whose angles are oblique and sides are equal (6)
- Rh (E3): a rhomboid having two adjacent sides equal (2)
- Rh (E4): a parallelogram with oblique angles with two adjacent sides equal (1)



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### Example 3

- Students' awareness of strengths and limitations of mathematical modeling
- With a focus on students' awareness of the assumptions made in formulating problem mathematically, as well as on their interpretation of mathematical results in relation to the real world situations



"The heart of key competencies is reflectiveness" (OECD, 1997)

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### Men's Marathon World Record

(2) The figure below shows the graph of world records in year in Table 1. Based on the graph, it would be safe to say that the points are roughly on the same line. If we regard the points are on the same line and so in the near future, we could predict the records in year 2020. Explain how to find the world record in year 2020.

Table 3: Men's marathon running world record evolution

Date	Time	Athlete	Marathon Event
Dec 9, 1981	02:08:18	Robert De Castella (Australia)	Fukuoka Marathon
Oct 21, 1984	02:08:05	Steve Jones (UK)	Chicago Marathon
April 20, 1985	02:07:12	Carlos Lopes (Portugal)	Rotterdam Marathon
April 12, 1988	02:06:05	Belaysayh Dinsamo (Ethiopia)	Rotterdam Marathon
Sept 20, 1998	02:06:05	Ronaldo da Costa (Brazil)	Bejin Marathon
Oct 24, 1999	02:05:42	Khalid Khannouchi (USA)	Chicago Marathon
April 14, 2002	02:05:38	Khalid Khannouchi (Morocco)	London Marathon
Sept 28, 2003	02:04:55	Paul Pierantoni (Kenya)	Bejin Marathon
Sept 30, 2007	02:04:26	Haile Gebreselassie (Ethiopia)	Bejin Marathon
Sept 28, 2008	02:03:59	Haile Gebreselassie (Ethiopia)	Bejin Marathon
Sept 25, 2011	02:03:58	Patrick Makau (Kenya)	Bejin Marathon
Sept 29, 2013	02:03:53	Wilson Kipsang (Kenya)	Bejin Marathon



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Men's marathon running world record

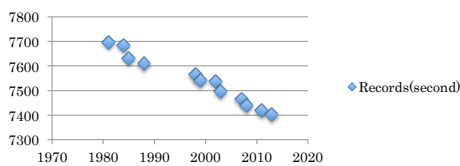


Figure 1: Scatter plots of the men's marathon running world record evolution



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(3) We predict the world records in the future, as the points of year and record on the graph are on the same line. If we would regard the points on the line, what can we say with the world records in the future. Choose one of the following statements and explain the reason you chose it.

- The world record will be improving further.
- The rate of change for the improvement of world record will be decreasing gradually
- The world record will approaching to a constant number
- The world record will have been improving, and



human can never run a marathon less than two hours

48



(3) We predict the world records in the future, as the points of year and record on the graph are on the same line. If we would regard the points on the line, what can we say with the world records in the future. Choose one of the following statements and explain the reason you chose it.

- A. The Rate of change for the world record will be decreasing gradually
- B. The world record will have been improving, and then it will be below two hours
- C. A human can never run a marathon less than two hours
- D. Other

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Table 4: Result of Marathon World Record Question (3).

Choice	A	B	C	D	NR
Response (%)	11.6	81.2	1.5	4.3	1.5

- Most students referred only to the formal mathematical terms such as linear function, constant (negative) rate of change, the line that is downward to the right, and so on.
- There are many physiological determinants of distance running performance factors such as  $\text{VO}_2$  max, lactate threshold, running economy, as well as the increase of prize money, advancement of sports technology, living environments, and the change of marathon course and so forth.

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### Students' explanation

- *"It took about ten years for breaking the record of 1985 and the pace of renewal came back later. This should be implying there was something happened such as progress of science. But it has a certain limitation".*
- *"There is a limitation in human abilities. Although the records may be improving as a result of technological advances, the rate of change will become smaller."*

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- Being aware of the assumptions in making mathematical model is important characteristics of those who are mathematically literate. The results of the small case study suggest that majority of students tend to not care this aspect of mathematical modeling.
- If we take definition of mathematical literacy as closely related to critical and reflective thinking, the assessment of mathematical literacy should focus on the recognition of strength and limitation of mathematical modeling.

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- For the development of assessment items for the secondary students, it is important to include those items that focus on the limitation of mathematical modeling and the importance of interpreting mathematical results in relation to the real world situations.
- A framework of assessing mathematical literacy needs to incorporate the dimension of this "meta-level" recognition on mathematical modeling.
- It should be noted that assessment of mathematical literacy should also focus on the aspects of mathematical activities within an intra-mathematical area.

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### Looking back

- **Perspectives on assessing "Mathematical Literacy"**
  - **The Japanese Contexts**
    - The National Course of Study
    - The New National Assessment
- **School Mathematics as a "Language Subject"**
  - Project: Science Literacy for All Japanese
  - Mathematics: The Language of Science
- **Examples (Activities)**
  - Constructing Definitions
  - Crest Cutting (Propositions)
  - Explaining (Proofs)
  - Marathon World Records (Describing, Modeling)

