How to develop and use constructed response items constructively

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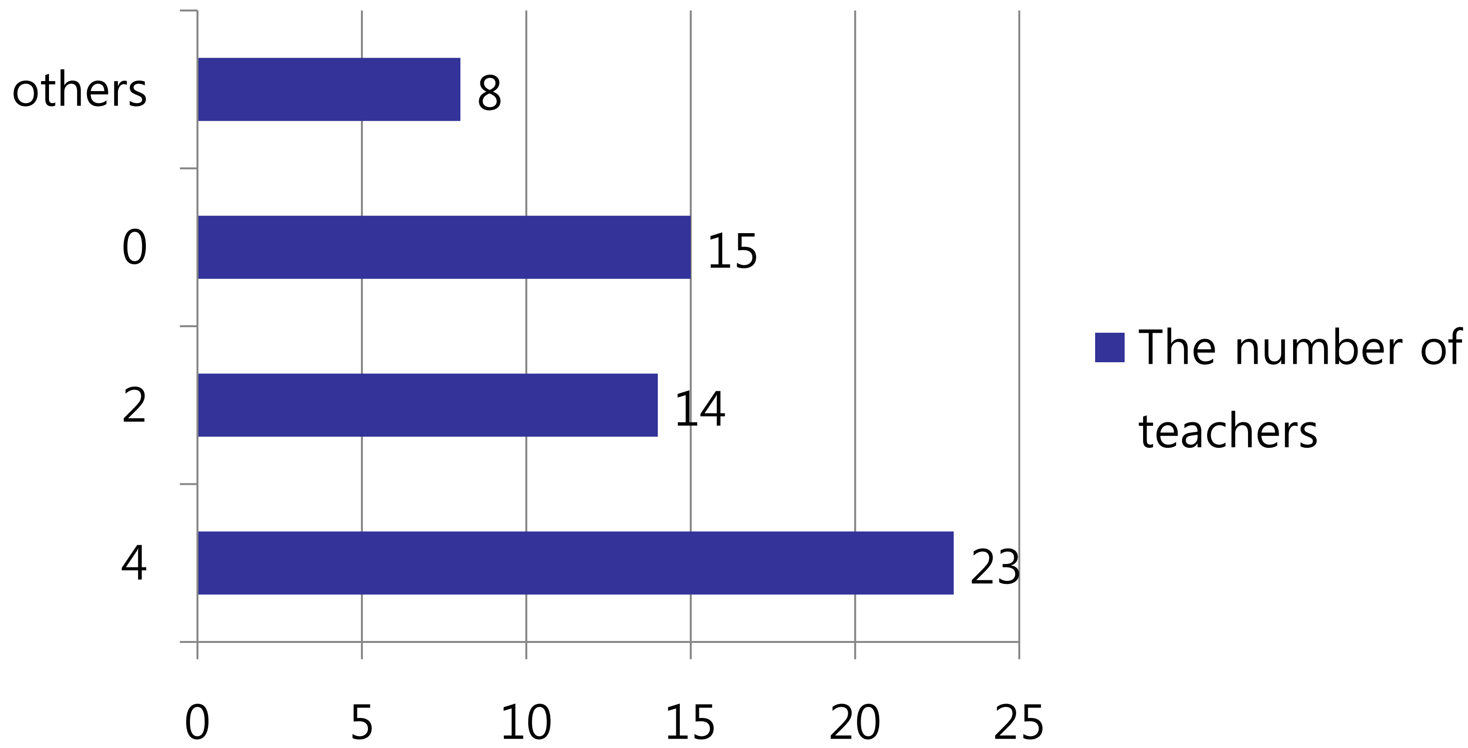
AME-SMS Conference 2014
Workshop

Find the value of $(-2)^2$ (4 points)

<student's answer> $(-2)^2 = -2^2 = 4$



If (x, y) *satisfies the three inequalities*

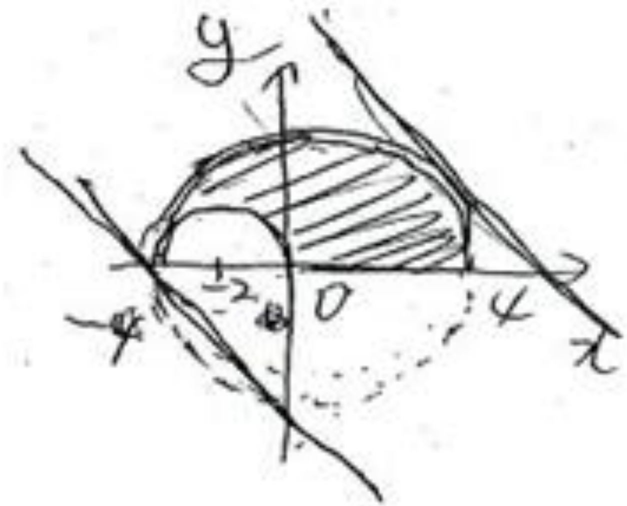
$$x^2 + y^2 \leq 16, (x+2)^2 + y^2 \geq 4, y \geq 0$$

Find the maximum and minimum of $3x + 4y$

(8 points)

Exemplary answer

The region satisfying the three inequalities is drawn in the right figure.



Let $3x + 4y = k$

When the line is tangent to the circle, k is maximum. The distance from (0, 0) to the line is same to the radius of circle,

$$3x + 4y - k = 0$$

$$\frac{|-k|}{5} = 4, \qquad Thus, \qquad k = \pm 20$$

Therefore, maximum is $k = 20$

When the line passes to (-4, 0), k is minimum. If $x = -4, y = 0,\ k = -12$.

Therefore, minimum is $k = -12$

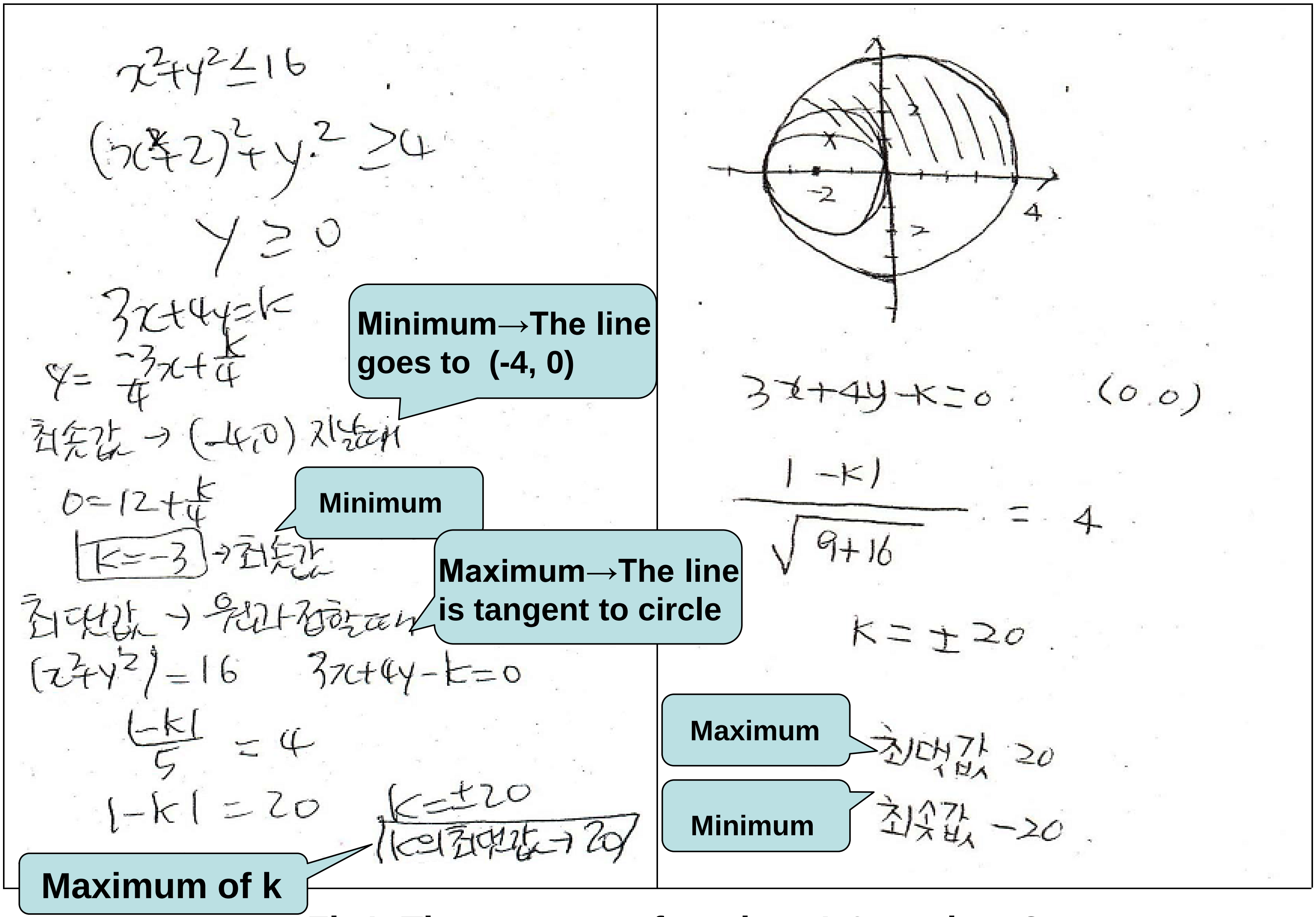


Fig1. The answers of student 1 & student 2

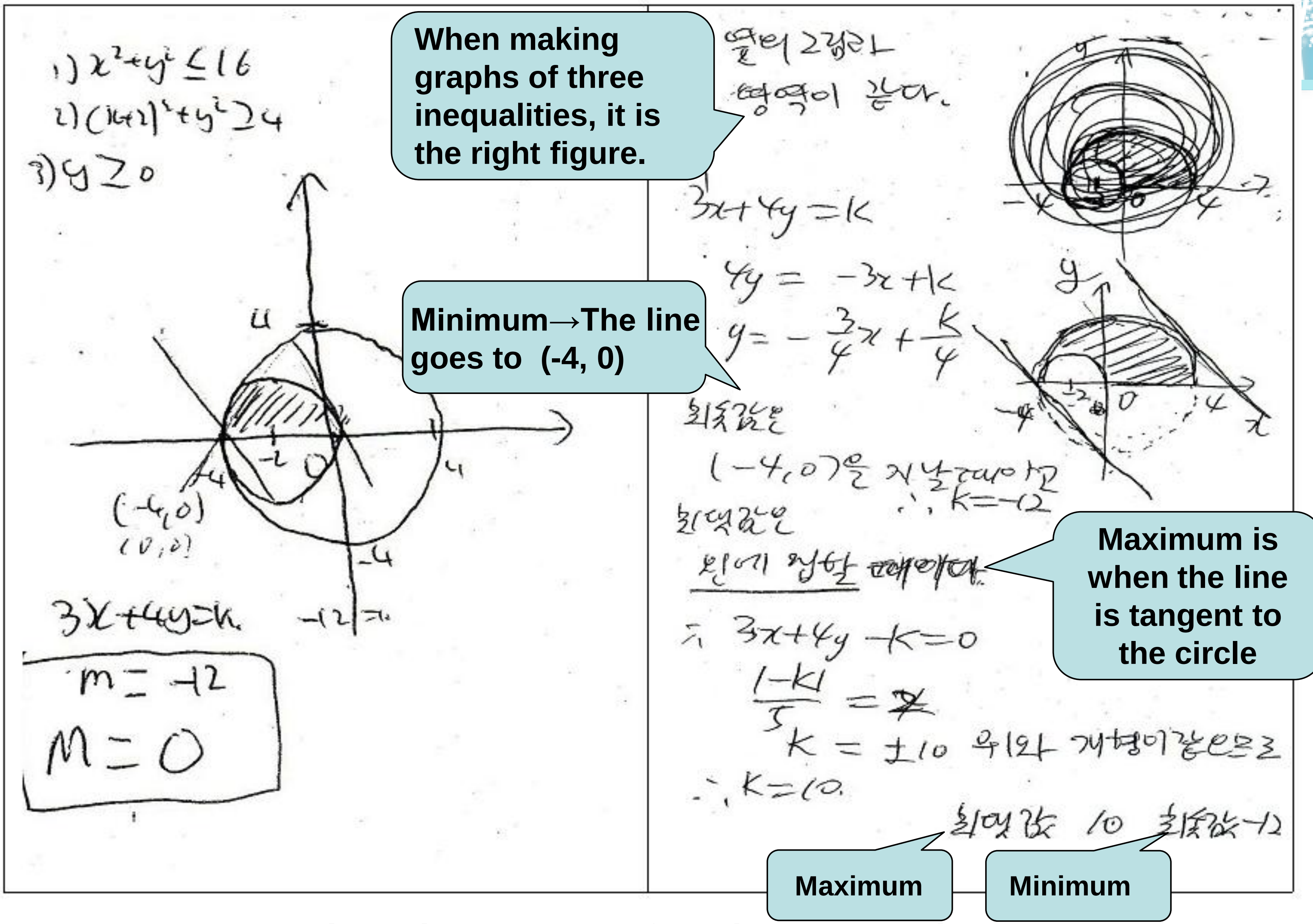


Fig2. The answers of student 3 & student 4

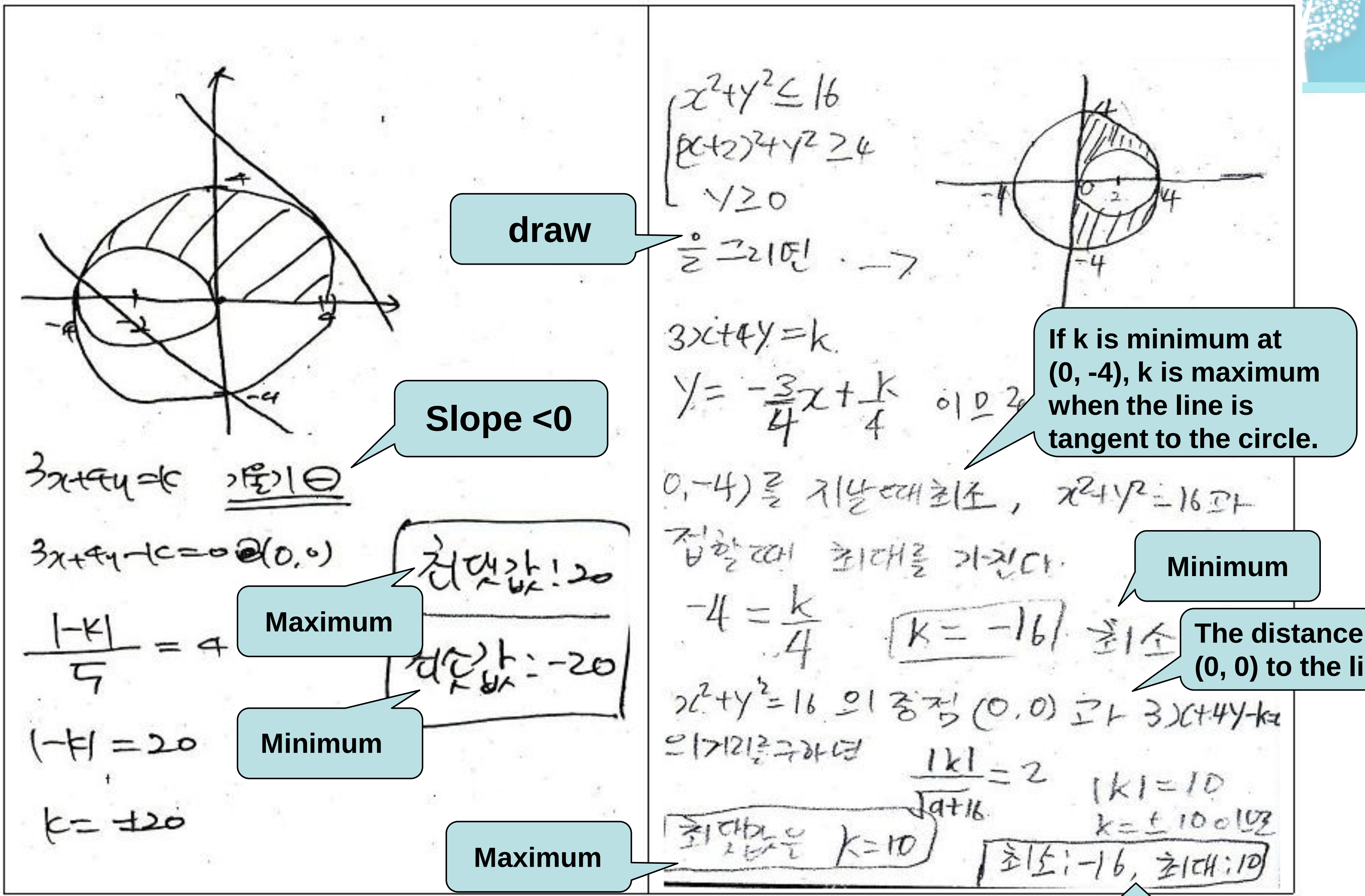


Fig3. The answers of student 5 & student 6

The scores given by pre-service teachers

Pre-service teacher	Student1	Student2	Student3	Student4	Student5	Student6
A	4	6	4	7	6	1
B	4.5	4	1	6	5.5	2.5
C	6	5	2	7	5	3
D	6	5	3	6	5	3
E	3	5	4	7	7	3
F	3.5	3.5	3.5	5.5	4.5	3
G	7	4	5	7	4	2
H	4.5	5.5	4.5	6.5	5.5	2
I	3	6	3	6	6	0
J	7	6	0	7	6	0
K	4.5	5	3	6.5	5	1.5
L	6	4	3	6	5	2
M	5.5	4	2.5	5.5	6	3

CONTENTS

Three types of assessments in Korea

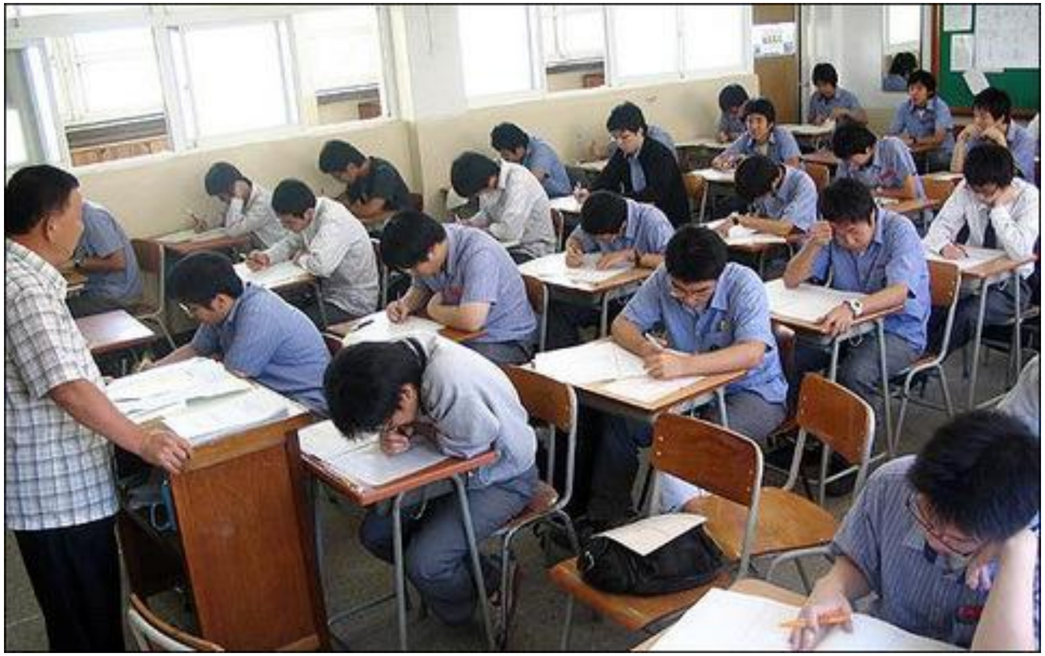
- College Scholastic Ability Test(CSAT)
  : nation-wide / college admission

- National Assessment of Educational Achievement (NAEA)
  : nation-wide

- Classroom-based Assessments

College Scholastic Ability Test
(大學數學能力評價)

College Scholastic Ability Test(CSAT)



College Scholastic Ability Test(CSAT)

❖ **Test period**

- **The third Thursday of November every year**

❖ **Type of test**

- **A type : Liberal Arts bound**
- **B type : Science bound**

❖ **Testing time and number of items**

- **100 minutes and 30 items**

College Scholastic Ability Test(CSAT)

Types of problems

❖ **Problem type**

- Multiple choice : 70% (21 problems)
- Short answer : 30% (9 problems)

❖ **Process**

- Calculation
- Comprehension
- Reasoning
- Problem Solving

College Scholastic Ability Test(CSAT)

2013 CSAT – B type

	Calculation	Comprehen -sion	Reasoning	Problem solving
M	1(11%)	3(33%)	3(33%)	2(22%)
S	4(20%)	9(42%)	2(10%)	6(28%)
T	5(17%)	12(40%)	5(17%)	8(27%)

Short-answer: Calculation

Evaluate $\lim_{x \to 2} \frac{(x-2)(x+3)}{x-2}$

College Scholastic Ability Test(CSAT)

Multiple-Choice: Comprehension

The maximum value of $f(x) = 2\cos^2 x + k \sin 2x - 1$, **is** $\sqrt{10}$. **What is the value of positive number** k**?**

① 1 ② 2 ③ 3 ④ 4 ⑤ 5

College Scholastic Ability Test(CSAT)

Multiple-Choice : Reasoning

Two matrices A and B satisfy the followings;

$$AB + A^2B = E,\ (A - E)^2 + B^2 = O$$

Which of the following is true? (E: identity matrix, O: zero matrix)

a. There exist an inverse matrix of B

b. $AB = BA$

c. $(A^3 - A)^2 + E = O$

① b ② c ③ a, b

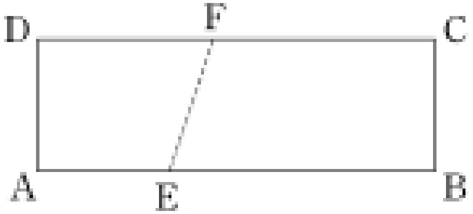
④ a, c ⑤ a, b, c

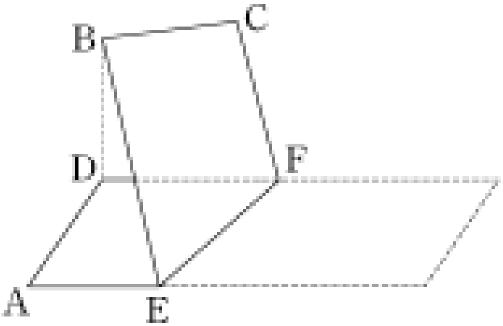
College Scholastic Ability Test(CSAT)

Short-answer: Problem solving

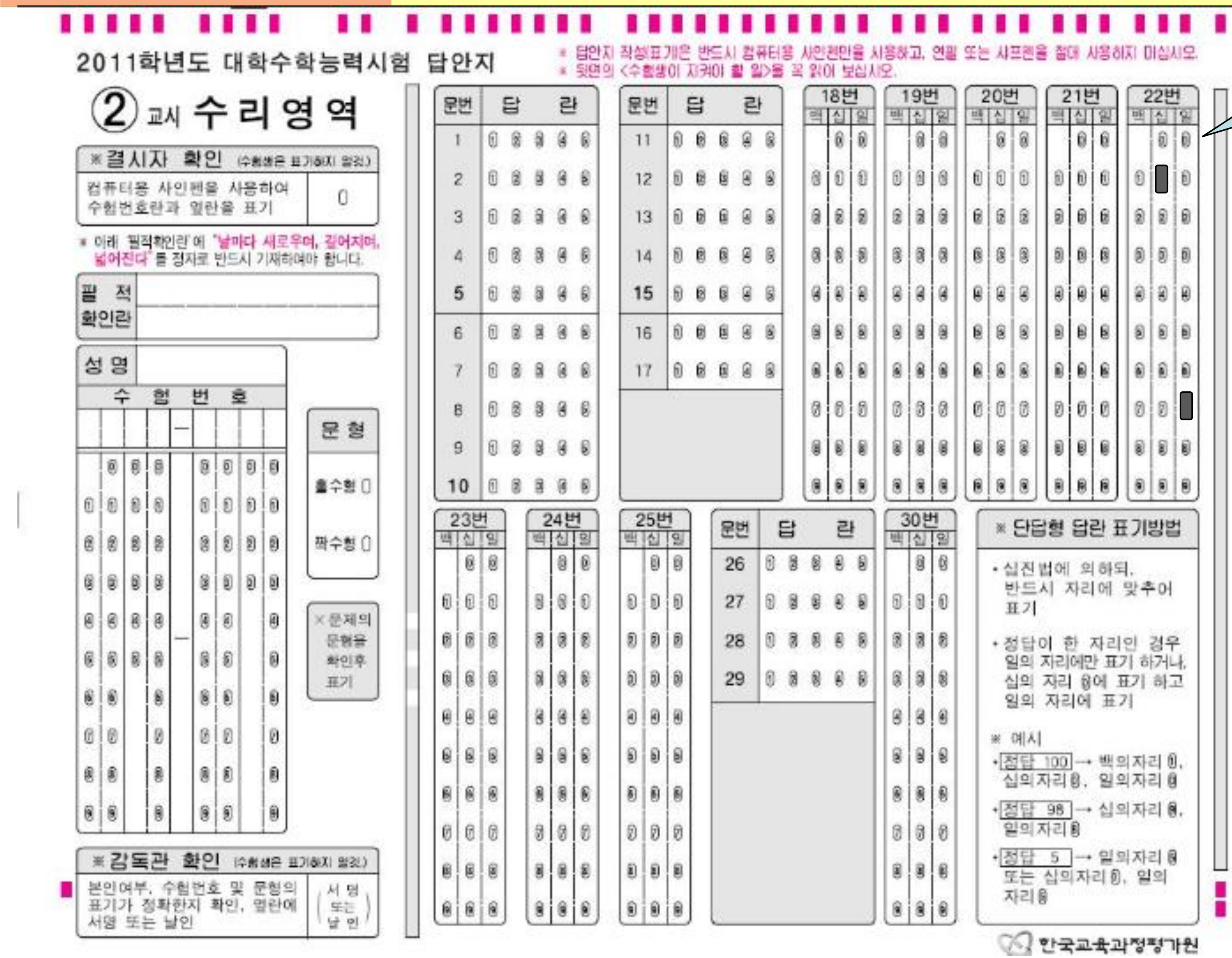
Rentangular paper ABCD has two sides $\overline{AB} = 9, \overline{AD} = 3$.

Then paper is folded up along the line $\overline{EF}$ so that the projection of point B onto the plain $AEFD$ is equal to the point D. Let θ $(0 < \theta < {}^{\pi}/_{2})$ be the angle between two plains $AEFD$ and $EFCB$, when $\overline{AE} = 3$. Evaluate $60\cos\theta$. (not to consider the thickness of the paper.)   (4pt)





Answer sheet for CSAT



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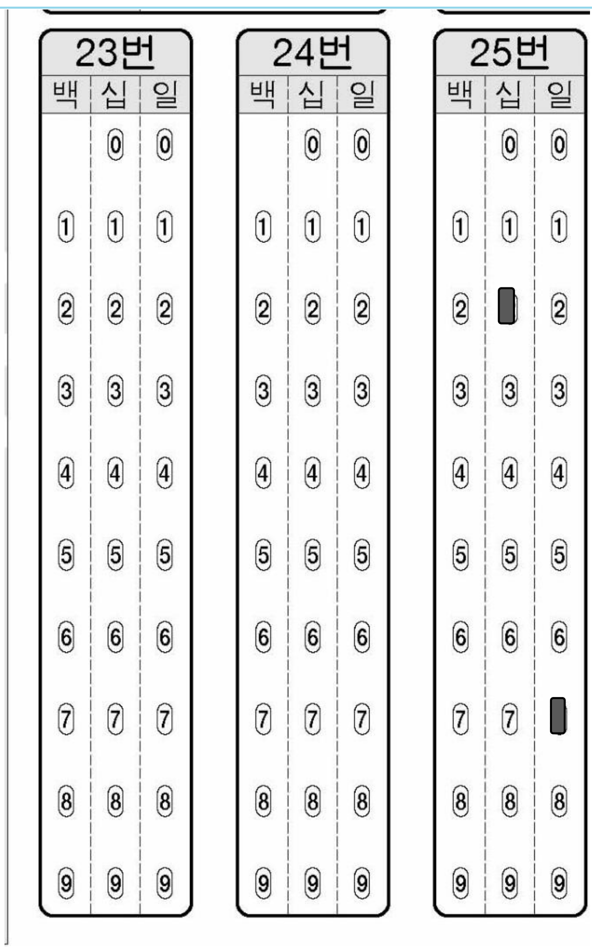
Answer sheet for CSAT



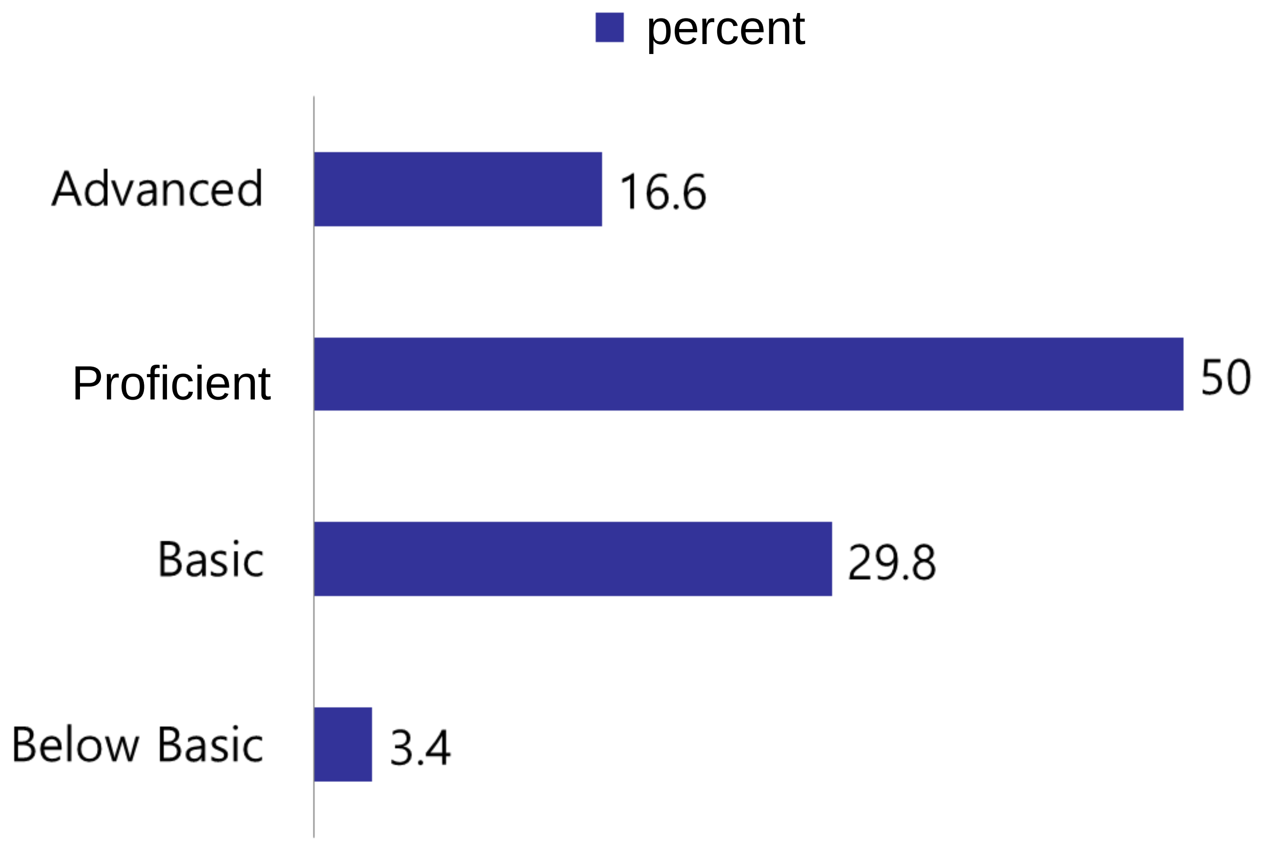
Marking in each digit according to students' answer for short-answer problems.

National Assessment of Educational Achievement (學業成就度評價)

National Assessment of Educational Achievement (NAEA)

- NAEA is a nationwide test that is implemented to evaluate elementary and secondary school students' achievements.
- The ministry will regularly analyze results of the NAEA, so as to better understand which factors impact academic ability and assist school efforts to raise student performance levels.

Result of NAEA in 2012 (middle school)



NAEA in Korea

❖ Testing time and number of items

- 60 minutes and 33 items

❖ Problem type

- Multiple choice : 29 items
- Short answer +descriptive problem : 4 items

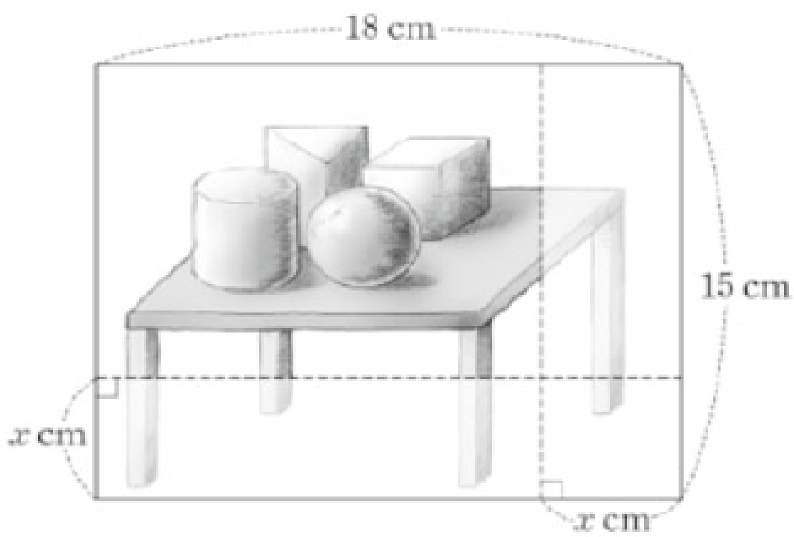
Example of multiple choice items from NAEA

Which of the following numbers is the rightmost number that is represented to the number line?

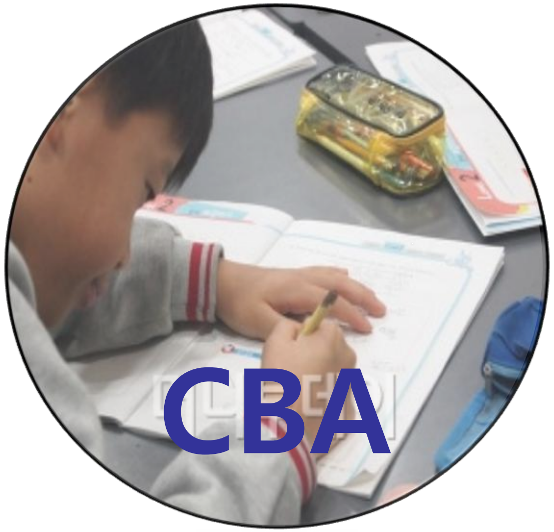
① $\frac{5}{2}$ ② $\sqrt{3}$ ③ 3

④ $2\sqrt{2}$ ⑤ $\sqrt{2}+1$

Example of descriptive problem from NAEA

[Essay question 4] To submit to a math newspaper, Hyeonji cut a rectangular picture from a magazine with width and length of 18cm and 15cm, respectively. As shown below, after cutting out x cm each from the width and length, the area of the remaining picture has become 2/3 of the original area. Answer the question. 	(1) Set up an equation to find the value of x. <Answer> $x^2 - 33x + 90 = 0$ (2) Describe the solving process and answer of finding the value of x. <Solving process> $(x-3)(x-30) = 0$ $x = 3 \text{ or } x = 30$ But $0 < x < 15,\ x = 3$ Therefore, $x = 3$

Classroom-based Assessment
教室基盤評價



Classroom-based assessment

Paper-pencil Test

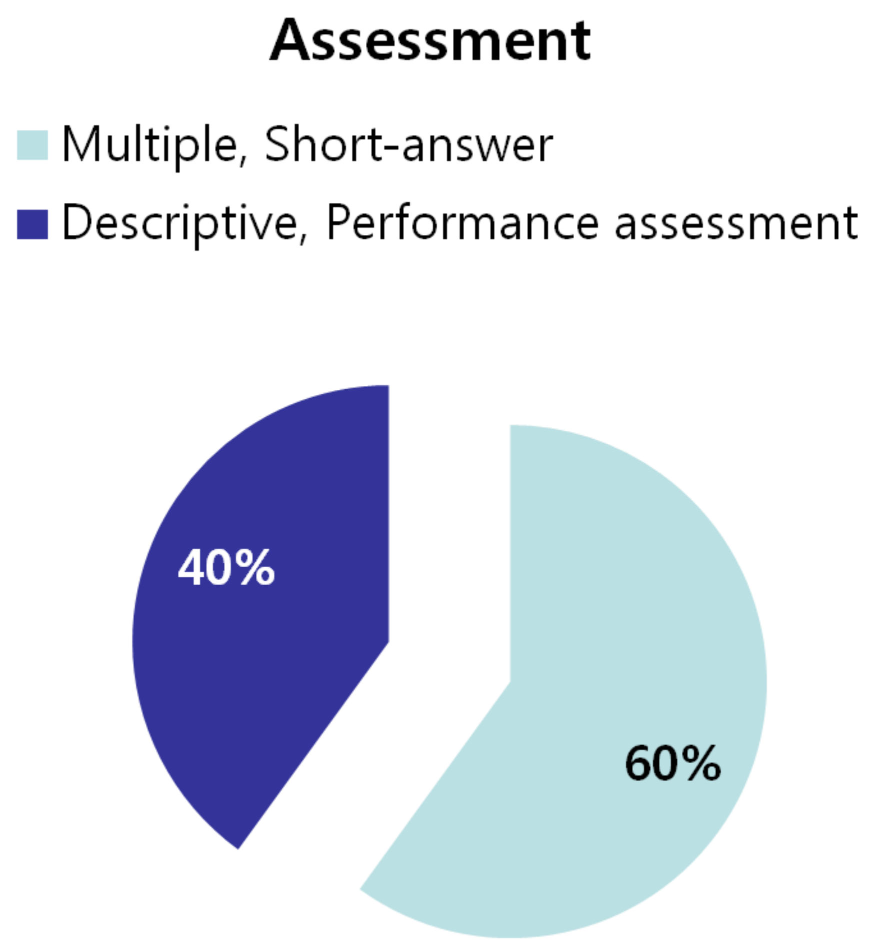
- Multiple choice
- Short-answer
- Descriptive(敍述形)
- Structured/ Long-answer

Performance assessment

- Project
- Quiz
- Research report
- Newspaper In Education(NIE)
- Observation & participation
- Presentation
- Self or peer evaluation report
- Essay

etc

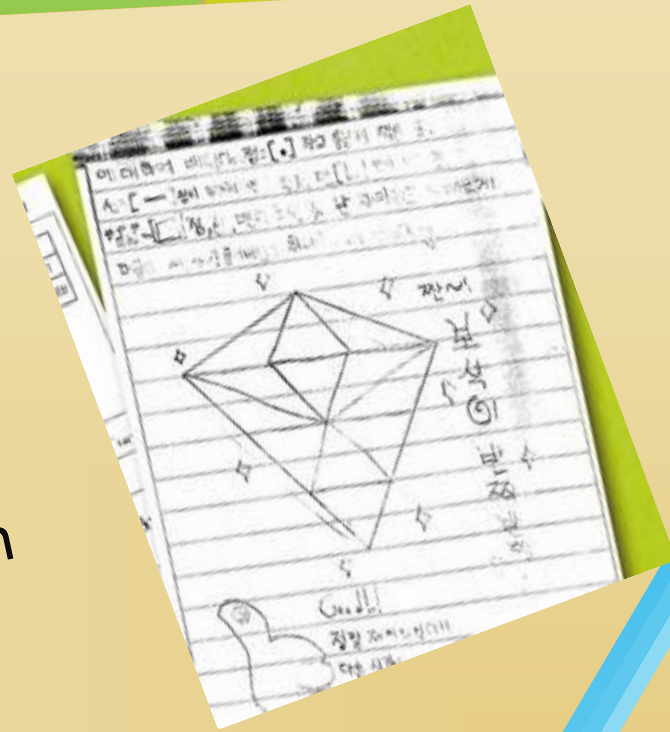
Classroom-based assessment



Descriptive(敍述形) problems

Descriptive

Problems that do not require lengthy descriptions but rather focus on the depth and the width of descriptions provided.



From 「Practicing Descriptive Problems in Academic Achievement Assessment」(Gyeonggi Provincial Office of Education_ Supervision Materials _2010-7)

Item Types in the PSLE Mathematics

❖ **Multiple-choice Question**

For each question, four options are provided of which only one is the correct answer. A candidate has to choose one of the options as his correct answer.

❖ **Short-answer Question**

For each question, a candidate has to write his answer in the space provided. Any unit required in an answer is provided and a candidate has to give his answer in that unit.

❖ **Structured / Long-answer Question**

For each question, a candidate has to show his method of solution (working steps) clearly and write his answer(s) in the space(s) provided.

Source: Singapore Examinations and Assessment Board

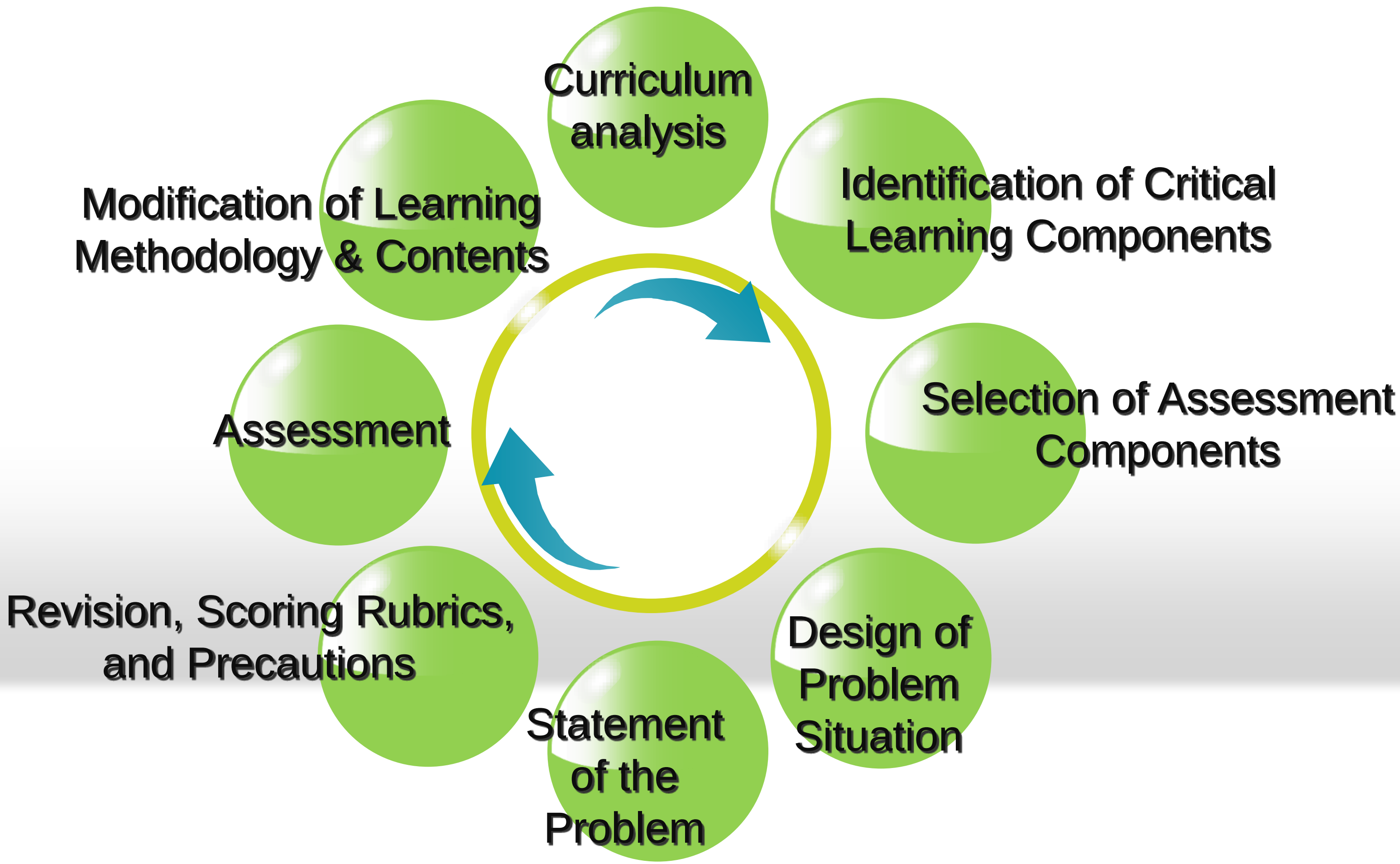
CONTENTS

Principles of designing descriptive problems

Principles

1. Consider the characteristics of the target student group.
2. Measure higher level thinking skills rather than knowledge from rote memory.
3. Specify problems to enable measurement of the learning outcome.
4. Provide the scoring criteria for point distributions per each problem.
5. Escalate the problems from low to higher complexity.
6. Avoid having big gaps of points between problems.
7. Do not allow students to select from a list of problems.

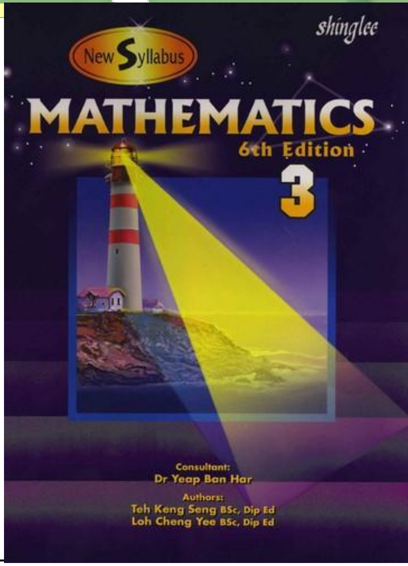
Process of developing descriptive problems



Curriculum analysis

Singapore

O-, N(A)- Level Mathematics



Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
G1 Trigonometric functions, identities and equations	
1.1 Six trigonometric functions for angles of any magnitude (in degrees or radians)	(a) Discuss the relationships between $\sin A, \cos A$ and $\tan A$, with respect to the line segments related to a unit circle. (b) Use a graphing software to display the graphs of trigonometric functions and
1.2 Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$	
1.3 Exact values of the trigonometric functions for special angles ($30°, 45°, 60°$) or ($\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$)	(c) (d)
1.4 Amplitude, periodicity and symmetries related to sine and cosine functions	
1.5 Graphs of $y = a\sin(bx) + c$, $y = a\sin(\frac{x}{b}) + c$, $y = a\cos(bx) + c$, $y = a\cos(\frac{x}{b}) + c$ and $y = a\tan(bx)$, where a is real, b is a positive integer and c is an integer.	(e)
1.6 Use of: * $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$,	

Content	Learning Experiences
GEOMETRY AND TRIGONOMETRY	**Students should have opportunities to:**
$\sin^2 A + \cos^2 A = 1$, $\sec^2 A = 1 + \tan^2 A$, $\operatorname{cosec}^2 A = 1 + \cot^2 A$ * the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ * the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$ * the expression of $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$	
1.7 Simplification of trigonometric expressions	
1.8 Solution of simple trigonometric equations in a given interval (excluding general solution)	
1.9 Proofs of simple trigonometric identities	

Curriculum analysis

Korea

<고등학교 수학>

4 삼각함수

① 일반각과 호도법의 뜻을 안다.

② 삼각함수의 뜻을 안다.

③ 사인함수, 코사인함수, 탄젠트함수의 그래프를 그릴 수 있고,
그 그래프의 성질을 이해한다.

④ 삼각함수의 성질을 이해한다.

⑤ 간단한 삼각방정식과 삼각부등식을 풀 수 있다.

To understand the laws of sine and cosine.

5 삼각형에의 응용

① 사인법칙과 코사인법칙을 이해한다.

② 삼각함수를 활용하여 삼각형의 넓이를 구할 수 있다.

To find the area of a triangle using trigonometry.

Mathematics

Identification of critical learning components

To understand the laws of sine and cosine.

To understand the relationship between angles and sides based on the laws of sine and cosine.

To find the area of a triangle using trigonometry.

To calculate the area of a triangle using trigonometry.

Selection of assessment components

Assessment Components

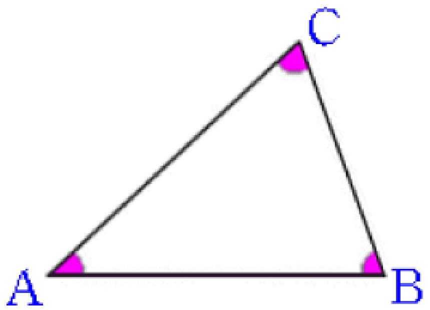
To understand the laws of sine and cosine and use them to find the area of a triangle.

Design of a problem

Provide the length of the sides and the angles to use the law of sine and cosine. To calculate the cosine value, use the second law of cosine, and then calculate the sine value

예시

사인법칙과 코사인법칙을 적용할 수 있도록 삼각형의 변의 길이와 각도를 제시한다. 세 변의 길이를 적당히 제시하여 코사인제2법칙을 이용하여 코사인의 값을 구하고 이를 통하여 사인값을 구할 수 있도록 구상한다.



Statement of the problem

Draft of the Problem

In ΔABC , $\overline{AB}=5,\ \overline{BC}=9,\ \overline{CA}=3$.
Answer the following questions. [10 points]

(1) Calculate $\sin A$.

(2) Calculate the area of ΔABC.

Revisions, scoring rubrics, and precautions

Revision Idea #1

Idea 1. Revise the lengths of the sides according to the Triangle Inequality Theorem.

Idea 2. Restate the problem so that students would use the second law of cosines and sine, instead of the Hero's formula, when calculating the area of the triangle.

Idea 3. Provide a diagram/picture to help the students better understand the problem.

Idea 4. Show how the 10 points are divided between the sub problems.

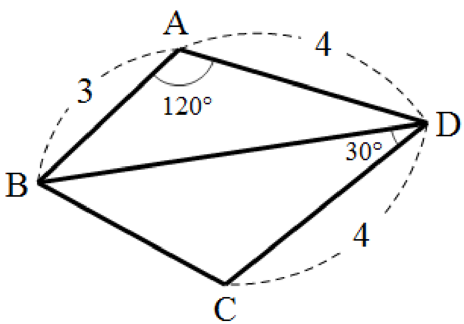
Revision of problems

Revision #1

In the following rectangle $ABCD$**,**

$$\angle A = 120°, \angle BDC = 30°, \overline{AB} = 3, \overline{AD} = 3, \overline{CD} = 4$$

Answer the following questions. [10 points]



(1) Calculate the length of side BD and show all your work. [3 points]

(2) Calculate the area of the rectangle $ABCD$ **and show all your work. [7 points]**

Revisions, scoring rubrics, and precautions

Revision Idea #2

Idea 5. Dividing into sub problems may serve as a hint for the students. Re-design the problem so that the sub problems do not provide clues to the students in terms of the solution.

Idea 6. Calculating the area of the rectangle in sub problem (2) requires students to repeat the same process. Revise the problem so that they calculate the triangle area using sine just once.

Idea 7. The length of BD is $\sqrt{37}$, while the area of BCD is also $\sqrt{37}$. Revise the problem so that they would be different.

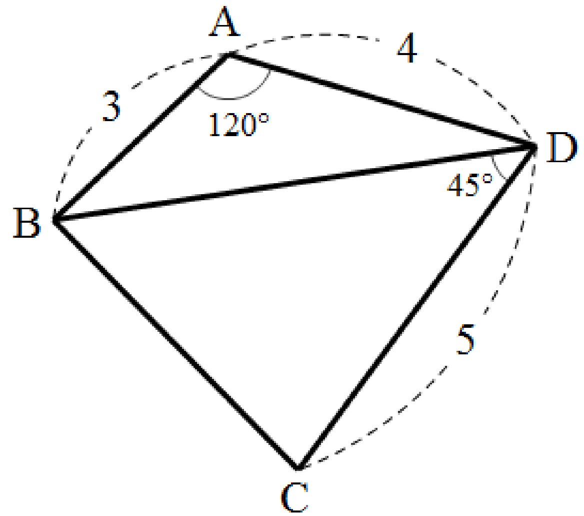
Revision of problems

Revision #2

In the rectangle $ABCD$,

$$\angle A = 120°, \angle BDC = 45°, \overline{AB} = 3, \overline{AD} = 4, \overline{CD} = 5.$$

Find the area of the triangle BCD **and describe the process. [10 points]**



CONTENTS

1 Three types of assessments in Korea

2 Designing Descriptive Problems (敍述形 問題製作)

3 Grading of Descriptive Problems (敍述形 問題 採點)

Scoring rubrics

The scoring scales used for correcting open-ended tasks vary considerably, and may range from general scoring scales to task-specific ones (Wiliam, 1993).

General scoring rubric

Task-specific rubric

- **Analytic scoring rubric**
- **Holistic scoring rubric**

From Wiliam, D. (1993). *Assessing authentic tasks: norms, criteria, and other referents.* A paper presented at the Nordic Symposium Research on Assessment in Mathematics Education, University of Goteborg, November 5, 1993

Scoring rubrics

Analytic scoring rubric

Involving a procedure whereby separate points are awarded for each aspect of the problem-solving process, i.e., understanding the problem, planning the solution, and getting an answer

ANLYTIC SCORING RUBRIC

• Understanding the problem
0: Complete misunderstanding of the problem.
3: Part of the problem misunderstood or misinterpreted.
6: Complete understanding of the problem

• Planning a solution
0: No attempt, or totally inappropriate plan.
3: Partly correct plan
6: Plan could lead to a correct solution.

From Lambdin Kroll D. et al., (1992). Cooperative Problem Solving: But What About Grading? *Arithmetic Teacher, 39*(6), 17-23.

Scoring rubrics

Holistic scoring rubric

Focusing on the solution as a whole, rather than on its various components

HOLISTIC SCORING RUBRIC

0 points:
•Problem is not attempted or the answer sheet is blank.
•The data copied are erroneous and no attempt has been made to use that data
•An incorrect answer is written and no work is shown

1 point
•The data in the problem are recopied but nothing is done
•A correct strategy is indicated but not applied in the problem
•The student tries to reach a subgoal but never does.

2 points:
•An inappropriate method is indicated and some work is done, but the correct answer is not reached.
•A correct strategy is followed but the student does not pursue the work sufficiently to get the solution
•The correct answer is written but the work either is not intelligible or is not shown

3 points:
•The student follows a correct strategy but commits a computational error in the middle

From Cross , L., and Hynes, M.C. (1994). Assessing Mathematics Learning for Students with Learning Differences. *Arithmetic Teacher, 41(7*), 371-377.

Scoring rubrics

General Scoring Rubric

Analytic scoring rubric
Holistic scoring rubric

❖**The advantage**

They can be used for a wide range of problems

❖ **The issue**

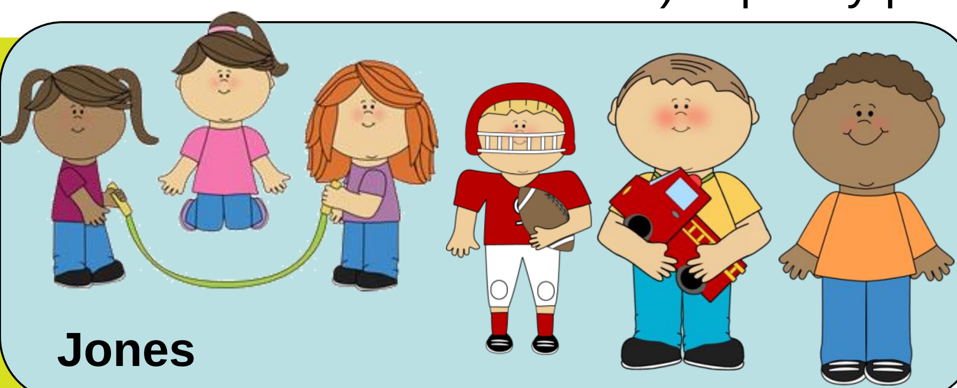
Do such general analyses actually provide sufficient footholds for further instruction ?

From Heuvel-Panhuizen, M.H.A.M. van den (1996). *Assessment and Realistic Mathematics Education*.Utrecht: Urecht University

Scoring rubrics

Task-specific scoring rubric

With respect to footholds for further instruction, more can be expected of the task-specific scoring scales, in which the categories of possible answers (often illustrated with student work) explicitly pertain to a specific problem.



Jones

King

Here is a picture of the children in two families. The Jones family has three girls and three boys and the King family has three girls and one boy. Which family has more girl?

Follow-up question:
Which family has more girls compared to boys?

From Lamon , S.J., and Lesh , R.(1992). Interpreting responses to problems with several levels and types of correct answers. In R. Lesh and S.J. Lamon (eds.), *Assessment of Authentic Performance in School Mathematics*(pp.319-342). Washington: AAAS Press.

Scoring rubrics

Task-specific scoring rubric

Scoring categories pertaining to the Families problem

0: The student reasons additively
1: The student reasons multiplicatively in some situations
when prompted to consider relative comparison
2: The student reasons multiplicatively in some situations without prompting
3: The student's initial response uses relative thinking
4: The student thinks relatively and explains his or her thinking
by making connections to other pertinent material or
by translating to an alternate form or representation

From Lamon , S.J., and Lesh , R.(1992). Interpreting responses to problems with several levels and types of correct answers. In R. Lesh and S.J. Lamon (eds.), *Assessment of Authentic Performance in School Mathematics*(pp.319-342). Washington: AAAS Press.

Scoring rubrics

Task-specific scoring rubric

Scoring categories

❖Analytic
In the sense that they involve various aspects of the solution,
Such as the forms of representation, solution strategies, reasoning strategies, solution errors, mathematical arguments, quality of description

❖General
For the forms or the representations where a distinction is made between explanations in words, pictures and symbols

❖Specific
The solution strategies

From Heuvel-Panhuizen, M.H.A.M. van den (1996). *Assessment and Realistic Mathematics Education*.Utrecht: Urecht University

Let’s make task specific scoring rubrics

.

Work with your colleagues and prepare a model answer for these problems.

Create task-specific scoring rubrics that includes partial scoring.

Sample answer

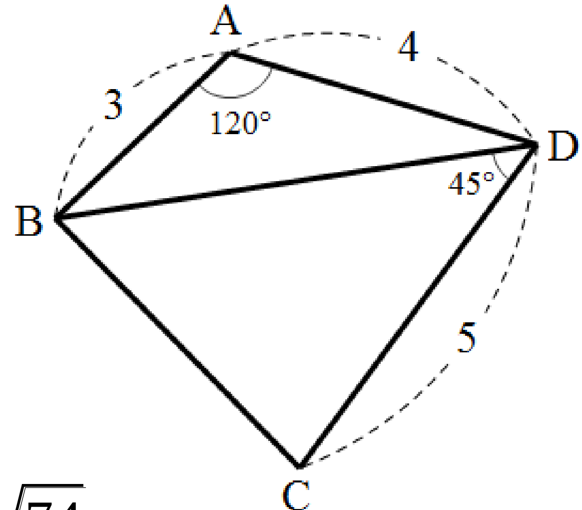
Revision # 2

In the triangle ABD,

$$\overline{BD}^2 = \overline{AB}^2 + \overline{AD}^2 - 2\overline{AB}^2 \cdot \overline{AD}^2 \cdot \cos A$$ **(second cosine formula)**

$$= 9 + 16 - 2 \cdot 3 \cdot 4 \cdot \cos A$$

$$= 9 + 16 - 2 \cdot 3 \cdot 4 \cdot \left(-\frac{1}{2} \right) = 37 \;.$$



Therefore, $\overline{BD} = \sqrt{37}$

Thus, $\Delta BCD = \frac{1}{2} \cdot \overline{BD} \cdot \overline{CD} \cdot \sin 45^\circ$

$$= \frac{1}{2} \cdot \sqrt{37} \cdot 5 \cdot \frac{\sqrt{2}}{2} = \frac{5\sqrt{74}}{4}$$

Task-specific rubric for the revision #2

		Scores
Calculation of the length of $\overline{BD}$	The student follows a correct strategy and find the correct answer $\sqrt{37}$.	5
	The student knows the cosine rule and how to calculate but she/he commits a computational error in the middle.	3
	The student knows the cosine rule but she/he didn't use the correct value of cosine function.	2
Calculation of the area of the triangle	The student follows a correct strategy and find the correct answer $\frac{5\sqrt{74}}{4}$.	5
	The student knows how to calculate the area of a triangle using sine function but she/he commit a computational error in the middle.	3
	The student knows how to calculate the area of a triangle using sine function but she/he didn't use the correct value of sine function.	2

Student answer

Example 1

According to the second cosine formula,

$$\overline{BD}=\sqrt{3^2+4^2-2\cdot 3\cdot 4\cdot \cos 120^\circ}$$

$$=\sqrt{25+12}$$

$$=\sqrt{37}.$$

$$\therefore \triangle BCD=\frac{1}{2}\cdot\overline{CD}\cdot\overline{BD}\cdot\sin 45^\circ$$

$$=\frac{5\sqrt{74}}{4}$$

Student answer

Example 2

According to the second cosine formula,

$$BD^2 = AB^2 + AD^2 - 2AB\cdot AD\cos 120^\circ$$

$$= 9 + 16 - 2\cdot 3\cdot 4\cdot\left(-\frac{1}{2}\right)$$

$$= 37$$

$$\therefore BD = \sqrt{37}$$

$$\triangle BCD = \frac{1}{2}\overline{BD}\cdot\overline{CD}\sin 45^\circ$$

$$= \frac{1}{2}\ 37\cdot 5\cdot\frac{1}{2}$$

$$= \frac{185}{4}$$

Student answer

Example 3

According to the second cosine formula,

$$\overline{BD} = \sqrt{3^2+4^2-2\cdot 3\cdot 4\cos 120^\circ} = \sqrt{25+12} = \sqrt{37}$$

$$S_{\triangle BCD} = \frac{1}{2}\overline{BD}\cdot\overline{DC}\cdot\sin 45^\circ = \frac{\sqrt{2}}{4}\cdot\sqrt{37}\cdot 5 = \boxed{\frac{5}{4}\sqrt{74}}$$

Student answer

Example 4

Using the second cosine formula to find the length of $\overline{BD}$

$$\overline{BD}^2 = 9 + 16 - 2 \times 3 \times 4 \cos 120^\circ$$

$$= 25 - 12 = 13$$

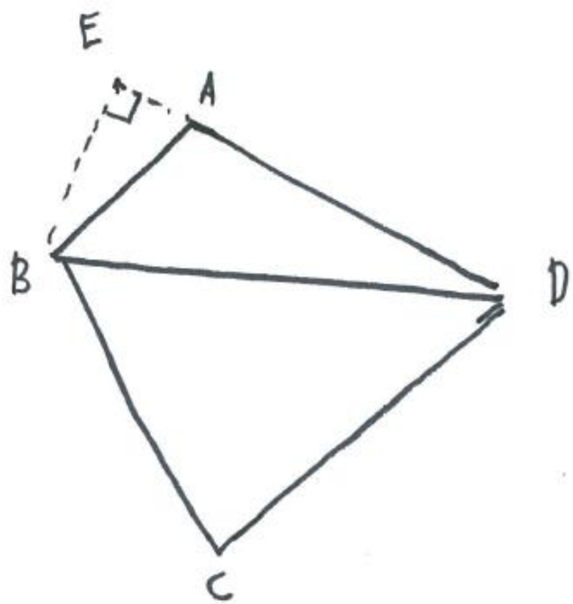
$$\therefore \overline{BD} = \sqrt{13}$$

$\therefore \triangle BCD$ [illegible] 의 넓이 $= \frac{1}{2} \times \overline{BD} \times \overline{CD} \times \sin 45^\circ = \frac{1}{2} \times \sqrt{13} \times 5 \times \frac{\sqrt{2}}{2}$

$$= \frac{5\sqrt{26}}{4}$$

Student answer

Example 5



$AE = AB \cos 60° = \frac{3}{2}$

$BE = AB \sin 60° = \frac{3\sqrt{3}}{2}$

$BD = \sqrt{DE^2 + BE^2}$

$= \sqrt{\frac{27}{4} + \frac{121}{4}} = \sqrt{37}$

넓이 $S = \overline{BD} \cdot \overline{DC} \sin 45°$

$= \frac{5\sqrt{74}}{2}$

Student answer

Example 6

To apply the second cosine formula on the triangle ABD

$$\therefore \overline{BD}^2 = 37$$

$$\therefore \overline{BD} = \sqrt{37}$$

Task-specific rubric for the revision #2

		Scores	Students' answers					
			1	2	3	4	5	6
Calculation of the length of $\overline{BD}$	The student follows a correct strategy and finds the correct answer $\sqrt{37}$.	5	5	3	5	2	5	?
	The student knows the cosine rule and how to calculate but she/he commits a computational error in the middle.	3						
	The student knows the cosine rule but she/he didn't use the correct value of cosine function.	2						
Calculation of the area of the triangle	The student follows a correct strategy and finds the correct answer $\frac{5\sqrt{74}}{4}$.	5	5	2	5	?	0	0
	The student knows how to calculate the area of a triangle using sine function but she/he commits a computational error in the middle.	3						
	The student knows how to calculate the area of a triangle using sine function but she/he didn't use the correct value of sine function.	2						

Agenda for discussion

Agenda for Discussion

Agenda 1. If a student makes a mistake calculating the length of BD and uses the wrong answer to find the area of the triangle BCD, he/she will automatically fail the second problem. How should this taken into account in scoring of the second problem (finding area of the triangle)?

Agenda 2. How should we score those students who do not describe the solution at all, but simply give the final answers?

Task-specific rubric for the revision #2

		Scores	Remarks
Calculation of the length of $\overline{BD}$	The student follows a correct strategy and finds the correct answer $\sqrt{37}$.	5	If a student shows only answer without any procedure, we give her/him 2 pt.
	The student knows the cosine rule and how to calculate but she/he commits a computational error in the middle.	3	
	The student knows the cosine rule but she/he don't use the correct value of cosine function.	2	
Calculation of the area of the triangle	The student follows a correct strategy and finds the correct answer $\frac{5\sqrt{74}}{4}$.	5	If a student gets a wrong answer of the length of $\overline{BD}$ and follows a correct strategy using that, then we give her/him 3 pt.
	The student knows how to calculate the area of a triangle using sine function but she/he commits a computational error in the middle.	3	
	The student knows how to calculate the area of a triangle using sine function but she/he don't use the correct value of sine function.	2	

Task-specific rubric for the revision #2

		Scores	Students' answer					
			1	2	3	4	5	6
Calculation of the length of $\overline{BD}$	The student follow a correct strategy and find the correct answer $\sqrt{37}$.	5	5	3	5	2	5	2
	The student knows the cosine rule and how to calculate but she/he commits a computational error in the middle.	3						
	The student knows the cosine rule but she/he didn't use the correct value of cosine function.	2						
Calculation of the area of the triangle	The student follow a correct strategy and find the correct answer $\frac{5\sqrt{74}}{4}$.	5	5	2	5	3	0	0
	The student knows how to calculate the area of a triangle using sine function but she/he commits a computational error in the middle.	3						
	The student knows how to calculate the area of a triangle using sine function but she/he didn't use the correct value of sine function.	2						

Using graded descriptive problems

How to constructively use students' answer for further instruction

Student answer

Example 2

According to the second cosine formula,

$$BD^2 = AB^2 + AD^2 - 2AB \cdot AD\cos 120^\circ$$

$$= 9 + 16 - 2 \cdot 3 \cdot 4 \cdot \left(-\frac{1}{2}\right)$$

$$= 37$$

$$\therefore BD = \sqrt{37}$$

use BD²

$$\triangle BCD = \frac{1}{2}\overline{BD} \cdot \overline{CD}\sin 45^\circ$$

$$= \frac{1}{2} \cdot 37 \cdot 5 \cdot \frac{1}{2}$$

$$= \frac{185}{4}$$

$$\boldsymbol{sin45^\circ = \frac{\sqrt{2}}{2}}$$

Student answer

Example 4

Using the second cosine formula to find the length of $\overline{BD}$

$$\overline{BD}^2 = 9 + 16 - 2\times 3\times 4\cos 120^\circ$$

$$= 25 - 12 = 13$$

$\cos 120^\circ = -\dfrac{1}{2}$

$$\therefore \overline{BD} = \sqrt{13}$$

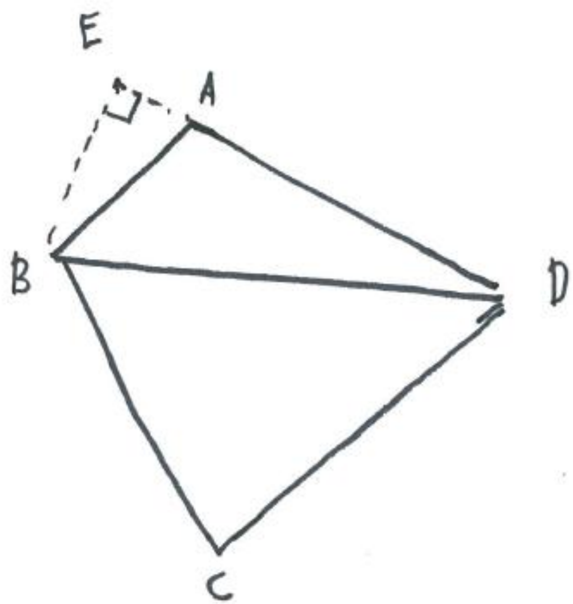
$\therefore \triangle BCD$ 의 넓이는 $= \frac{1}{2}\times \overline{BD}\times \overline{CD}\times \sin 45^\circ = \frac{1}{2}\times\sqrt{13}\times 5\times\frac{\sqrt{2}}{2}$

$$= \frac{5\sqrt{26}}{4}$$

Student answer

Example 5

Alternative method



$$AE = AB\cos 60^\circ = \frac{3}{2}$$

$$BE = AB\sin 60^\circ = \frac{3\sqrt{3}}{2}$$

$$BD = \sqrt{DE^2 + BE^2}$$

$$= \sqrt{\frac{27}{4} + \frac{121}{4}} = \sqrt{37}$$

넓이 $S = \overline{BD}\cdot\overline{DC}\,\sin 45^\circ$

$$= \frac{5\sqrt{74}}{2}$$

Student answer

Example 6

The student didn't calculate the area of triangle using trigonometry

It was not enough for him to solve the problem.

To apply the second cosine formula on the triangle ABD

$$\therefore \overline{BD}^2 = 37$$

$$\therefore \overline{BD} = \sqrt{37}$$

To sum up,

Thank you

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Comments/ Questions are welcomed anytime via

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