## Concepts

| Key Ideas | Questions (general) | Examples (specific) | Possible Student Responses |
| :---: | :---: | :---: | :---: |
| Meanings, definitions, representations | C1: What does ... mean? <br> (Separate meaning from rule.) | What is the highest common factor (HCF) of two numbers? | C: The largest number that can divide the two numbers evenly (without remainder). |
|  |  |  | W : The larger of the two given numbers. <br> W: Explain the "division" method to find HCF. |
|  | C2: Show ... on a diagram. | Show the value of $c$ on the graph of $y=m x+c$. | C: Mark the point ( $0, c$ ). <br> W: Mark other points, e.g., $x$ intercept. |
| Examples, non-examples, facts | C3: Give me an example of ... Another example, another one ... | Give me an example of a quadratic equation. | C: Any correct equation. <br> W: Quadratic expression instead of quadratic equation. <br> Q: What about $y=2 x^{2}+x-3$; equation of a quadratic function? |
|  | C4: Give me an example that is not of ... | Give me an example that is not a quadratic equation. | C: W: |
| Connections | C5: How is ... similar to (different from) ...? | How is a rhombus different from a square? | C: W: |

## Skills (Methods)

| Key Ideas | Questions (general) | Examples (specific) | Possible Student Responses |
| :---: | :---: | :---: | :---: |
| Steps, procedures | S1: What is the formula for ...? | What is the formula for $\sin 2 A$ ? <br> (May be useful for students to memorise important formulae instead of looking them up from formula list.) | C: $2 \sin A \cos A$ <br> W: $2 \sin A$ |
|  | S2: What is the first step? Next step? How do you begin? <br> (To avoid student tendency to think of just any formula to solve a problem, ask them questions about what to find before asking this question.) | Prove the identity: $\tan x+\cot x=(\sec x)(\operatorname{cosec} x)$ | C: <br> W: |
|  | S3: What are the missing steps? <br> (How many intermediate steps are required?) | What is the gradient of the line with equation $3 x-4 y+5=0$. Answer: 3/4 | C: <br> W: |
| Conditions of use | S4: What are the conditions for applying ... ? <br> Have you checked them? | Before you use Pythagoras Theorem, what must you check first? | C: Right-angled triangle; given sides must be for the same triangle. |

## Processes

| Key Ideas | Questions (general) | Examples (specific) | Possible Student Responses |
| :---: | :---: | :---: | :---: |
| Reasoning, inductive justification, deductive proofs | P1: How do you know this is true? <br> (How do you convince others that this is true? Why do you believe in ...?) | A parallelogram is not a rectangle? Why? | C: <br> W: |
|  | P2: Why do you use ...? <br> (Could be under Skills) | You use Sine rule to solve this problem. Why? | C: <br> W: |
| Communication | P3: What is the correct word or symbol for ...? <br> (Could be under Concepts) |  | C: W: |
|  | P4: Explain what you are doing here ... (in your own words) <br> (Could be under Skills) |  | C: W: |
| Applications | P5: What maths can be used in this real-life context ...? | As a reporter, you want to show the number of litterbugs caught in the past five years. How? | C: W: |

$\qquad$

## Metacognition

| Key Ideas | Questions (general) | Examples (specific) | Possible Student Responses |
| :---: | :---: | :---: | :---: |
| Monitor problem solving process, unstuck | M1: What does ... remind you of? | Solve: $x^{2}-3 x=x-3$. What does this remind you of? | C: Solve quadratic equation with zero on one side. |
|  |  |  | W: Solve equation by cancelling factors. |
|  | M2: If you continue to do this, do you think you are on the right track? |  | C: W: |
|  | (Encourage students to re-read question.) |  |  |
|  | M3: What heuristic would you try when you are stuck? | Three people were at a coffee shop. There were 20 sugar cubes. Each person put an odd number of cubes in his cup. They used all 20 sugar cubes. How many cubes did each person use? (modified from Zazkis \& Liljedahl, p. 80) | C: <br> W: |
| Look back, make sense, extend | M4: Does the answer make sense? | The average of 24 and 26 is 35 . Does this make sense? | C: Answer should be between 24 and 26. |
|  | (Students need to know about real-life contexts.) |  | W : This is what I get from calculator (copy wrongly?) |
|  | M5: What would you do if ... is changed? | A restaurant gives a discount equal to the age of the paying customer. If the bill is $\$ 250$ and the paying customer is 40 years old, how much is the discount? What is the discount if the paying customer is 60 years old? 110 years old? | C: W: |
| Self-regulated learning | M6: How would you avoid similar mistakes? |  | C: |
|  |  |  | W: |

## Attitudes



## Questioning Sequence: Consecutive Numbers: Algebraic Method

Find 3 consecutive numbers that add up to 60.

| Steps \& Purposes | Instructions \& Questions | Possible Student Responses |
| :---: | :---: | :---: |
| Concepts | - What does "consecutive numbers" mean? <br> - Give |  |
| Skills | - We are going to solve this using algebra. <br> - In algebra method, what is the first thing to think of? |  |
| Metacognition | - Does your answer satisfy the condition? [check] <br> - Can |  |
| Attitude | - Do you |  |

## Q\&A: Consecutive Numbers: Logical Argument

Find 3 consecutive numbers that add up to 60.

| Steps \& Purposes | Instructions \& Questions | Possible Student <br> Responses |
| :--- | :--- | :--- |
| Concepts | • What does "consecutive numbers" mean? |  |
|  | $\bullet$ |  |
| Reasoning | - They add up to 60. |  |
| Metacognition | • Does your answer satisfy the condition? [check] |  |
| Attitude | $\bullet$ |  |

