Assessment for Learning: Using Open-Ended Tasks in the Lower Primary Mathematics Lessons

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Outline

- Introduction: Assessment
- Task Design: Comparisons
- Difference between Closed Task and Open-Ended Task
- Open-Ended Task: Problem with missing data or hidden assumptions
- Designing Short Open-Ended Task

Write your thoughts about the following here…

- ✓What is assessment?

- ✓Are there differences between assessment OF, and assessment FOR learning? Explain

What is Assessment?

Assessment is an integral part of teaching and learning. A well designed assessment can support the development of students' problem-solving ability by assessing progress in the development of mathematics concepts, skills, processes, metacognition and attitudes. Assessment also gives focus to the content that is important and the aims that are to be achieved. It can clarify expectations (e.g. rubrics), check students' prior knowledge (e.g. diagnostic test), provide feedback on students' progress (e.g. formative assessment), and check for mastery (e.g. summative assessment).

[Curriculum Planning and Development Division, MOE, Singapore, 2006]

Assessment for/of Learning

- Assessment ***of*** Learning
  - to ***measure pupil achievement and report evidence of learning***
  - for accountability purposes → grading, ranking and certification
  - tends to be summative in nature
  - carried out at the end of the unit, semester or year
- Assessment ***for*** Learning
  - to ***support classroom learning and teaching***
  - to redirect learning in ways that help pupils master learning goals
  - ***formative*** in nature
  - takes place all the time in the classroom, a ***process*** that is ***embedded in instruction***

[Curriculum Planning and Development Division, MOE, Singapore 2010]

Compare the four: Comment on

Similarities and/or Differences in terms of:

(A) Assessment Objectives

- e.g. concept, skills, processes, attitude, metacognition?

(B) Expected Outcomes

- nature of work displayed

Practice 4 Comparing, Ordering and Number Patterns

1. Fill in the missing numbers.

19 28 82 63

Task 1

(a) 49 = ____ tens ____ ones

(b) 28 = ____ tens ____ ones

(c) 82 = ____ tens ____ ones

(d) 63 = ____ tens ____ ones

(e) ____ is a smaller number than 63 but a greater number than ____.

(f) The greatest number is ____.

(g) The smallest number is ____.

(h) Arrange from the smallest to the greatest number.

____ ____ ____ ____

smallest

Source: Yeo, K. K. J.(2014). Amazing Mathematics: Practice Makes Perfect 1B. Page 9

Task 2

Fill in the blanks.

(a) Write three numbers greater than 27.
They must be smaller than 33.

_____

Arrange these three numbers from the smallest to the greatest.

_____, _____, _____

Smallest

Source: Yeo, K. K. J.(2014). Amazing Mathematics: Practice Makes Perfect 1B. Page 11

Task 3

Count backwards from 30 to 0.
Write the numbers.
Choose three numbers and order them from smallest to largest.

[Taken from Yeo, K. K. (2010). Research on P1 & P2 Authentic Tasks.]

Task 4

Look at the magazines and newspapers.

Cut out at least 5 numbers.

Arrange and paste your numbers in order from the greatest to the smallest on a piece of paper.

Explain your ordering process.

Compare the Four Tasks Again: Comment on…

(C) Task Design

(D) Task Potentials

Open-Endedness?

(E) Task Difficulties

Learning Points for Students from the Task?

Question:

Tasks 1, 2, 3 and 4 – which of these can be classified as "Assessment FOR Learning" task?

Why do AfL?

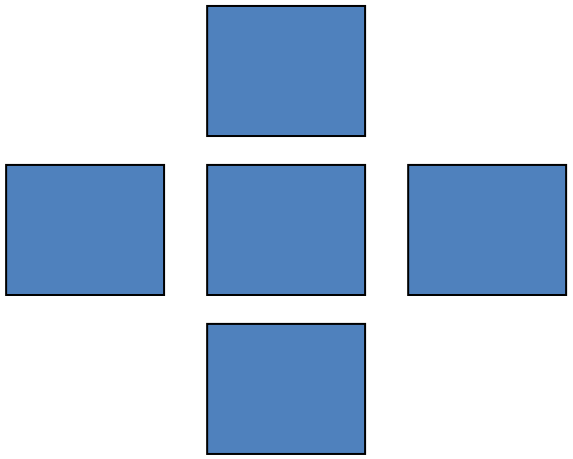
Research says it works

When implemented well, formative assessment can effectively ***double*** the speed of student learning.

Source: William D. (2007), Ahead of the curve: The power of assessment to transform teaching and learning (pp 183-204)

Number Bond – Same Sum

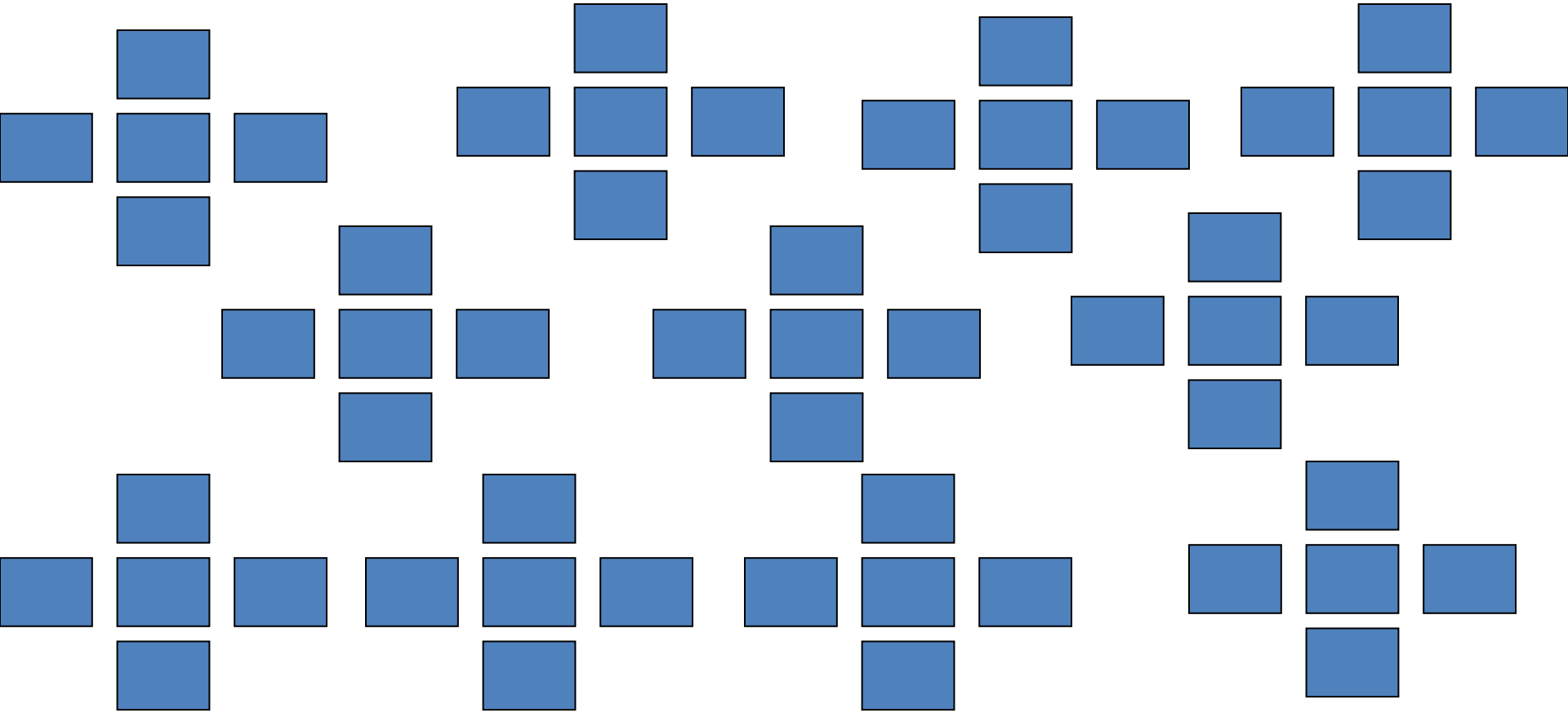
- Give 10 digit cards (0 to 9), put any 5 of the digit cards, one in each square, so that the sum of the row of cards equals the sum of the column of cards.



- Write down as many combinations of 5 digit cards that give equal row and column sums.

Put it up in the class Maths Corner or Notice board

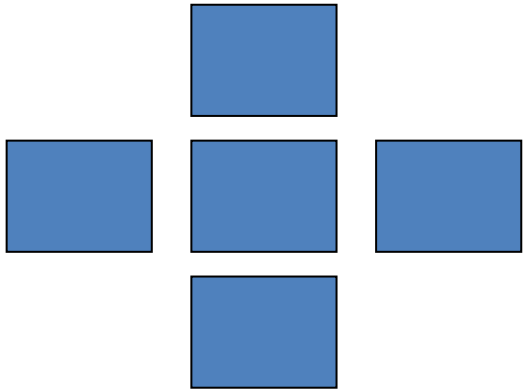
- **Give 10 digit cards (0 to 9), put any 5 of the digit cards, one in each square, so that the sum of the row of cards equals the sum of the column of the of cards.**



- **Write down as many combinations of 5 digit cards that give equal row and column sums.**
- **Find the greatest possible sum and the smallest possible sum**

Number Bond – Same Sum

- Give 10 digit cards (0 to 9), put any 5 of the digit cards, one in each square, so that the sum of the row of cards equals the sum of the column of the cards.



1) **Write down as many combinations of 5 digit cards that give equal row and column sums.**
2) **What is the greatest possible sum of the row of cards?**
3) **What is the smallest possible sum of the column of cards?**
4) **What will be the sum of the row of cards when the 5-digit cards are even numbers?**
5) **What will be the sum of the row of cards when the 5-digit cards are odd numbers?**

Why Open-Ended Tasks?

1. Engage all students in mathematics learning.
2. Enable a wide range of student responses.
3. Enable students to participate more actively in lessons and express their ideas more frequently.
4. Provide opportunity for teachers to probe and enhance students' mathematical thinking.

Difference between Closed and Open-Ended Tasks

Closed Task

There are 27 apples on the table and 23 apples in the basket. How many apples are there in all?

Open-Ended Task

There are some apples on the table and some apples in a small basket. If there are 50 apples altogether, how many apples are on the table?

Explain your answer.

Definition of Open-Ended Tasks

- Considered as ill-structured problems as they lack clear formulation
  - May contain missing data
  - No fixed procedure that guarantees a correct solution

Orton & Frobisher (1997)

Whole Numbers
Primary 2

There are some apples on the table and some apples in a small basket. If there are 50 apples altogether, how many apples are on the table?
Explain your answer.

Difference between Closed-Ended Tasks and Open-Ended Tasks

Closed-ended Tasks	Open-ended Tasks
Routine textbook sums	Non-routine
One expected correct answer	A variety of correct responses
Structured pre-taught procedures	A variety of solution strategies
Require pupils to give a specific and predetermined answer in the form of a single number, figure or mathematical object.	Require pupils to explain concepts and solution processes using various modes: diagram, symbols and words
Allow teachers to check if pupils have learned certain solution methods taught by them	Allow pupils to demonstrate their own ways of approaching and solving the problem

Features of Open-Ended Problems

- No fixed method
- No fixed answer/many possible answers
- Solved in different ways and on different levels (accessible to mixed abilities)
- Encourage divergent thinking
- Offer pupils room for own decision making and natural mathematical way of thinking
- Develop reasoning and communication skills
- Open to pupils' creativity and imagination

Example of a Open-Ended Task

There are some apples on the table and some apples in a small basket.
If there are 50 apples altogether, how many apples are on the table?
Explain your answer.

Higher-Level of Cognitive demands:

- **Pupils to make own assumptions about the missing data**
- **Pupils to access relevant knowledge as they see fit e.g; addition and subtraction within 100, division, etc..**
- **Pupils to display number sense and equal grouping patterns**
- **Pupils to use the strategy of draw a picture, model drawing and guess and check.**
- **Pupils to communicate their reasoning using multiple modes of representation**
- **Pupils to display creativity in as many possible strategies and solutions**

Primary One

Problem with missing data or hidden assumptions

Problems to Solve with Missing Data/Hidden Assumption

A can holds up to 3 tennis balls.

What is the fewest number of cans you would need to hold 16 tennis balls?

Explain your answer.

Possible Solution

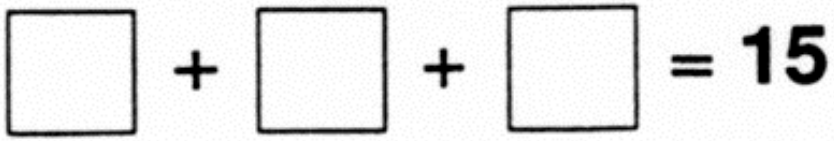
Six cans are needed.

There will be five full cans and one can with only one tennis balls

Pupils may draw diagrams

Whole Numbers
Lower Primary

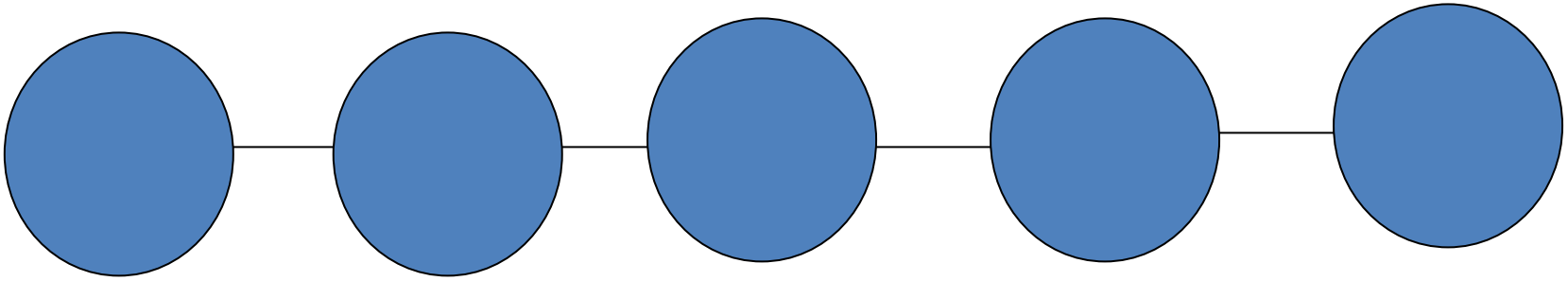
Use the numbers 1 to 9 to complete the following number sentences.



Do it in as many ways as you can.

Problems to Solve with Missing Data/Hidden Assumption

Can you put the numbers 1 to 5 in the circles so that the difference between each pair of joined numbers is more than 1?



Problems to Solve with Missing Data/Hidden Assumption

- Write a story about the heights of the people in your family. Be sure to use the words shorter and taller.
- *Stories will vary*
- *Teachers may integrate with English lesson.*

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Primary TWO

Problem with missing data or hidden assumptions

Problems with Missing Data or Hidden Assumptions

Steve is twice as old as Maria.

If their ages are whole numbers, list five ages that Steve and Maria could be.

If Steve is _____, then Maria is _____

If the sum of their ages is 27, how old is Steve?

Numbers in blanks will vary but the number in the first blank of each sentence must be twice the number in the second blank.

For example, if Steve is 16, then Maria is 8. If the sum of their ages is 27, then Steve is 18 and Maria is 9.

Pupils may use systematic listing or guess and check.

Problems with Missing Data or Hidden Assumptions

Name:_____ Class:_____

Multiplication

_____ groups of _____

Draw diagrams to show how you represent it

Write down two other ways of showing _____ groups of _____ and the answers.

1. _____
2. _____

Open-Ended Task

- Mathematics version of a cloze passage.
- A set of numbers is provided and pupils determine where to place each number so that the situation makes sense.

Complete the story using appropriate numbers

John had _____marbles. He put his marbles in some boxes. He put _____ marble(s) in each box and had _____ marble(s) left. He kept some marbles for himself and gave _____ box(es) or _____ marble(s) to his brother.

Designing Open-Ended Tasks

General considerations

- The Open-Ended Tasks must be mathematically meaningful.
- They must serve important curriculum goals.
- They are often open-ended and contextualized.
- They are equally accessible to all the students.
- They can be completed in a reasonable length of time.

Method 1: Using the Answer

1. Identify a mathematical idea or concept.
2. Think of a closed question and write down the answer.
3. Make up a new open-ended question that includes (or addresses) the answer.

Example

Closed Question

In 784, the digit _____ is in the tens place.

Open-Ended Question

Write five 3-digit numbers that have digit 8 in the tens place.

Activity 4 (LOWER Primary, P1 and P2)

Consider the following:

(a) What is the mathematics focus of the closed question?

(b) Does the new open-ended question have the same mathematical focus?

(c) Is the new open-ended question clear in its wording?

(d) Is the new question actually open ended?

Method 1: Using the Answer

1. Identify a mathematical idea or concept.
2. Think of a closed question and write down the answer.
3. Make up a new open-ended question that includes (or addresses) the answer.

Example

Closed Question

Add $\frac{6}{10}$ **+** $\frac{3}{10}$ **=** $\square$

Open-Ended Question

List *two* fractions that added up to $\frac{9}{10}$

Method 2: Adapting a Routine Textbook Item

1. Identify a mathematical idea or concept.
2. Think of a routine textbook item question
3. Adapt it to make an open-ended question.

Example

Closed Question

Which number is greater 189 or 212?

Open-Ended Question

Write five whole numbers between 189 and 212

Implications

- By giving high level task will not automatically result in pupils' engagement in high level thinking.
- Teachers must have a paradigm shift towards a more process-based approach.
- Teachers' knowledge and understanding of high-order Open-Ended Tasks.
- Teachers' knowledge of classroom-based factors that maintain pupils' high level engagement.

Conclusions

1. Open-Ended Task should serve the purpose of making informed decision to improve teaching and learning.

2. Open-Ended Task should be an integral part of teaching and learning.

2. There is a strong need for teachers to improve mathematics teaching and learning through effective classroom assessment, for example, using Open-Ended Task.