-----Workshop-----
Characteristics of good mathematics test items

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Introduction

- Assessment is critical to the teaching and learning process.
- An accurate assessment of knowledge is necessary for educators to either provide feedback about student's ability or for them to tailor their instruction to meet the needs of their students.
- So, what are the characteristics of good test items?
  - ????
  - ????
  - ????

Characteristics of good test items

- Has a clear test objective

- These are derived from instructional objectives

- Related to content of instruction

- This is from the topics that we teach

- Are pitched at a cognitive level

- This is pre-determined by the test designer

Frameworks of Cognitive Domain

- Bloom's Taxonomy of Critical Thought
  (knowledge level, comprehension level, application level, analysis level, synthesis level, evaluation level)

- TIMSS Assessment Cognitive Framework
  (knowing, applying, reasoning)

- Multi-dimensional approach to understanding – SPUR
  (S-skills, P-properties, U-uses, R-representations)

Blooms Taxonomy of Critical Thought

Bloom's Taxonomy of Critical Thought

KNOWLEDGE LEVEL: Learn the information.

Sample Verbs: Define, find, follow directions, identify, know, label, list, memorise, name, quote, read, recall, recite, recognise, select, state, write.

COMPREHENSION LEVEL: Understand the information.

Sample Verbs: Account for, explain, express in other terms, give examples, give in own words, group, illustrate, infer, interpret, paraphrase, recognise, retell, show, simplify, summarise, translate.

APPLICATION LEVEL: Use the information.

Sample Verbs: Apply, compute, construct, construct using, convert (in maths), demonstrate, derive, develop, discuss, generalise, interview, investigate, keep records, model, participate, perform, plan, produce, prove (in maths), solve, use, utilise.

ANALYSIS LEVEL: Break the information down into its component parts.

Sample Verbs: Analyse, compare, contrast, criticise, debate, determine, diagram, differentiate, discover, draw conclusions, examine, infer, relate, search, sort, survey, take apart, uncover.

SYNTHESIS LEVEL: Put information together in new and different ways.

Sample Verbs: Build, combine, create, design, imagine, invent, make up, present, produce, propose.

EVALUATION LEVEL: Judge the information.

Sample Verbs: Assess, defend, evaluate, grade, judge, measure, perform a critique, rank, recommend, select, test, validate, verify.

TIMSS Cognitive Framework - Knowing

1	**Recall**	Recall definitions; terminology; number properties; geometric properties; and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
2	**Recognize**	Recognize mathematical objects, e.g., shapes, numbers, expressions, and quantities. Recognize mathematical entities that are mathematically equivalent (e.g., equivalent familiar fractions, decimals and percents; different orientations of simple geometric figures).
3	**Compute**	Carry out algorithmic procedures for +, −, ×, ÷, or a combination of these with whole numbers, fractions, decimals and integers. Approximate numbers to estimate computations. Carry out routine algebraic procedures.
4	**Retrieve**	Retrieve information from graphs, tables, or other sources; read simple scales.
5	**Measure**	Use measuring instruments; choose appropriate units of measurement.
6	**Classify/Order**	Classify/group objects, shapes, numbers, and expressions according to common properties; make correct decisions about class membership; and order numbers and objects by attributes.

TIMSS Cognitive Framework - Applying

1	**Select**	Select an efficient/appropriate operation, method, or strategy for solving problems where there is a known procedure, algorithm, or method of solution.
2	**Represent**	Display mathematical information and data in diagrams, tables, charts, or graphs, and generate equivalent representations for a given mathematical entity or relationship.
3	**Model**	Generate an appropriate model, such as an equation, geometric figure, or diagram for solving a routine problem.
4	**Implement**	Implement a set of mathematical instructions (e.g., draw shapes and diagrams to given specifications).
5	**Solve Routine Problems**	Solve standard problems similar to those encountered in class. The problems can be in familiar contexts or purely mathematical.

TIMSS Cognitive Framework - Reasoning

1	**Analyze**	Determine, describe, or use relationships between variables or objects in mathematical situations, and make valid inferences from given information.
2	**Generalize/ Specialize**	Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.
3	**Integrate/ Synthesize**	Make connections between different elements of knowledge and related representations, and make linkages between related mathematical ideas. Combine mathematical facts, concepts, and procedures to establish results, and combine results to produce a further result.
4	**Justify**	Provide a justification by reference to known mathematical results or properties.
5	**Solve Non-routine Problems**	Solve problems set in mathematical or real life contexts where students are unlikely to have encountered closely similar items, and apply mathematical facts, concepts, and procedures in unfamiliar or complex contexts.

What is the SPUR Approach
(Thompson & Senk, 2008; Usiskin, 2003)

- ***Skills*** represent those procedures that students should master with fluency; they range from applications of standard algorithms to the selection and comparison of algorithms to the discovery or invention of algorithms, including procedures with technology.
- ***Properties*** are the principles underlying the mathematics, ranging from the naming of properties used to justify conclusions to derivations and proofs.
- ***Uses*** are the applications of the concepts to the real world or to other concepts in mathematics and range from routine "word problems" to the development and use of mathematical models.
- ***Representations*** are graphs, pictures, and other visual depictions of the concepts, including standard representations of concepts and relations to the discovery of new ways to represent concepts.

Let's examine some test items

TIMSS 2011 – Grade 4 Test Item

Three thousand tickets for a basketball game are numbered 1 to 3000. People with ticket numbers ending with 112 receive a prize.

Write down all the prize-winning numbers.

Prize-winning numbers:_____

What is the test objective?

What is the content domain?

What is the cognitive domain?

Let's examine some test items

TIMSS 2011 – Grade 4 Test Item

Three thousand tickets for a basketball game are numbered 1 to 3000. People with ticket numbers ending with 112 receive a prize.

Write down all the prize-winning numbers.

Prize-winning numbers: **112, 1112, 2112**

What is the test objective?

Use place value, make a systematic list / create a number pattern

What is the content domain?

Number

What is the cognitive domain?

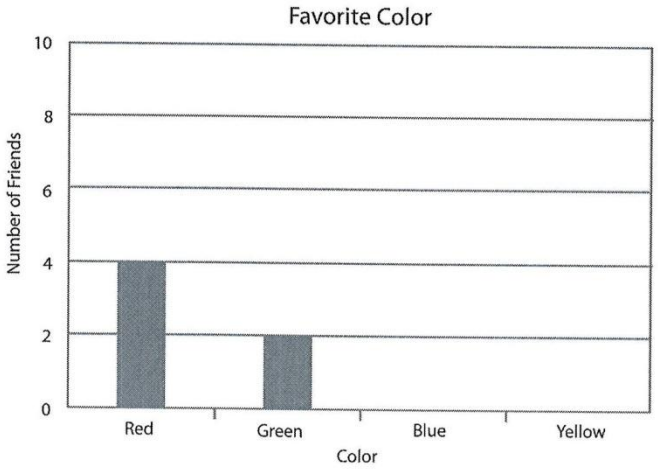
Reasoning

TIMSS 2011 - Grade 4 Item

Darin asked his friends to name their favorite color. He collected the information in the table shown below.

Favorite Color	Number of Friends
Red	4
Green	2
Blue	6
Yellow	7

Then Darin started to draw a graph to show the information. Complete Darin's graph.



- What is the test objective?
- What is the content domain?
- What is the cognitive domain?

TIMSS 2011 - Grade 4 Item

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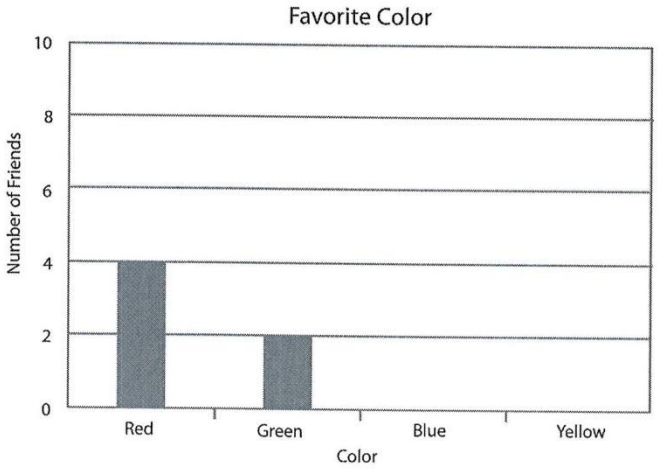


- What is the test objective? To represent data given in a table as a bar graph
- What is the content domain? - Data representation & interpretation
- What is the cognitive domain? - Applying

TIMSS 2011 - Grade 4 Item

4 x ■ = 28

What number goes in the box to make this number sequence true?

Answer:_____

- What is the test objective?
- What is the content domain?
- What is the cognitive domain?

TIMSS 2011 - Grade 4 Item

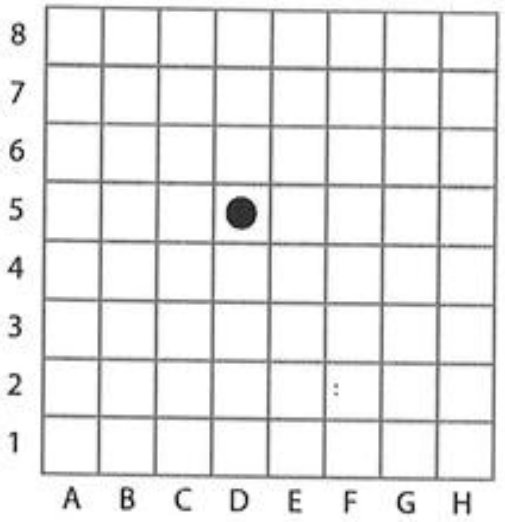
4 x ■ = 28

What number goes in the box to make this number sequence true?

Answer:_____

- What is the test objective? To find how many groups of 4 make 28
- What is the content domain? Number
- What is the cognitive domain? Knowing

TIMSS 2011 - Grade 4 Item



Jamie is playing a board game. His counter is on square D5. Which of these moves would put his counter on square G7?

Ⓐ 2 squares to the right and 3 squares up

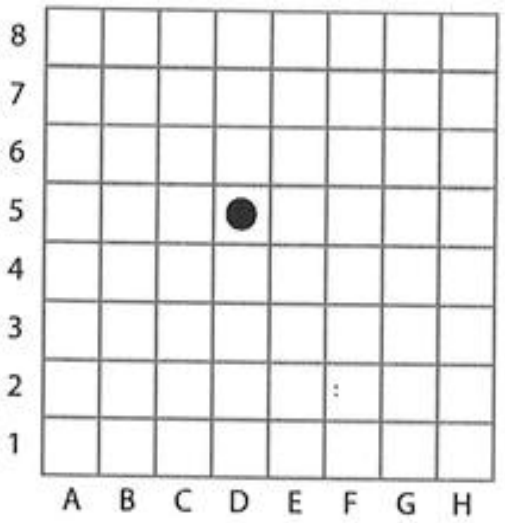
Ⓑ 2 squares to the left and 3 squares up

Ⓒ 3 squares to the right and 2 squares up

Ⓓ 3 squares to the left and 2 squares up

- What is the test objective?
- What is the content domain?
- What is the cognitive domain?

TIMSS 2011 - Grade 4 Item



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- What is the test objective? To describe the movement of a point in a 2-dimensional grid (cartesian plane)
- What is the content domain? Geometry – translation
- What is the cognitive domain? Applying

What is the difference between the two items given?

- Find the area of a rectangle 8 cm by 5 cm?
- Draw a rectangle with an area of 24 square cm.
- Find the average of the following numbers:

  3, 5, 12 and 6
- The average of 4 numbers is 8, what can the four numbers be?

WORKSHOP
Time to write some test items

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Let's do some work!

- Find the sum of 36 and 64.
- Find the difference between 64 and 36.
- Find the perimeter of a rectangle 8 cm by 5 cm.
- 2000 paper clips weigh 6.5 kg. Find the mass of 50 clips.
- Find the volume of a cuboid with length 8cm, breadth 6 cm and height 4 cm.
- Beth's garden has 12 pink and 15 yellow roses. What is the ratio of the number of pink roses to the number of yellow roses?

Group Presentations

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Reflections

- What new knowledge have I gained today?
- What would I do differently the next time I am writing a test item?

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THANK YOU

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