Assessment
&
Undergraduate Mathematics

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5 June 2014 (Singapore)

Err … I'm a JC Teacher …

Why Should I Care!!??

I like math,
but not proofs

I like calculations, but these are so tedious

Like That Lor!

Strengths of Undergraduate Mathematics Education

- Logical Thinking
- Analytical Skills
- Problem Solving

Desirable Learning Outcomes

- Logical Thinking
- Analytical Skills
- Problem Solving
- Domain Knowledge & Ability to Apply
- Computational & IT Skills
- Communication
- Teamwork

Logical Thinking

- Reasoning
- Proofs
- Algorithms

- Formulae
- Final Answer
- "Cleaned-up" Solution

- Process
- First Principles & Definitions

Logical Thinking

A plane π is given by the equation $\mathbf{r} \cdot \begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix} = 3$. Let A be the point $(2, -1, 3)$ and let B be the point $(4, 1, -3)$. Find the length of the projection $\overrightarrow{AB}$ onto the normal of π.

$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$$

$$\therefore \text{ Projection} = \left| \frac{\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix}}{\left| \begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix} \right|} \right| = \frac{36}{\sqrt{30}} = \frac{6}{5}\sqrt{30}$$

Require derivation once in a while?

Logical Thinking

Find the solutions of $3x^2 + 5x - 1 = 0$, giving your answer in surd form.

$$a = 3 \qquad b = 5 \qquad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{37}}{6}$$

Completing the square sometimes?

Logical Thinking

Show that $\lim\limits_{x \to 5} 2x + 1 = 11$.

$$\text{If} \quad |x-5| < \tfrac{1}{2}\varepsilon,$$

$$\text{then} \quad |(2x+1) - 11|$$

$$= |2x - 10|$$

$$= 2|x-5| < 2 \cdot \tfrac{1}{2}\varepsilon = \varepsilon.$$

Like Magic! How would we know it should be this?!

More Process Than Answer

Analytical Skills, Problem Solving, Domain Knowledge & Ability to Apply

"Guide students to learn mathematics in a way that helps them to better understand its place in society: its meaning, its history, and its uses."

Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004, The Mathematical Association of America, 2004

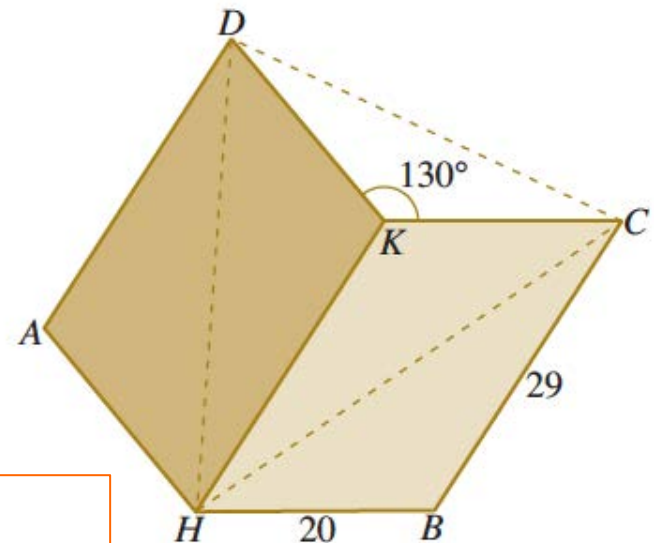# Analytical Skills, Problem Solving, Domain Knowledge & Ability to Apply

- Content Knowledge
- Applications
- Modelling

- Fake Applications
- Unrealistic Examples

- Realistic Scenarios
- Open-ended Problems

Analytical Skills, Problem Solving, Domain Knowledge & Ability to Apply

In the diagram, $AHKD$ and $BCKH$ are two rectangular pages of a café menu, $HB = 20$ cm and $BC = 29$ cm. The page $AHKD$ is turned about the edge HK and makes an angle of 130° with the page $BCKH$.

Calculate

(a)   the length of CH,

(b)   the distance between C and D,

(c) $\angle CHD$.

Why Would I Care About These?

Analytical Skills, Problem Solving, Domain Knowledge & Ability to Apply

Your class is organizing a game of chance at your school's funfair. Each player pays $1 to roll three dice. The total score is recorded and there are cash prizes for high scores.

(Relevant Scenario)

(a) A classmate has proposed a payout of $10 for a score of 18, $5 for a score of 17, 16 or 15, and $2 for a score of 14, 13 or 12. Do you think this is sufficient incentive for people to play the game, and for you to make enough profits?

(Content Knowledge & Application)

(b) Would you have any proposals to improve upon the suggestion in (a)?

(Open-ended)

Analytical Skills, Problem Solving, Domain Knowledge & Ability to Apply

Get Real!
Think!

Analytical Skills, Problem Solving, Domain Knowledge & Ability to Apply

"Useful mathematical skills include a broad training in the core of mathematics, statistics, mathematical modeling, and numerical simulation, as well as depth in an appropriate specialty ...

Computational & IT Skills

"Computational skills include, at a minimum, experience in programming in one or more languages … sufficient to … bridge the gap between theory and practical implementation."

Mathematics in Industry, SIAM, 2012

Computational & IT Skills

- Use of Software
- Programming
- New Technologies

- Small Unrealistic Examples
- Gap between Theory & Reality

- "Large" Problems from Real-life
- Modelling Problems
- Projects

Computational & IT Skills

Hisham took a bank loan of $400,000 to buy a flat. The bank charges an annual interest rate of 3% on the outstanding loan at the end of each year. Hisham pays $2,000 at the beginning of each month until he completes the payment of his loan (plus interest).

(a) Find the minimum number of years Hisham needs to repay his loan.

(b) If the interest rate increases to 5% per annum at the end of the third year and stays at that rate thereafter, how long would Hisham need to repay his loan?

Relevant & Realistic Scenario

Good Use of Spreadsheet

Allows for Variations

Computational & IT Skills

A tea company claims that a new tea can lower cholesterol level by as much as 20 mg/dL. Over a period of time, a random sample of 60 volunteers drink this new tea. The decrease in cholesterol level, x in mg/dL, is summarised by

$$\sum x = 1020, \quad \sum x^2 = 21164.$$

Could have started with raw data & have a more complete problem

60 is ridiculously small for any commercial testing!

Computational & IT Skills

(The education minister) has already started to act, by looking "quite seriously" at introducing computer programming in schools to prepare people for the new technological age.

The Straits Times
28 Jan 2014

Communication

"... skill in communicating is commonly listed as the most important quality employers seek in a prospective employee. However, many students expect mathematics classes to be wordless islands ..."

Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004, The Mathematical Association of America, 2004

Communication

- Written Exposition
- Oral Presentation
- Explanation

- "Focus Only on the Math"

- Thought Process
- Ideas
- Communicating the above

Communication

Find the solutions of $3x^2 + 5x - 1 = 0$, giving your answer in surd form.

$$a = 3 \qquad b = 5 \qquad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{37}}{6}$$

Sorry, but what are these?

Communication

How many distinct 6-digit numbers can be formed by using the digits 0, 2, 2, 3, 3 and 5 exactly once each?

$$\# \; 6\text{-digit} \; \text{numbers}$$

$$= \frac{5!}{2!} + \frac{5!}{2!} + \frac{5!}{2!\,2!}$$

$$= 60 + 60 + 30$$

$$= 150$$

What are these?
Where do they come from?
How?

Communication

Show by mathematical induction that

$$(2^2)(1) + (3^2)(2) + \cdots + (n+1)^2(2^{n-1}) = (n^2+2)2^n - 2.$$

$n = 1$

$LHS = (2^2)(1) = 4$

$RHS = (1^2+2)\,2^1 - 2 = 6-2 = 4 = LHS.$

Assume OK for $n = k$.

$n = k+1$ $LHS = (k^2+2)2^k - 2 + ((k+1)+1)^2 (2^{(k+1)-1})$

$= (2k^2 + 4k + 6)\,2^k - 2$

$= (k^2 + 2k + 1 + 2)\,2^{k+1} - 2$
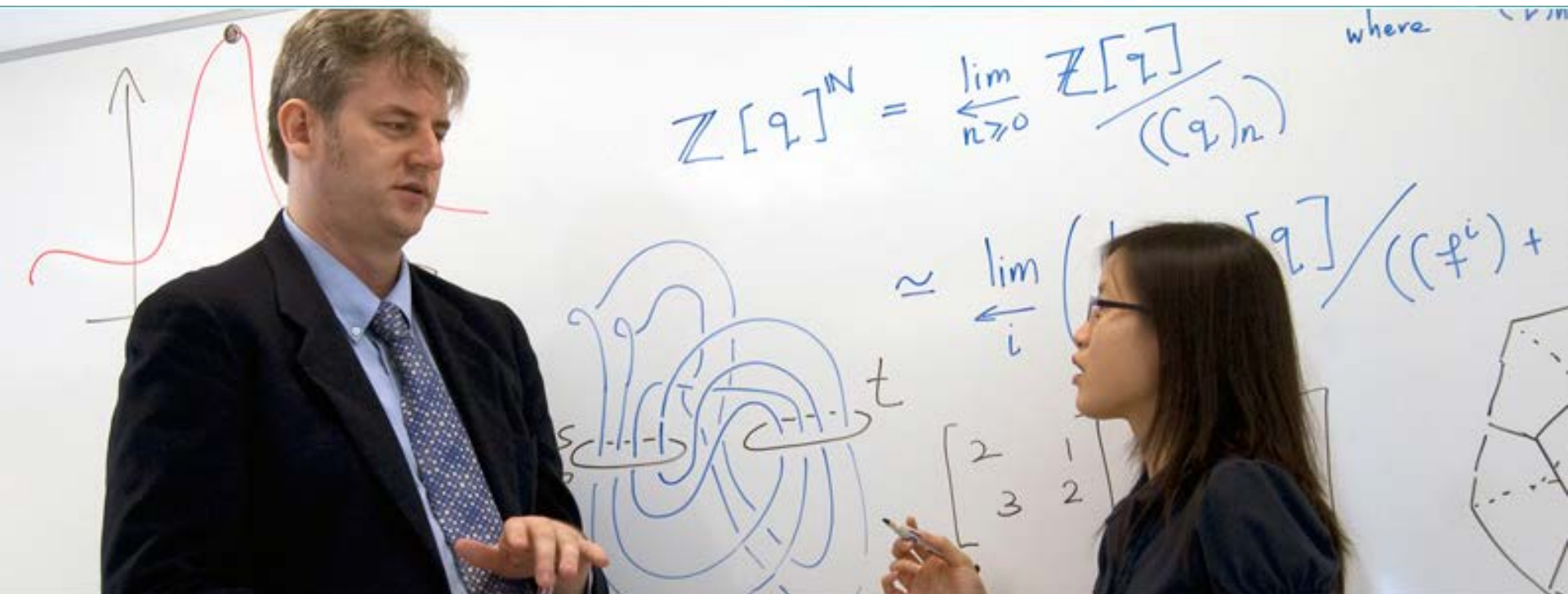
$= ((k+1)^2 + 2)\,2^{k+1} - 2$

$\therefore$ Proved.

Communication

Explain your ideas!!!

Teamwork

Group Projects

Conclusion

- Process vs Answer
- First Principles vs Formulae
- Impact on Real-life Problems
- Power of Modelling & IT tools
- Communication of Ideas
- Working with Others
- Old Habits Die Hard!!!

Thank You