AME-SMS Conference 2014
5th June 2014 (Thursday)

Assessment in Mathematics
http://math.nie.edu.sg/ame/amesms14/

Association of Mathematics Educators

Singapore Mathematical Society

Probing and assessing students during student-teacher interactions

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Assessment in Mathematics

Documenting, usually in measurable terms, knowledge, skills, attitudes …

it can focus on the individual learner, the learning community (class, workshop, or other organized group of learners), the institution, or the educational system as a whole.

http://en.wikipedia.org/wiki/Educational_assessment

Assessment in Mathematics

Gathering information…

- to help determine what students know and can do → to decide **what** to teach
- to help determine how students learn → to decide **how** to teach

http://www.gov.mb.ca/fs/imd/edu_assess.html

Assessment in Mathematics

Documenting
Measuring

Accountability
Comparisons
Policy

Determining knowledge and learning

Monitor learning
Decisions on what / how to teach

Assessment in Mathematics

Pervasiveness of assessment

Teaching $\cong$ Assessment

- Documenting, measuring
- Probing students
- Making decisions
  - → mathematical space
  - → goals
  - → curriculum, texts, tasks, sequencing
  - → resolution of dilemmas

Assessment in Mathematics

Pervasiveness of assessment

Teaching $\cong$ Assessment

- Documenting, measuring
- Probing students
- Taking decisions

Can these be learned?developed? taught? And what would it entail?

- Probing students

- Taking decisions

What is it?

Why is it important?

To whom?

How may it be learned?

Learning to listen

What is ‘listening to students’?

(beyond mere physiology and beyond passiveness)

Giving careful attention to what students say and do, trying to understand it, its possible sources and its entailments.

It should include:

- Detecting and creating opportunities in which students are likely to engage in expressing freely their mathematical ideas;
- Questioning students in order to uncover the essence and sources of their ideas;
- Analyzing what one hears making the intellectual effort to take the ‘other’s perspective’ in order to understand it on its own merits; and
- Deciding in which ways to productively integrate students’ ideas.

Why is it important?

- Constructivism

- Caring, receptive, empathic conversations

- Internalized technique for learning and for interpersonal relationships

- For ourselves, the listeners

"Thinking ourselves into other persons leads us to reflect about our own relationship to mathematics"

Jahnke, H.N.: 1994, 'The historical dimension of mathematical understanding – Objectifying the subjective', in J.P. da Ponte & J.F. Matos (Eds.), *Proceedings of the 18th International Conference for the Psychology of Mathematics Education*, vol. 1, Lisbon, Portugal, pp. 139-156.

“Confessions” of two mathematicians

“…what surprised me most was that I learnt mathematics. Actually, a lot of it.”

Aharoni, R.: 2003, ‘What I learnt in primary school’, Invited talk at the 55th British Mathematics Colloquium (BCM), University of Birmingham, Great Britain, available at http://www.math.technion.ac.il/~ra/education.html

“At first I was surprised – How could I, an expert in geometry, learn from students? But this learning has continued for twenty years and I now expect its occurrence. In fact as I expect it more and more and learn to listen more effectively to them, I find that a larger portion of the students in the class are showing me something about geometry that I have never seen before.”

Henderson, D.W.: 1996, ‘I learn mathematics from my students -- multiculturalism in action’, For the Learning of Mathematics 16(2), 46-52.

The challenges of ‘listening’

- “Packaged” knowledge

“I have observed, not only with other people but also with myself…that sources of insight can be clogged by automatisms. One finally masters an activity so perfectly that the question of how and why is not asked any more, cannot be asked any more, and is not even understood any more as a meaningful and relevant question.”

Freudenthal, H.: 1983, *The Didactical Phenomenology of Mathematical Structures*, D. Reidel, Dordrecht, p.469.

“Highly practiced cognitive and perceptual processes become automatized so there is nothing in memory for experts to “replay”, verbalize, and reflect upon”

Nathan, M.J. and Petrosino, A.: 2003, ‘Expert blind spot among preservice teachers’, American Educational Research Journal, 40(4), pp.905-928.

Challenge: Unpacking, unclogging, even “unlearning”

The challenges of ‘listening’

- “Packaged” knowledge

- “Decentering” capabilities

“Making sense of children ideas is not so easy. Children use their own words and their own frames in ways that do not necessarily map into the teacher’s ways of thinking.” “The ability to hear what children are saying transcend disposition, aural acuity, and knowledge, although it also depends on all of these.”

Ball, D. L. and Cohen, D. K.: 1999, ‘Developing practice, developing practitioners - Toward a practice-based theory of professional education’, in L. Darling-Hammond and G. Sykes (Eds.), *Teaching as the Learning Profession. Handbook of Policy and Practice*, Jossey-Bass, San Francisco, CA, pp. 3-32.

Challenge: to adopt the other’s perspective, to ‘wear her conceptual spectacles’ (keeping away as much as possible our own perspectives), to test in iterative cycles our understanding of what we hear, and possibly to pursue it and apply it for a while.

The challenges of ‘listening’

- “Packaged” knowledge

- “Decentering” capabilities

- Different ways

“Evaluative” listening: listening against the background of an expected answer. It implies a virtual ‘measurement’ of the ‘distance’ between the student present state of knowledge and the desirable goal, providing straightforward feedback and applying subsequent ‘fixing’ strategies.

“Attentive” listening: I targeted at where the students stand, the sources of their idiosyncratic ideas and their potential as a source for learning.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education*, 28(3), 355–376.

The challenges of ‘listening’

- “Packaged” knowledge

- “Decentering” capabilities

- Different ways

Challenge: distinguish between the two modes, learn to emphasize attentive over evaluative listening

The challenges of ‘listening’

- “Packaged” knowledge
- “Decentering” capabilities
- Different ways
- Timing

Challenge: learn to reflect in real time

Proposed goals:

- Recognize that listening is important
- Recognize that listening is productive
- Develop listening an enacted and often practiced competency
- Cope with the challenges

Questions:

What kind of “curriculum for professional learning” and “pedagogy for professional development” should be developed?

What kind of experiences should be orchestrated in order to develop desirable listening capabilities?

How such a curriculum may work in teacher courses and to what extent the goal of learning to listen can be attained?

Some proposed answers:

- History of mathematics
- Students unexpected answers
- Lesson Study
- A Video Project in brief

History of Mathematics

Learning History of mathematics

and

Learning to listen

Imply

Learning **to interpret**

a) In order to fully understand the ideas behind a historical (mathematical) source we need a similar kind of unpacking and decentering needed for listening to students

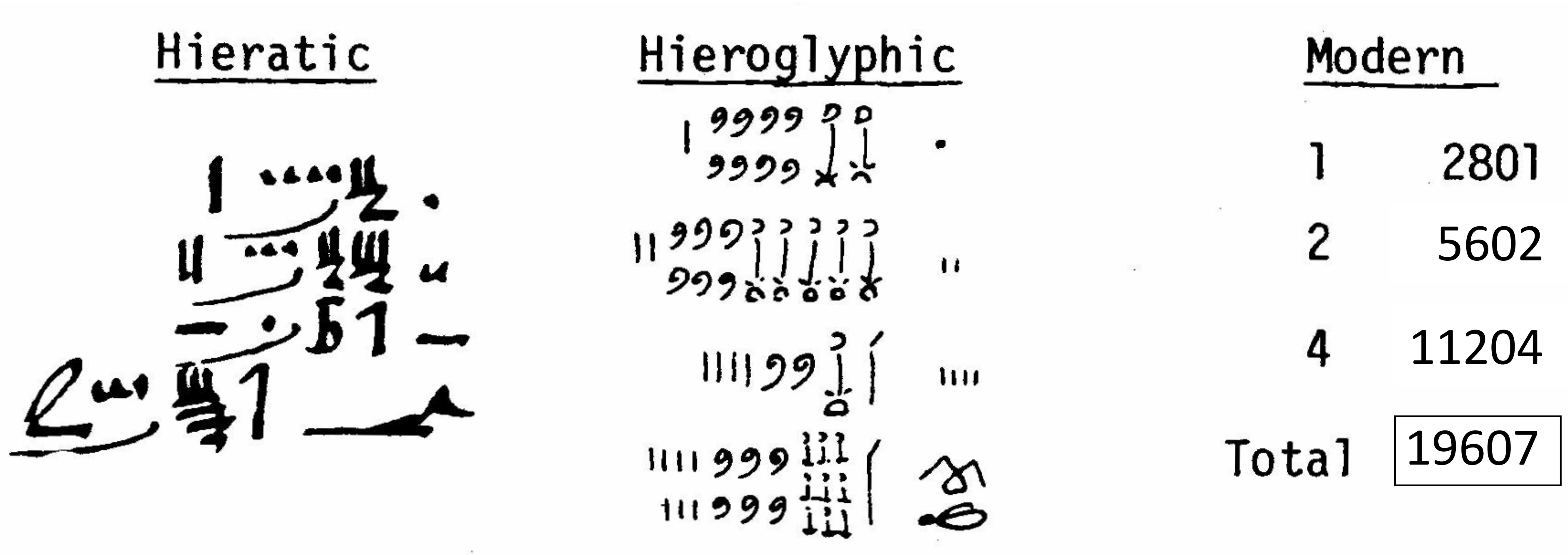
b) Suitable historical context and source

c) “Hermeneutic tools”

Hermeneutic tools

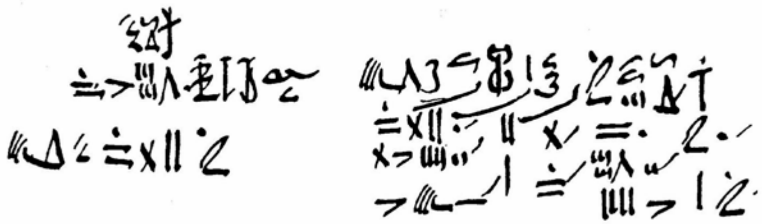
- Parsing the source
- Posing questions to oneself (or to a peer) : What is really written? Why did the author write in such a way? What are the hidden assumptions? If this text says A, and A entails B – where is B in the text? This questioning may lead to adopt the ‘writers’ perspective’;
- Paraphrasing parts of the text in our words and notations;
- Summarizing partial understandings, locating and verbalizing what it is still to be clarified;
- Contrasting different pieces for coherence.
- Corroborating with a recursive process (e.g. applying our understandings to similar texts, examples or problems).

EXAMPLE

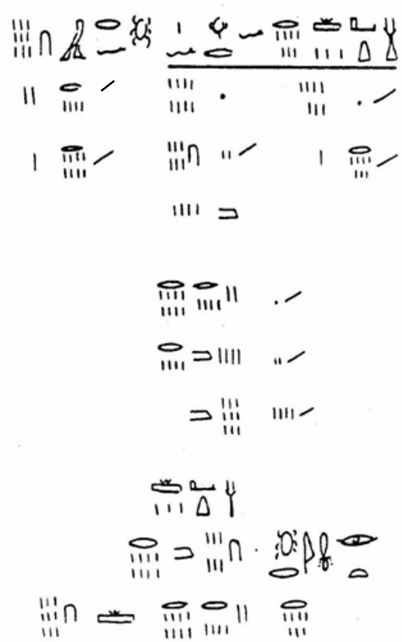

Hieratic
Hieroglyphic
Modern
1 2801
2 5602
4 11204
Total 19607

The following is the solution to Problem # 24 (Peet, 1970). The omissions are for the purpose of this task.

Hieratic*



Hieroglyphic**



A quantity whose seventh part is added to it becomes 19.

/ 1	7
/ $\frac{1}{7}$	- - -
1	8
/ 2	- - -
$\frac{1}{2}$	- - -
/ $\frac{1}{4}$	- - -
/ $\frac{1}{8}$	- - -
/ 1	$2 + \frac{1}{4} + \frac{1}{8}$
/ 2	- - - - -
/ 4	- - - - -

The doing as it occurs:—The quantity is $16\frac{1}{2} + \frac{1}{8}$

one seventh is - - -

Total - - -

- Generate questions such that the answers would help you understand the solution process to the linear equation.

- Write, in modern notation, the equation corresponding to the first sentence, and solve it.

- Reproduce and summarize the method of solution and explain it.

- Write down the solution of the problem, as it would have appeared in the Papyrus (but in modern notation), if the first trial number had been 14 instead of 7.

Supporting questions

- ‘dictionary’ translations of notations
- description of a calculation (surface and deep structure)
- solution with our method
- parsing of the text, ‘local’ understanding of small parts
- ‘pasting pieces together’ towards a global understanding
- applying the Egyptian method to a new problem
- clarifying previous knowledge, hidden assumptions, conditions of applicability
- investigating mathematical properties, generality.

A. Arcavi, & M. Isoda, 2007. “Learning to listen: from historical sources to classroom practice” *Educational Studies in Mathematics*, 66(2), 111-129.

Students unexpected answers

Find fractions between two given fractions

“Sir, you don’t have to go to all that trouble to find a fraction between two fractions,

all you have to do is add the tops and the bottoms”

“No, that’s not the way it’s done”

$$0 < \frac{a}{b} < \frac{c}{d} \qquad \frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$$

Sherzer, L. (1973) “McKay’s Theorem” *The Mathematics Teacher*, 66(3), 229-230.

Arithmetic Sequence

$$S_{10} = 65$$

$$a_{10} = 20$$

$$a_1 = ?$$

$$d = ?$$

$S_{10} = 65$

$a_{10} = 20$

$a_1 = ?$

$d = ?$

$$S_n = na_1 + \frac{n(n-1)}{2}d$$

$$A_n = a_1 + (n-1)d$$

$$65 = 10a_1 + 45d$$

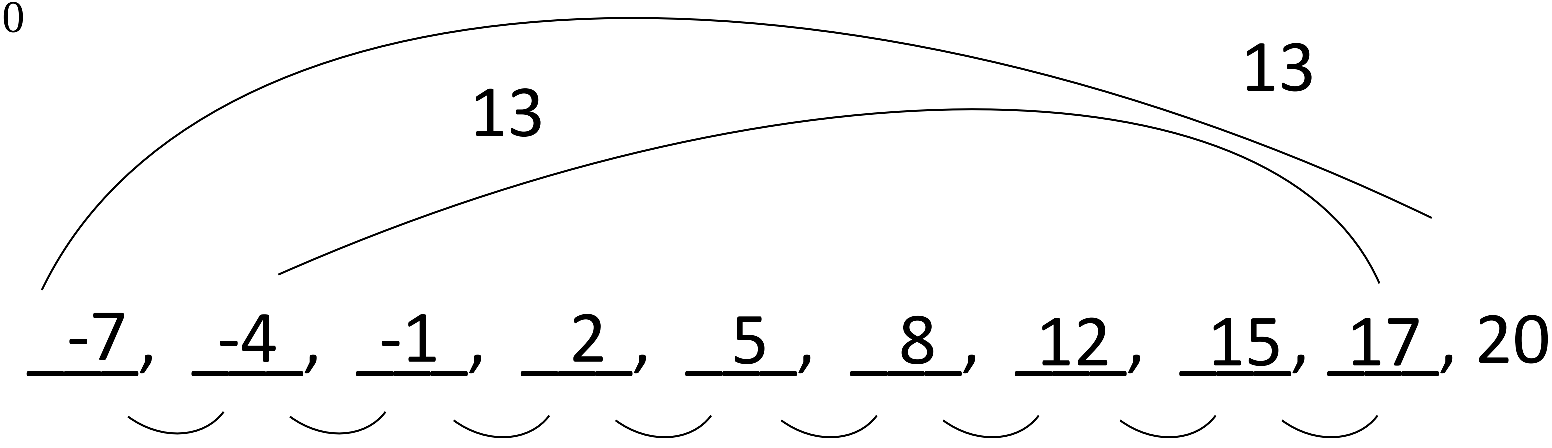
$$20 = a_1 + 9d$$

$S_{10} = 65$

$a_{10} = 20$

$a_1 = ?$

$d = ?$



$$\frac{65}{5} = 13$$

Five arcs are 65, so one arc is 13

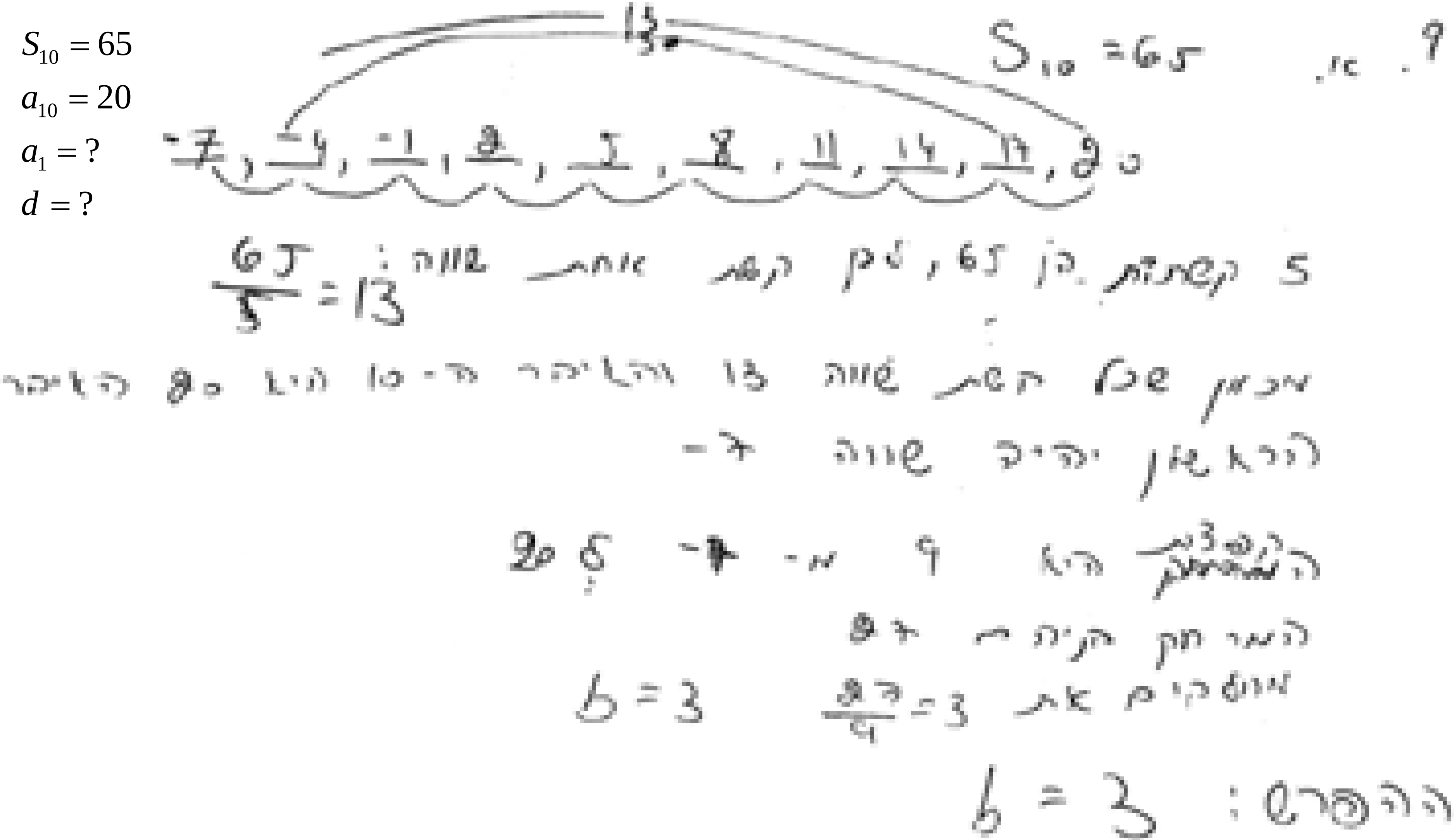
So the first one must be -7

There are 9 jumps

There distance is 27

Each jump is 3

$S_{10} = 65$
$a_{10} = 20$
$a_1 = ?$
$d = ?$



Ahuva, a fifth grade teacher, wanted to assess whether her students know how to find the whole when a part is given.

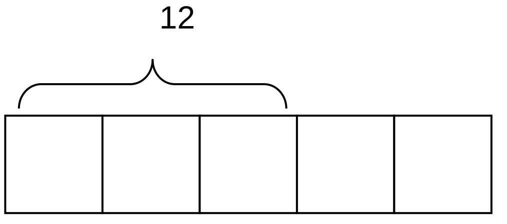
She administered her students a quiz that included the following problem:
"**3/5 of a number is 12, what is the number explain your solution**."

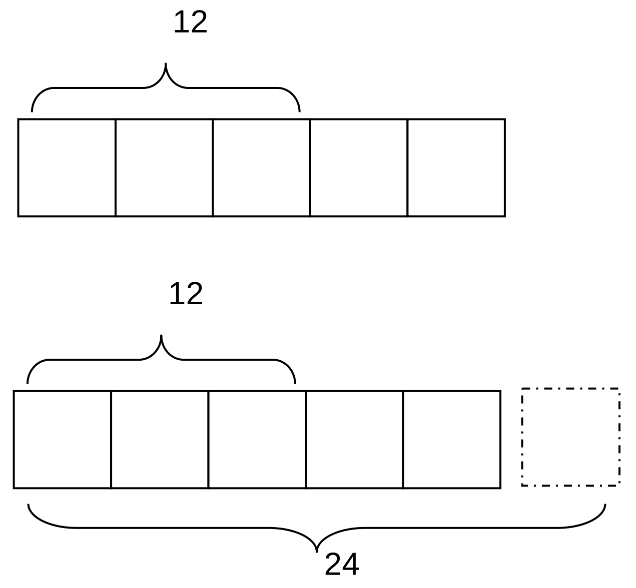
Ron wrote:

"12 * 2 = 24, 24:6 = 4, 24 – 4 = 20"

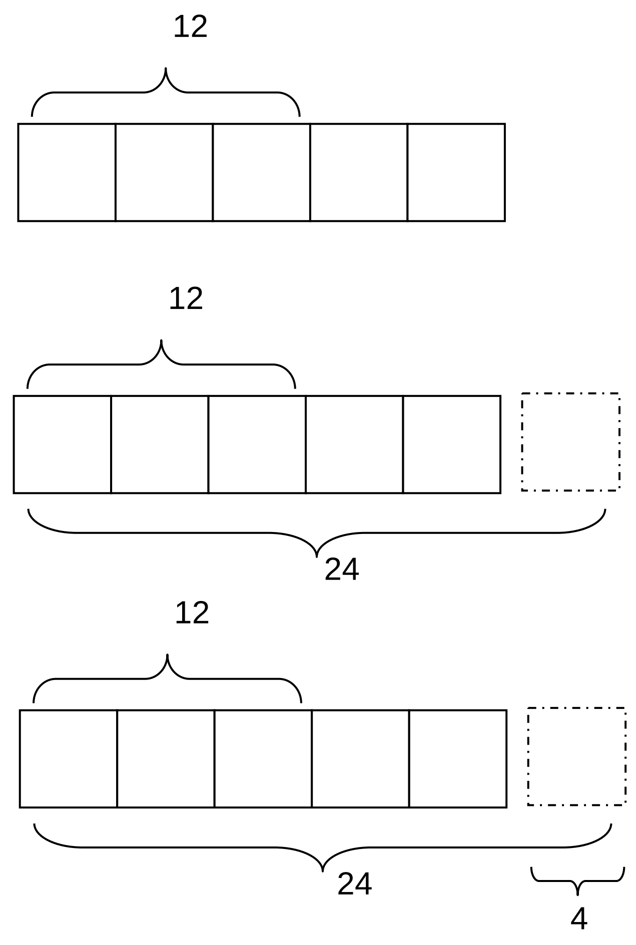
- Is Ron's solution correct?

- What would be your assessment of Ron's knowledge?"


12



12
12
24


12
12
24
12
24
4

Some proposed answers:

- History of mathematics

- Students unexpected answers

- Lesson Study

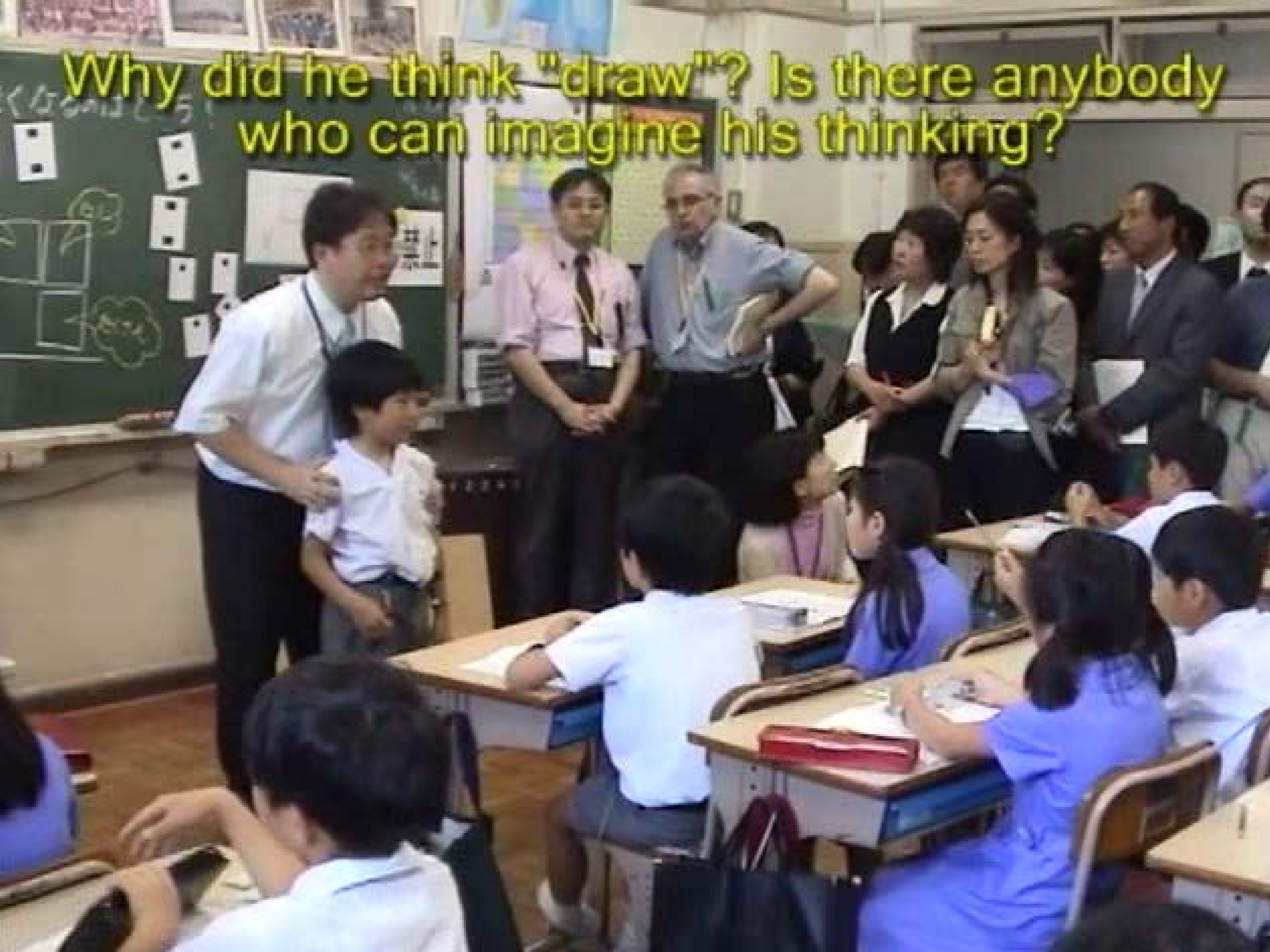
Role models – Lesson Study


91
×
1
×
9
こたえが大きくなるのはどっち？


Who said "draw", now?
こたえが大きくなるのはどっち?
9
1


Do you understand why he said 'draw'?
こたえが大きくなるのはどっち?
9
1


I see. But, it can't be a draw.


Why did he think "draw"? Is there anybody
who can imagine his thinking?

Some proposed answers:

- History of mathematics
- Students unexpected answers
- Lesson Study
- A Video Project in brief

THE TRUMP FOUNDATION
קרן טראמפ

VIDEO-LM
Viewing, Investigating and Discussing Environments Of Learning Mathematics

עדש"ה
עמיתים דנים בשיעורי המתמטיקה

מכון ויצמן למדע
WEIZMANN INSTITUTE OF SCIENCE

Teacher-centered discussions around videotaped mathematics lessons: What can be learned?

The framework of analysis

Mathematical and meta-mathematical ideas

The framework of analysis

Mathematical and meta-mathematical ideas

Given the topic of the lesson, there is a range of relevant concepts, procedures and ideas.

- What can we include in this “span of ideas”?
- Which of these ideas, or others, did the teacher bring forward in the lesson?
- Which meta-mathematical notions were evident within the lesson?

The framework of analysis

Mathematical and meta-mathematical ideas

Explicit and implicit goals

The framework of analysis

Explicit and implicit goals

The rich span of mathematical ideas around a given topic enables various teachers' **choices** of the goals they wish to pursue within a lesson.

One of the reasons that lessons given by different teachers on the same topic do not resemble one another, is that these teachers derived different goals from the range of mathematical ideas relevant to the topic.

Explicit and implicit goals

Exercise: Ascribing goals

What can be gained?

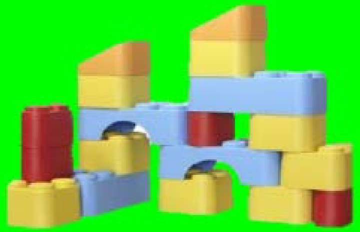
- Promoting the skill of articulating goals
- Enhancing awareness to the fact that there are alternative (sometimes even competing) goals to teaching a certain mathematical subject

The framework of analysis

Mathematical and meta-mathematical ideas

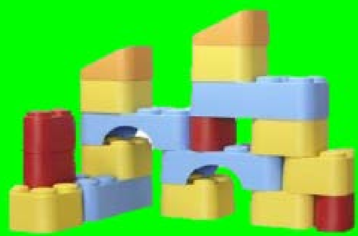
Explicit and implicit goals

Tasks and activities



The framework of analysis

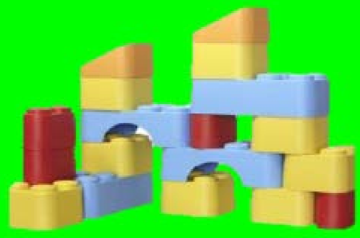
Tasks and activities



The means by which the teacher’s goals are fulfilled.

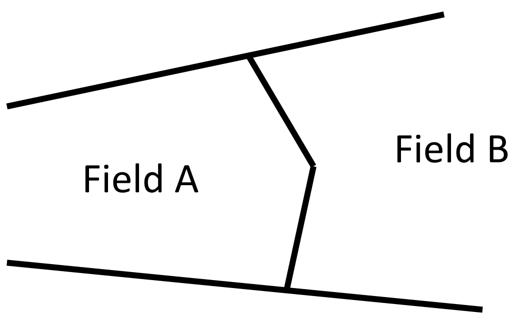
The video enables teachers to watch a “task in action”, how it is implemented, the nuances in introducing it and how the teacher addresses the students’ reactions.

Tasks and activities



Example:

Junior high school geometry



Mathematical ideas

goals

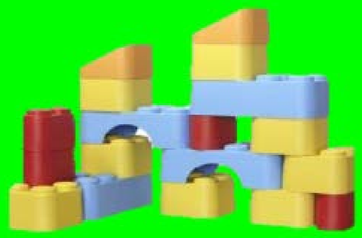
Tasks

The framework of analysis

Mathematical and meta-mathematical ideas

Explicit and implicit goals

Tasks and activities



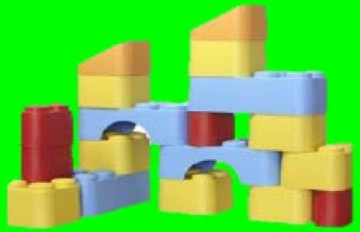
Interactions with students

The framework of analysis

Mathematical and meta-mathematical ideas

Explicit and implicit goals

Tasks and activities



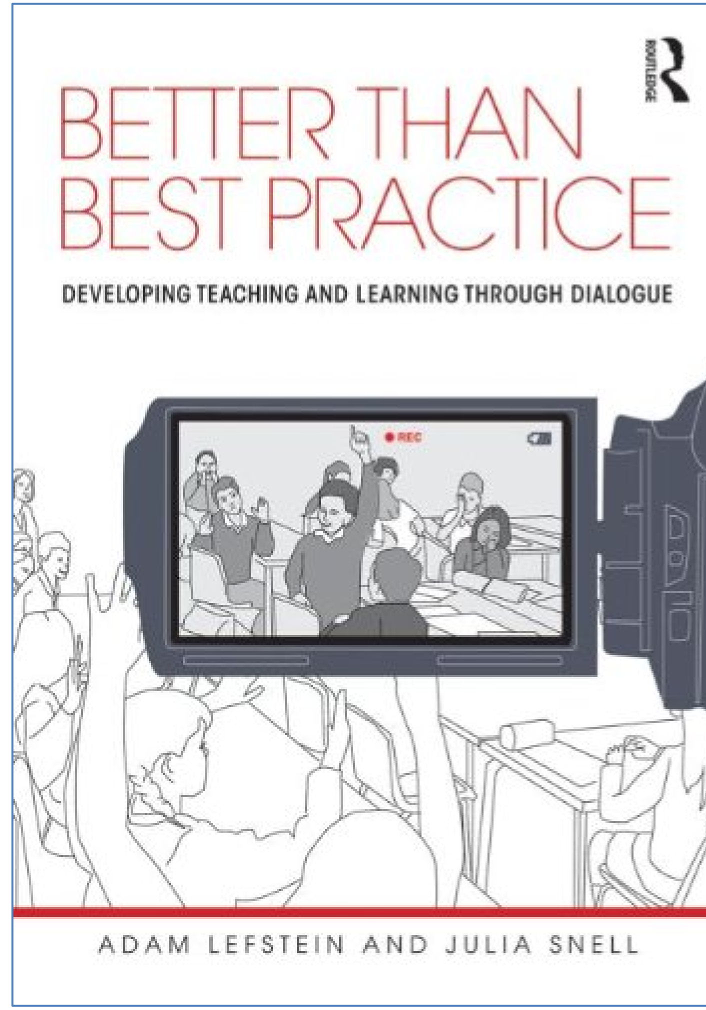
Dilemmas and decision-making processes

Interactions with students

Dilemmas and decision-making processes

“We all teach fine, the point is to understand what you’re doing, why do you do it, and do you really agree with what you decided to do. If you agree, fine, but if you don’t – go and fix it! But be aware of what you did. I never thought about that”

(a participant in the summer course)

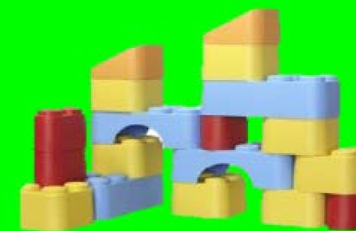


The framework of analysis

Mathematical and meta-mathematical ideas

Explicit and implicit goals

Tasks and activities



Dilemmas and decision-making processes

Beliefs about mathematics teaching

Interactions with students

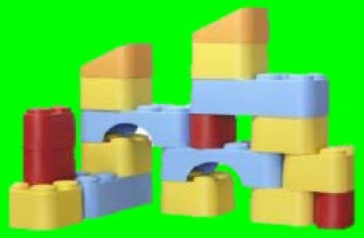
The framework of analysis

Beliefs about mathematics teaching

Attention to questions such as

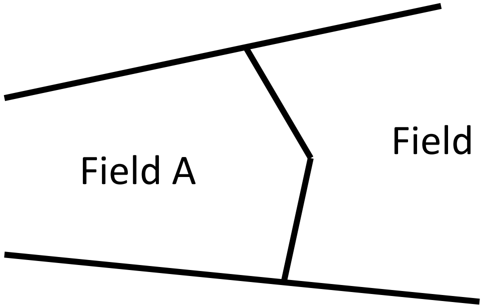
- What may be the filmed teacher’s views about the nature of mathematics as a discipline?
- How does the teacher perceive his/her role?
- What may be his/her ideas about what “good mathematics teaching” is?
- What does s/he think about the students’ role as learners?

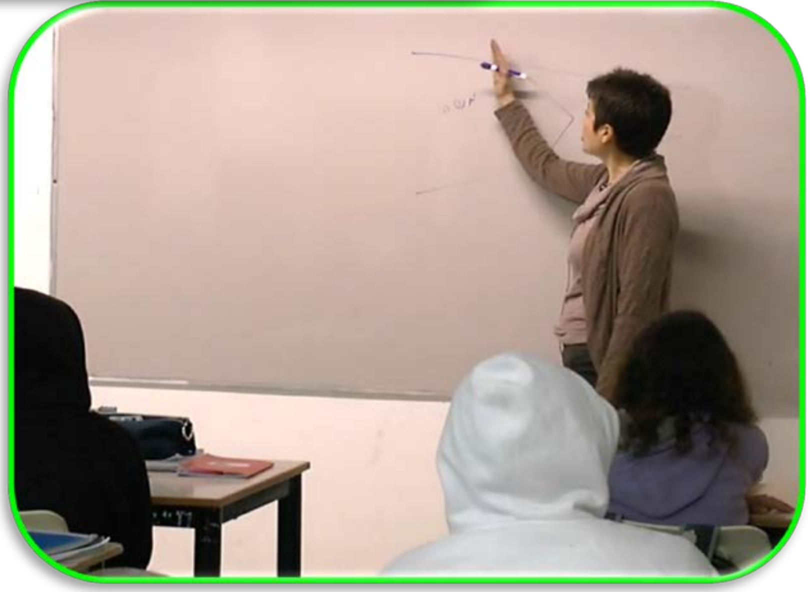
Tasks and activities



Example:

Junior high school geometry





http://wws.weizmann.ac.il/conferences/video-lm2014/lectures

Assessment in Mathematics

Probing students – Decision making

Learning to listen, what and how?

Proposals for teacher development

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