Reflecting on calculation: when drilling becomes fulfilling

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Think of a number …

- Think of a number (single digits are easiest)

Reflection to see patterns

- Patterns are evidence of underlying mathematical structure
- Patterns of same-ness; patterns of difference

Different → same

- Different starting numbers → same relationship

Same ⟶ different

60 ÷ 1 = 60

Expectations

- Teacher – generated (deliberate or accidental)

Creating expectations deliberately

1 ÷ 9 = 0.111 …

Creating student-generated expectations accidentally

- 23 ÷ 10 = 2.3

Long division

- 222222 ÷ 13 =

Expectations

- Generated from past experience

Reflection on the effects of actions

- Patterns in the effects of actions can be evidence of mathematical meaning

Patterns in equivalent fractions

$$\frac{1}{2} \sim \frac{1}{2} = \frac{1}{2}$$

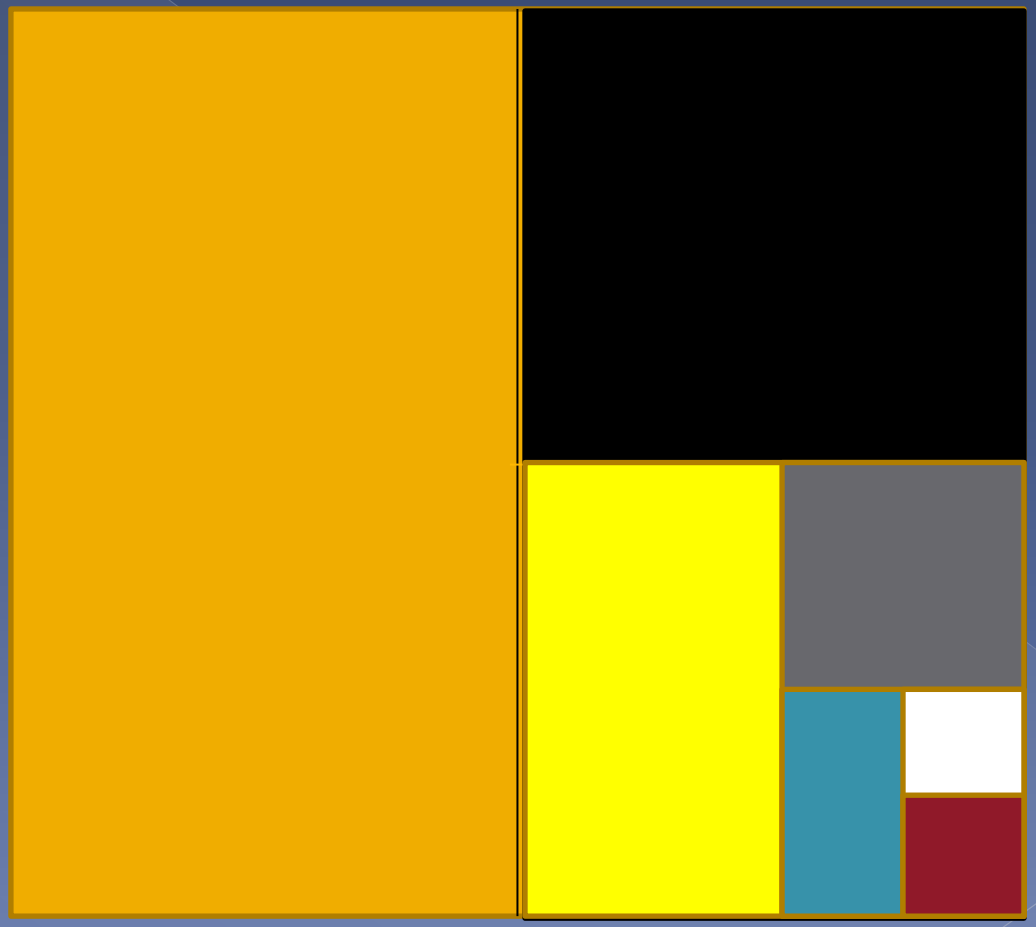
Patterns in fractions

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

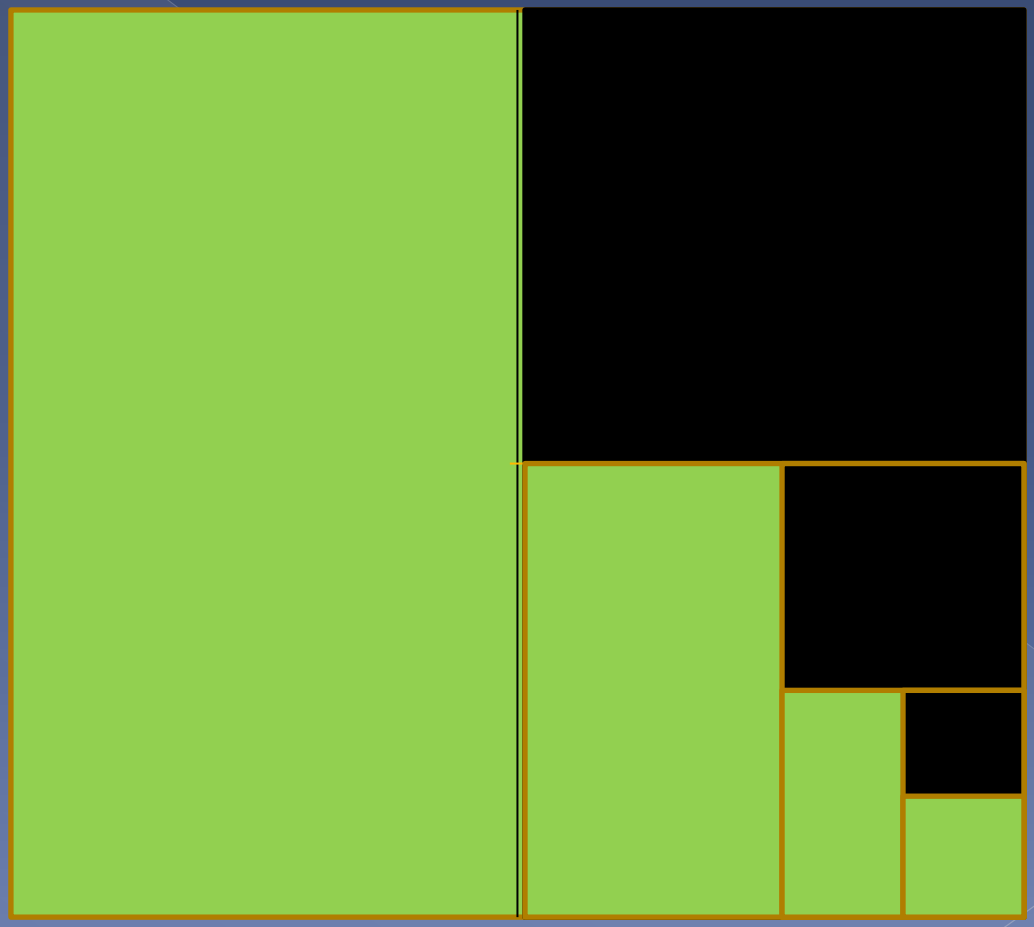
Patterns in Fractions

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$$

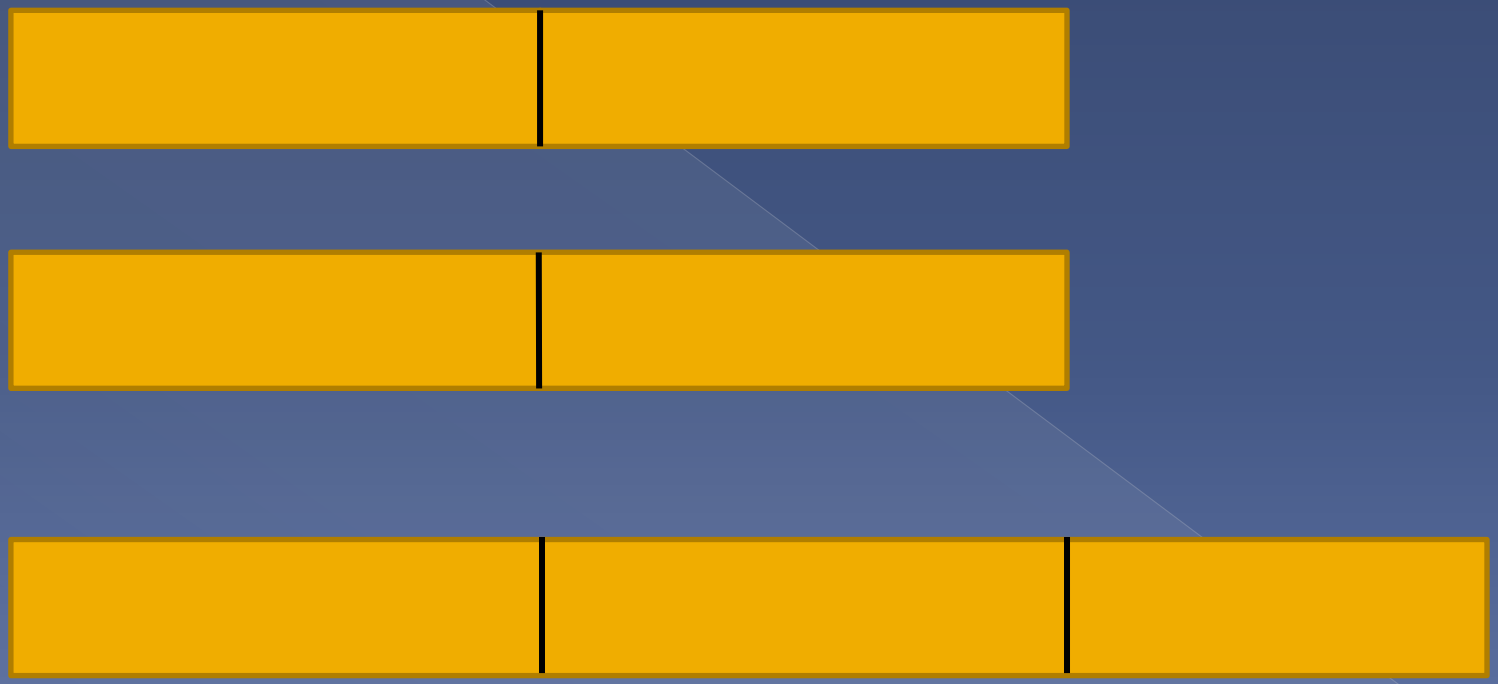
Patterns in fractions



Patterns in fractions



Reflecting on actions: stretching and shrinking



Reflection/analysis of a whole piece of work

- Reflection on a whole piece of work can reveal similarities and differences with other objects

A structured collection of questions

- Think of a number
  - Multiply it by 12 and divide by 3
  - Now multiply the output by 3 and divide by 12

Reflection on a whole object

- Two triangles enlarged additively and multiplicatively

Reflection on the learning process

- Reflection can trigger awareness of change and growth in knowledge
  - What do I know now that I did not know an hour ago?
  - What could I do now that I would not have thought of an hour ago?
  - How did I come to know these new things?

Therefore learners need to:

- reflect on what they have produced, to look for patterns
- reflect on the effects of their actions, so they can make predictions, check their work, invent short cuts
- know that reasoning from experience can lead to false conjectures so we need logical reasoning too

Therefore teachers need to:

- provide sequences of tasks that reveal mathematical patterns: different/same
- encourage learners to reflect on the effects of their actions, so they can make predictions, check their work, invent short cuts
- provide tasks that show how reasoning from experience can sometimes be misleading

Summary

- Well-structured tasks and prompts can make reflection a habit
- Methods and concepts of the multiplicative relationship interact
- Reflection can transform knowledge about calculation
- Engaging the reflective mind can enrich understanding