Mathematics Lessons Stimulating Reflective Learning: Japanese Perspective

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Mathematics Education in Japan
# National Course of Study (CS)

## History of CS after the World War II

<table>
<thead>
<tr>
<th>Year</th>
<th>Theme</th>
</tr>
</thead>
<tbody>
<tr>
<td>1947</td>
<td>Experiences in Daily Life</td>
</tr>
<tr>
<td>1958</td>
<td>System of Contents in Mathematics</td>
</tr>
<tr>
<td>1968</td>
<td>Mathematics Modernization</td>
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<tr>
<td>1977</td>
<td>Basics in Mathematics</td>
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<tr>
<td>1989</td>
<td>New Vision for Scholarship</td>
</tr>
<tr>
<td>1998</td>
<td>Zest for Living and Integration of Learning</td>
</tr>
<tr>
<td>2008</td>
<td>Mathematical activity</td>
</tr>
</tbody>
</table>

- Mathematical thinking
- Central concept
- Unification, Extension
- Advantage of mathematics
- Mathematical problem solving
- Mathematical activity
Current objectives of mathematics for elementary school

Through mathematical activities students will

(1) acquire **basic and fundamental knowledge and skills** about numbers, quantities and geometric figures,

(2) cultivate their ability to consider phenomena from their daily lives with foresight to generate and organize logical thinking steps to follow through, and to **represent** those phenomena,

(3) recognize the **joy of mathematical activities** and the merit of mathematical manipulation, and

(4) foster a disposition to willingly **make use of mathematics in daily life and studies**. (MEXT, 2008)
Basic policies for the latest revision in mathematics

- Mathematical activities must be further enriched so that students will acquire fundamental and basic knowledge and skills, develop the ability to think and express mathematically, and increase their motivation to learn.

- Curriculum should be organized by incorporating repetitions of some topics according to the developmental stages and grade levels of students.
• In mathematics, there are many ways to express ideas, using numbers, mathematical expressions, figures, tables, or graphs. It is important to enrich learning activities where these methods are used to teach how to think, explain, and express one's ideas.

• To increase the motivation for learning, curriculum should incorporate appropriate repetitions of topics, and to help students apply what they have learned and acquired to activities in their daily life, in the study of other subjects, or in learning more advanced mathematics.

• To enrich mathematical activities, instructional activities should be described in each grade.
Mathematics textbook

• In the elementary and lower secondary schools, one company publishes one textbook for each grade-level. **Six textbooks companies publish six elementary textbooks** for the elementary schools.

• Editing textbooks involve a number of people engaged in mathematics education. The editing process generally starts three years before the textbooks used. They first decide the principles and points of emphasis, then write and revise drafts, and submit the final draft to MEXT for examination. **MEXT approval is imperative in order to authorize the text for educational use.**

• **Textbook selection is conducted in each district** for elementary and lower secondary schools under the supervisory board of education in local government.
Professional development of teachers

• By law, teachers are required to develop their professional knowledge and skills by engaging in research and training. Training opportunities are guaranteed for public teacher training courses at the national, prefectural, and municipal levels.

• Universities offer a variety of programs for in-service teacher education. Graduate school programs are open to schoolteachers as well.

• Teachers voluntarily organize groups for more private meetings. There are many such types of meetings at the municipal and prefectural levels.

• Importance of teacher training within schools has also been recognized.
Lesson Study

- **Lesson study** is a method to improve teaching practices in which a group of teachers jointly plan, teach, discuss, revise, and re-teach a lesson by carefully observing real classroom situations.

- The traditional lesson study system in different communities of teachers is employed at the individual schools and local/national levels.

- The processes and outcomes of lesson study are often open to the public. The results of these studies are also presented in a variety of research meetings and conferences. Many schools affiliated with universities have exhibition research lessons, post-lesson conferences, and presentations on current research activities. Teachers come together from all over Japan; for them, it is a rich opportunity to observe lessons and exchange information.
Teaching Mathematics in the Classroom: Problem Solving Approach
Japanese lesson pattern
(Stigler & Hiebert, 1999)

- Reviewing the previous lesson
- Presenting the problem for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the major points

Structured Problem Solving
Critical roles of teacher

a) *Posing a problem* (about 5 min.)

Teachers present the problem of the day by using different teaching materials to elicit children interest. The teacher asks questions to make children understand the problem and remember previous knowledge to solve the problem.

- Are there any keywords or numbers that help you to understand the problem?
- Which part do you think is different from the problem in the previous lessons?
- How can you solve this problem?
- What operations can you use to find the answer?
b) During problem solving by children (about 15 min.)

The teacher walks around the room listening to children’s ideas, offering hints to children who are unable to find ways to solve the problem, encourage children to think of various ways for solving the problem, and make a mental note of several children who made the expected approaches to the problem.

Which solution method should be presented first?

How should I summarize based on the discussion of different solution methods?

In this period, another purpose of the teacher is analyzing children’s thinking to solve the problem.
c) *During presenting the solutions (about 10 min.)*

The teacher invites children to present their method of solving the problem on the blackboard, by selecting children in a particular order for encouraging those children who found simple methods and showing children’s ideas that will be discussed later. In some cases, even an incorrect method may be presented.

Teachers promote children to explain their thoughts using logical steps with words (“first, “next”, because”), manipulative materials, graphs, and mathematical expressions. In this period, teachers praise children’s thinking and ask children these questions:

- Who can explain this solution?
- How did you think?
- Do you agree with Kaori’s way of solving the problem?
- Do you understand Taro’s explanation?
- Who solved the problem in the same was as Risa?
- Can you solve the problem differently?
d) **Comparing and discussing solution methods** *(about 10 min.)*

In this period the teacher encourages children to realize the differences and similarities between their own ideas and others. Teachers do not tell children if their method presented is right or wrong, instead they prefer to let children discuss amongst themselves the validity of different solutions always looking for a better way of thinking: e.g., faster, easier, and precise.

- Where is the same and where is different?
- Which solution method is simpler?
- Which is the easiest to understand?
- Which solution method can be used for another problems?
e) **Summary of the lesson (about 5 min.)**

The teacher leads children to reflect on what they have learned during the lesson and encourages children to write in their notebook about their own ideas and friends’ ideas discussed during the lesson.

The teacher summarizes the lessons to help children achieve the objective of the lesson and writes conclusions on the blackboard using children’s words by asking this question:

*What did you understand from today’s lesson?*
Important Aspects of Classroom Activities for the Purpose of Reflective Thinking and Learning
Stimulating reflective thinking and learning through structured problem solving

• “Kizuki” (noticing) and “Toi” (question): Teacher plans lesson to let children notice something new to them. Here, it is important that children pose their own questions and explore the questions by making connections with the knowledge they have learned. Teacher needs to suggest the points of reflection and to encourage children to ask such points by themselves.
Stimulating reflective thinking and learning through structured problem solving

• “Neriage” (kneading up): Reflection takes place naturally and substantially in the collaborative work with other people in class. Powerful individual learning becomes possible in the dynamics of learning as a classroom community.

How to organize the order of presentation or the discussion after presentation.
Stimulating reflective thinking and learning through structured problem solving

• “Yosa” (advantage of mathematical idea): Mathematics makes important contribution to reflective thinking. Teacher plans to teach mathematical concepts by providing the situation in which mathematical ideas behind the concepts become powerful means of reflection.
There are four rabbit cages, A, B, C, and D. Investigate how to order cages A, B, C, and D according to how crowded they are.

<table>
<thead>
<tr>
<th></th>
<th>Area (㎡)</th>
<th>Number of rabbits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>D</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>
Pedagogy for reflective thinking and learning

- Clear lesson objectives
- Use of open-ended problems to achieve lesson objectives
- Purposeful shift between individual and collective problem solving
- Organizing central questions to children (Hatsumon)
- Use of blackboard writing for reflection (Bansho)
Examples from Practice-Based Researches
Setting a foothold (Niimura et al., 2009)

Is the area of this figure a half of the rectangle?
Do you think you want to find the area of the third quadrilateral?

- It looks a little bit difficult.
- It looks like it is not a half of the rectangle. What a surprise!
- How can I find the area?
- Can I find the area by using the method for rhombus?
Different ways of kneading up children’s solutions

- Four principles of organizing discussion (Koto et al., 1998, 2010)
  - Examination of the validity of each solution
  - Examination of the relationship among solutions
  - Comparison of different solutions from the point of view of relevance, generality, or utility
  - Looking back the solutions from self-evaluation by each child
Are you impressed to know that the same idea for rhombus is used for finding the area of the third quadrilateral?

- I was surprised to know that different figures can be solved by the idea for rhombus.
- It is interesting. It is amazing.
- Why can similar ways be used?
- I want to use the idea for the other quadrilaterals.
Teaching with notebooks

• “Bubbling” method (Kameoka, 2009)

• Let children express their thinking using bubbles (Use notebook and blackboard writing)
• Understand children’s thinking and make appropriate support
• Encourage children’s reflection and self assessment by using bubbles
• Nurturing classroom atmosphere to accept different thinking and learn each other

Is this 360° ... ?

Are there any other ways?
The 1st stage of the notebook

**Space for thinking and working**

**Space for friends’ ideas and diary of learning**

Friends’ ideas

Today’s problem

What I thought about today’s study:

(i) What I found
(ii) What I think is good about my friends
(iii) What I want to do more
(iv) What I worked hard
(v) What I felt and wondered
The 2\textsuperscript{nd} stage of the notebook

Notebook toward mutual learning and portfolio

<table>
<thead>
<tr>
<th>Learning Task</th>
<th>Memo of friends’ idea</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space for work</td>
<td>Space for thinking</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Diary of learning</td>
</tr>
</tbody>
</table>
Stimulating reflection through interaction

Analysis of solutions by other people: e.g.,

- (Given a mathematical expression by A), let’s explain how A solved the problem.
- (Given solutions by B and by C), explain the idea of C to make the solution simpler.
- (Given an idea by D), do you think what D says is correct or not? Why?
- (Given a solution by E), E’s solution is not correct. Let’s find the mistake and correct it.
Stimulating reflection through interaction

• Encouraging mutual learning by generating and solving problems (Sudo & Furuhashi, 2012)

• Let’s write a mathematical expression that shows how to find the total number of marbles.
Generating problems: Individual work
Solving the problem: Pair work

Analyzing the mathematical expression of your partner:

Clue 1:

Clue 2:
Final Remarks
• In the lesson, it is important for the teacher to provide opportunity for reflection and to propose points and methods of reflection. It is also important that the teacher connect the result of reflection to the new problems. Children need to experience both the process of reflection and the advantage of reflective thinking.

• The form of structured problem solving is not the solution for fostering reflective thinking and learning. What is important is the awareness of the teacher that reflective thinking is so fundamental in learning mathematics and developing such attitude empowers children in their future lives.
Thank you very much
References

• English Translation of the Japanese Mathematics Curricula in the Course of Study, March, 2008 (Copying and posting is an infringement of copyright. www.globaledresources.com)
• Mathematics 1-6 set, DVD-ROM. (English translation of elementary school textbook published by Tokyo Shoseki. Copyright c 2011 by Tokyo Shoseki Co., Ltd., Tokyo)
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• Aguirre, K. (2012). (2012). *Comparison of mathematics teaching methodology in first and second grade of elementary education between Japan and Peru.* In Utsunomiya University International In-Service Teacher Training Program Report (pp. 55-74). Graduate School of Education, Utsunomiya University.


Use of open ended problems

• *Japanese mathematics lessons* include a variety of open ended problems (problems that have several correct answers). This approach provides children with the opportunities to think and apply their own knowledge to solve problems.

Let’s find calculations that satisfy □ ÷ △

4 ÷ 1  8 ÷ 2  36 ÷ 9  40 ÷ 10

Open ended problems provide children with the opportunities to think and apply their own knowledge to solve problems. Those problems also provide them with the opportunities to reflect their thinking.  

Pedagogy
Recording children’s thinking for reflection