Making mathematics more meaningful through reflective learning

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Nurturing reflective learners

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- Nurturing (The Australian Oxford Dictionary)

  noun

  1. nurture the process of bringing up or training (esp children); fostering care
  2. nourishment
  3. sociological factors as an influence on or determinant of personality

  Verb

  1 .bring up, rear 2. nourish

- Reflective learners

  Reflection, reflective thinking, reflective learning, metacognition…For us as teachers as well as our students

Ginger Meggs thinks about numbers

The West Australian 12 April 2008

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Thinking

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If one learns from others but does not think, one will be bewildered. If on the other hand, one thinks but does not learn from others, one will be in peril.

Confucian Analects 11.15 (Fisher, p.155)

As a child said ' I learn maths best when I have someone to talk it at' (Fisher, p.166)

Fisher, R. (2008) *Teaching thinking: Philosohical Enquiry in the Classroom.* London: Continuum International Publishing Group

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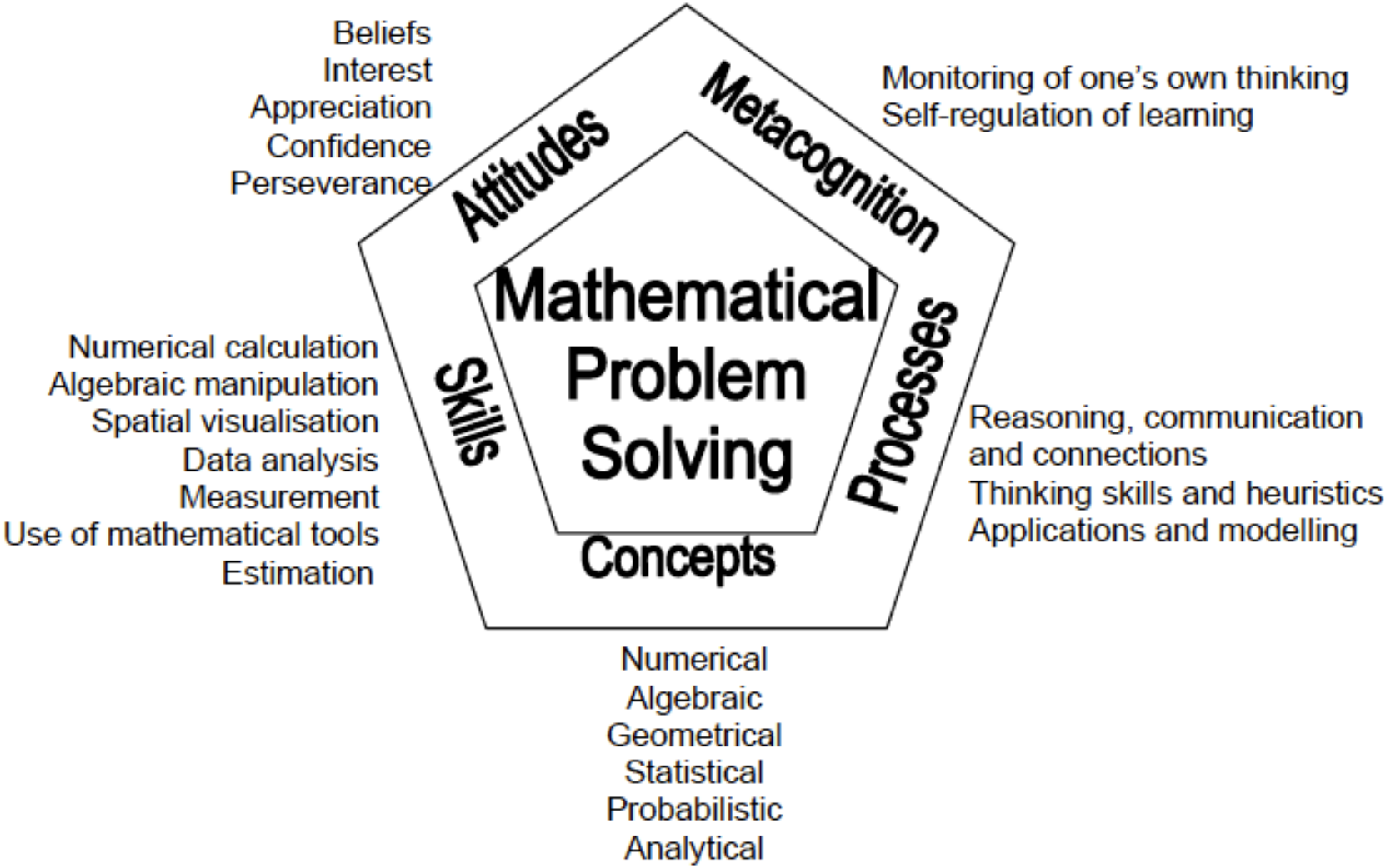

Secondary Mathematics in Singapore

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- Mathematical problem solving is central to mathematics learning. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems.
- The development of mathematical problem solving ability is dependent on five inter- related components, namely, *Concepts, Skills, Processes, Attitudes and Metacognition.*

  MOE (2006)

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Problem solving as part of the mathematics curriculum

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- Mathematical problem solving is a critical component of a Mathematics Education and should not only be a goal in mathematics learning but also a way to acquire new content (NTCM, 2000)
- When problem solving is appropriately and innovatively infused into the curriculum, the teacher can greatly enhance the experience of the students (Polya, 1973)

Cited in Quek et al (2007). Mathematical problem solving for integrated programme students: Beliefs anf performances in non-routine problems Earcome 4

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Secondary syllabus:Metacognition

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The provision of metacognitive experience is necessary to help students develop their problem solving abilities. The following activities may be used to develop the metacognitive awareness of students and to enrich their metacognitive experience:

- Expose students to general problem solving skills, thinking skills and heuristics, and how these skills can be applied to solve problems.
- Encourage students to think aloud the strategies and methods they use to solve particular problems.
- Provide students with problems that require planning (before solving) and evaluation (after solving).
- Encourage students to seek alternative ways of solving the same problem and to check the appropriateness and reasonableness of the answer.
- Allow students to discuss how to solve a particular problem and to explain the different methods that they use for solving the problem.

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Secondary syllabus:Metacognition

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Metacognition, or “thinking about thinking”, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

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Reflection & reflective learning

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- Reflective thinking ..is a part of the critical thinking process that refers specifically to the process of analyzing and making judgements about what has happened.

  McDonald & Dominguez (2009, p. 46)

- Dewey (1933) suggests that reflective thinking is an active, persistent, and careful consideration of a belief or supposed form of knowledge, and the further conclusions to which that knowledge leads. Learners are aware of and control their learning by actively participating in reflective thinking – assessing what they know, what they mneed to know, and how they bridge the gap during learning situations.

  Cited in McDonald & Dominguez (2009, p. 46)

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Reflection and Learning

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- Reflection plays a critically important role in mathematics learning and that just completeing tasks is insufficient. We must encourage students to reflect on their activity –ask them to justify a solution which will promote reflection. Ask in a small group or whole class: will it work?

  (Wheatley, 1992)

- In mathematics learning reflection is characterised by distancing oneself from the action of doing mathematics

  (Sigel, 1981)

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From a science perspective

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Reflection helps students improve their basic communications skills both orally and in written form. It assists each student in self-examining his or her learning experience and leads to the development of better critical-thinking skills. Students integrate their knowledge through the experience of reflection and begin to build a strong, basic understanding of underlying concepts and theories. The process of discussing and reflecting on learning experiences assists students in exchanging critical ideas and insights about the information being shared. Deliberate and guided reflection leads to expanded learning and understanding. **In short, reflection creates meaning.**

McDonald & Dominguez (2009, p. 49)Reflective writing. The science Teacher

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Reflection: how can we promote it?

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Feedback:

- Assignment/test/exam feedback, written and oral –encourage students to not just look at the mark
- In pairs or group work in problem solving, investigations or practical work
- Teacher-student
- Student-teacher
- Student-students

- Talking and writing about what you have learnt and how you learnt it

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Reflection: cognitive questions

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Start with the teacher asking cognitive questions in a supportive manner:

- How do you know it works?
- How is this one different?
- Is that surprising to you?
- Will it always be true?

Ask questions when things are going well, so a question from the teacher does not necessarily mean they are wrong.

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Reflection: cognitive questions

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Shift students to reflecting about their learning:

- How did I get that result?
- How did I solve that problem?
- What process did I use??
- What did I discovered for the first time?
- What did I find that surprised me?
- What happened reminds me of….?
- What am I wondering about now?

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Reflection and learning content

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Research on teaching of specific content of whole and rational number systems by Kramarski, Mevarech & Arami (2002). They had 2 groups for the treatment group they gave metacognitive instruction

- Comprehension
- Connections
- Strategies
- **Reflection**

This group outperformed the control in both standard tasks and problem solving; they were better able to reorganise, process information and justify reasoning.

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Reflection and learning: summary

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- For new material reflective learning occurs:
  - when learning is relatively ill-structured or is challenging to the learner
  - when the learner is intent on meaningful learning/wants to understand the material for him/herself in a manner that is meaningful to her/him (takes a deep approach)
- When new material is not involved the learner may try to develop understanding on the basis of what is known already and reconsider exoisting ideas that may be meaningful to further develop meaning.

Moon, 2004, p.87

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Links to problem solving

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Lester (1983, 231-232) defined a problem as a task for which:

1. The individual or group confronting it wants or needs to find a solution
2. There is not a readily accessible procedure that guarantees or completely determines the solution
3. The individual or group must make an attempt to find a solution

Lester, F. K. (1983)

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Problems

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- Krulik and Rudnik (1980) … the individual sees no obvious means or path to a solution
- Schoenfeld (1985, p.74)"...being a problem is not a property of a mathematical task. Rather it is a particular relationship between the individual and the task that makes the task a problem for that person”.
- A problem for one person may simply be an exercise for another person

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Problem solving: Polya

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- Understand the problem
- Devise a plan
  - Have you seen it before
  - Do you know a related problem
- Carry out the plan
  - Check each step
- Looking back
  - Examine the solution
  - Check the result

- Polya, G. (1954). *How to solve it*. Princeto: Princeton University Press

Schoenfeld: Components of problem solving

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- Cognitive resources – thebody of facts and procures at one's disposal
- Heuristics – the 'rules of thumb' for making progress in difficult situations
- Control– having to do with the efficiency with which individuals utilise the knowledge at their disposal
- Belief systems – one's perspectives regarding the nature of the discipline and how one goes about working on it

Toh et al (2011) cite Schoenfeld (1985) *Mathematical Problem Solving*. Orlando, FL: Academic Press

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Control

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- Control, also called metacognition, is a major determinant of problem-solving success or failure (Schoenfeld, 1985)
- Toh Tin Lam et al stress that much of the the difficulty in teaching problem solving is that schools **underemphasize the importance of metacognition.** They teach heuristics but do not teach problem solving with all of its aspects.
- Students need to think about what they are doing, reflect on what has or has not worked and not just look for a quick fix.

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Problem solving approach

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- This does not mean that everything that students do in their classrooms is related to solving problems.
- Teachers can use problems as an introduction to a topic. Carefully selected to be relevant they are a good way to get students engaged and talking about the mathematics involved.

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Japanese lessons

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Shimizu tells us in the 2009 Yearbook (p.91) that Japanese lessons have the format of:

1. Posing a problem
2. Students solve on their own, in pairs or small groups, the teacher moves round the room to help/question
3. Whole class discussion
4. Summing up
5. Exercises or extensions (depend on time)

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Japanese lesson seen in Tokyo

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- I have a three dimensional shape in my hand and when I look at only one face (with the rest of the shape behind it) I see a square
- What could the shape look like?
- Discuss with someone near you

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Problems and when would you use them?

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The new school has exactly 343 lockers numbered 1 to 343, and exactly 343 students. On the first day of school the students meet outside the building and agree on the following plan. The first student will enter the building and open all the lockers. The second students will enter the school and close every locker that has an even number. The third student will reverse every third locker (open if closed, close if open).The fourth student will reverse every fourth one and so on until all 343 students in turn have entered and reversed the relevant lockers. Which lockers will remain open?

Making Mathematics Practical Toh Tin Lam et al

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Nice numbers

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- A “nice” number is a number that can be expressed as the sum of a string of two or more consecutive positive integers. Determine which of the following numbers from 50 to 70 are “nice”.

- Derek Holton

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Another example: for what kind of lesson?

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- Two bullets are placed in two consecutive chambers of a 6-chamber revolver. The cylinder is then spun. Two people play a “safe” version of Russian roulette which involves them shooting their phones and not themselves. The first person points his gun at his mobile phone and pulls the trigger. The shot is **blank**. Suppose you are the second person and it is now your turn to point the gun at your hand phone and pull the trigger. Should you pull the trigger or spin the cylinder another time before you pull the trigger?

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Starting on probability

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- Students can play horse racing games involving development of understandings of probablity and odds and the calculation of the winnings if desired.
- These two games give the students the opportunity to see empirically that in the long run when tossing one dice the numbers from 1 to 6 are equally likely but that when tossing two dice and finding the sum the numbers from 1 to 12 are not equally likely.
- Each race can take as long as you wish; four moves for the horse to finish is adequate

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Run a race

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Horse's Number	Horse's Name	
1	Swift Girl	
2	Likely Lad	
3	Princess	
4	Black Beauty	
5	Mr Brown	
6	Red Heart	

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Horse racing

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Horse's Number	Horse's Name	
1	Swift Girl	
2	Likely Lad	
3	Princess	
4	Black Beauty	
5	Mr Brown	
6	Red Heart	
7	Poppy	
8	Pink Diamond	
9	French Lace	
10	Marmaduke	
11	Bright Eyes	
12	Pat's choice	

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Cocolate orange peel

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- You are going to make chocolate covered orange peel as gifts for your 5 friends. What would you need to do?
- Work with another person to talk about the process and when you have made a plan and done some work reflect on what you have done.

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Greedy Pig: A dice game

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- This game can be used as an excellent start to get students thinking about statistics and probability.
- Teachers have used this game because:
  - it engages students in thinking about their own strategies and decision making and
  - it uses students' own data to develop concepts from fairly basic ones of mean, median and quartiles through to inference and statistical tests.

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Greedy Pig: a dice game

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The rules are:

In each game the dice is rolled to determine the score and your score is the sum of the numbers on the dice for consecutive roles

For each roll of the dice the number is added to your previous score UNLESS the number on the dice is a “killer 2”, then your score is reduced to zero.

You choose how long to participate in the game by standing up; when you sit down you do not continue to add to your score but then you cannot lose it either!!

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Greedy pig: ready to roll

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- All stand up and we will roll a dice. Everyone write the number down in game 1 on the score card. This is your score so far. In the first game you can have the first 2 numbers “for free” even if one of them is a 2.
- Choose whether to remain standing- if we roll a '2' (the 'killer' number) and you are still standing, your entire score for the game will be zero!
- So you have to decide when to quit – you show this by sitting down.
- Complete your score card as you go!!
- Some of you may have a score at the end of a game, some of you will have zero
- Find your grand total after 5 games

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Recording

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- Record your score on the stem and leaf plot on the board

- How did you decide when to sit down? Suggestions?

- Here you would have students contribute ideas but be non-judgemental

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More games

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- Another 5 games - find the grand total and record on the other side of the stem and leaf plot. Let us look and see!

Let's choose strategy rolls or points

Vote:

Rolls	2	3	4	5	6	7	8	9	10
Points	10	15	20	25	30	35	40	45	50

Repeat until students have some chosen strategies

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- Students can calculate grand totals for the five games, means, medians and quartiles, look at the long run across games played by small groups, construct stem and leaf plots, box plots and histograms.
- The probabilities which can be calculated by older students as part of the decision making process. The following slide shows a graph of the expected score given the first 2 rolls are “free”.

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Distribution

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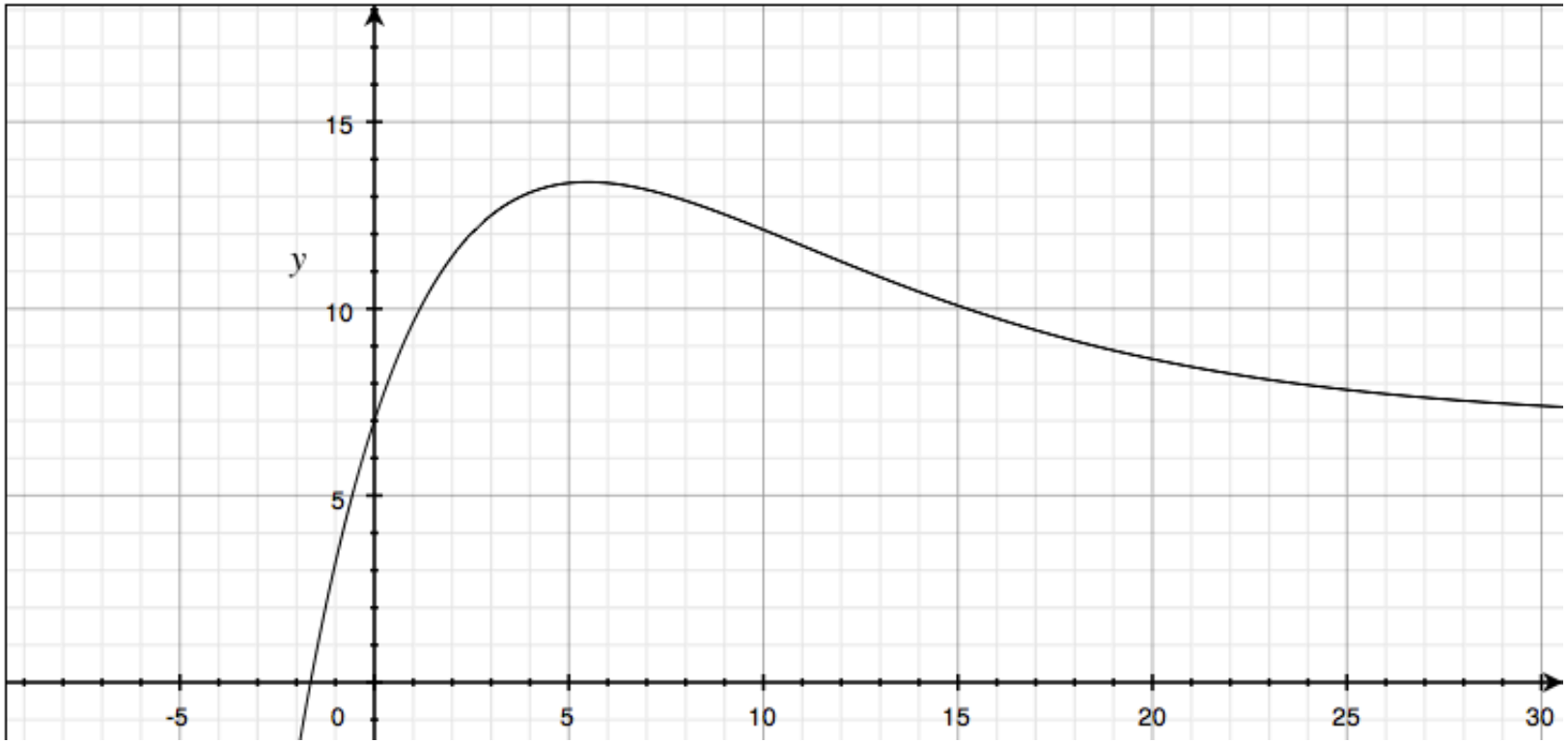
$$y=7+\frac{19x}{6}\cdot\left(\frac{5}{6}\right)^x$$



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Level of mathematics

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- This might suit young children who can get an empirical sense of how long to stand up
- It can motivate lower secondary students to make comparisons between means, medians with their own data. Make some hypotheses that can be tested by playing more games, look for
- Graphs such as stem and leaf, box plots, bar charts and so on. Outliers can be identified.
- It can lead into more sophisticated mathematics:
  - Inference and difference between samples
  - Expected values of the totals or averages
-

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Using the computer program

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- The advantage of the computer software developed by Doug Williams is that we can do multiple samples and look and see what happems in the long run.
- The software is available through maths300. The email address is maths300@esa.edu.au M.ore information in the next slide.

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Maths 300

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- Maths300 is available to teachers in any country around the globe.

  There are sample lessons on our website at http://www.maths300.esa.edu.au/index.php/free-sample-tour.html

  Prices at http://www.maths300.esa.edu.au/index.php/subscribe.html

  Order form at http://www.maths300.esa.edu.au/media/m300/documents/maths300subscribeform2012.pdf

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Secondary One

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Include:

- data collection methods such as:
  - * taking measurements
  - * conducting surveys
  - * classifying data
  - * reading results of observations/outcomes of events
- construction and interpretation of:

  tables, bar graphs, pictograms, line graphs,pie charts, histograms
- purposes and use, advantages and disadvantages of the different forms of statistical representationsand
- drawing simple inference from statistical diagrams

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Secondary two

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Data analysis-Include:

- interpretation and analysis of:
  - dot diagrams
  - stem-and-leaf diagrams
- mean, mode and median as averages
- purposes and use of mean, mode and median
- calculation of the mean for grouped data

Probability-Include:

- probability as a measure of chance
- probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)

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Secondary three/four

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Data analysis - Include:

- quartiles and percentiles
- range, interquartile range and standard deviation as measures of spread for a set of data
- interpretation and analysis of:
  - * cumulative frequency diagrams
  - * box-and-whisker plots
- calculate standard deviation for a set of data (grouped/ungrouped)
- use mean and standard deviation to compare two sets of data

Probability - Include:

- probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate)
- addition and multiplication of probabilities
- mutually exclusive events and independent events

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Conclusions-reflective thinking

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- Lots of different ways of thinking about reflective learning
- Relationship between problem solving and metacognition
- Students can develop their awareness of their own thinking and awareness of the processes
- It is hard to “fit it in” but worthwhile in the long run.

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Thank you