Nurturing reflective learners of rational number concepts

Kim Beswick

University of Tasmania
Australian Association of Mathematics Teachers

Outline

- What is a reflective learner?
- ‘Short view’ and ‘long view’ reflection
- Examples of reflection (short view)
  - Identifying reflection
  - What encouraged reflection?
  - What might the teacher know/believe?
- What these tell us about reflective learners and how teachers can nurture them
- What teachers know and believe to do these things.
- An example of ‘long view’ reflection

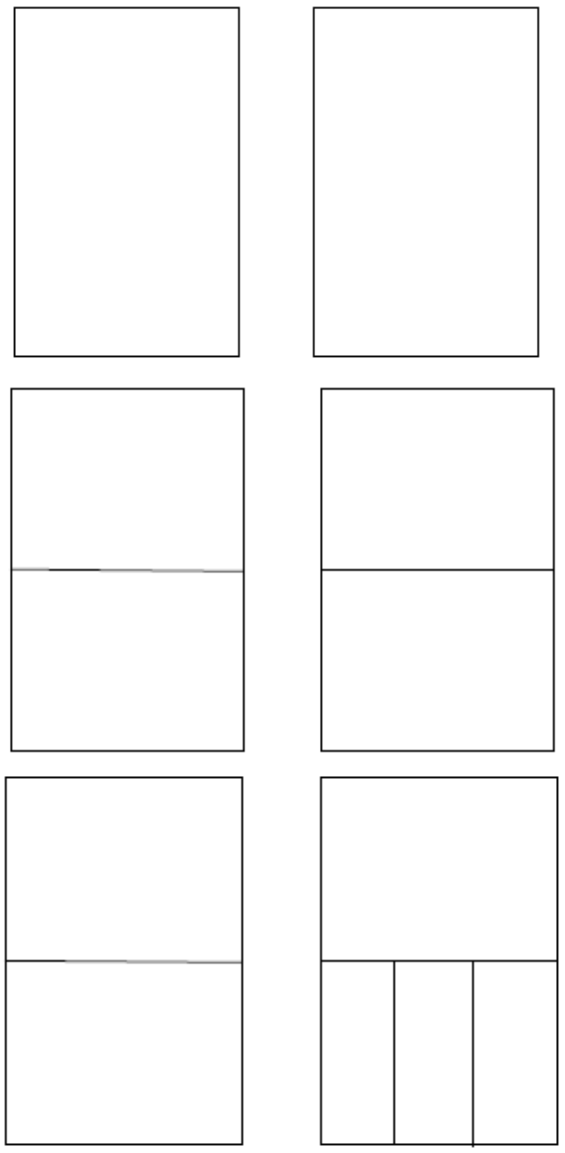
What is a reflective learner?

- Reflection:
  - "concentration of the mind; careful consideration"
- In relation to education:
  - "with a view to learning about the world or oneself"
  - "Looking back on experiences with a view to learning from them"

‘Short view’ and ‘long view’ reflection

- Looking back “as one goes”
  - Learning about the specifics of the current problem, mathematical ideas, oneself in this situation
- Looking back over a longer period of time
  - Learning about general principles to do with mathematics and oneself as a learner

Year 2, 7 years old



Three children and only 2 chocolate bars ...
You could cut both in half, that would give you 4 pieces so it still wouldn't be even ...
You'd have a leftover ...
You could cut it into 4 quarters ... 3 quarters
Is it all done?

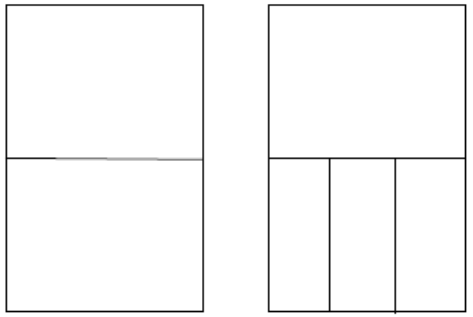
Evidence of reflection

- Thinking hard about the problem
- Realising that halving alone would not do the job
- Self-correcting
- Judging the solution to be satisfactory
- Looking to the teacher for feedback

What encouraged reflection?

- Task characteristics
  - challenging but accessible
- Teacher actions
  - chose the task, provided materials, provided wait time, refrained from commenting, provided some vocabulary “leftovers” which the child took up, prompted evaluation of the solution
- Teacher knowledge/beliefs
  - how children learn generally and about fractions in particular
  - young children can learn about fractions

Year 2,
7 years old



Wasn’t that a third?...
What’s three thirds?
Half ... A whole....

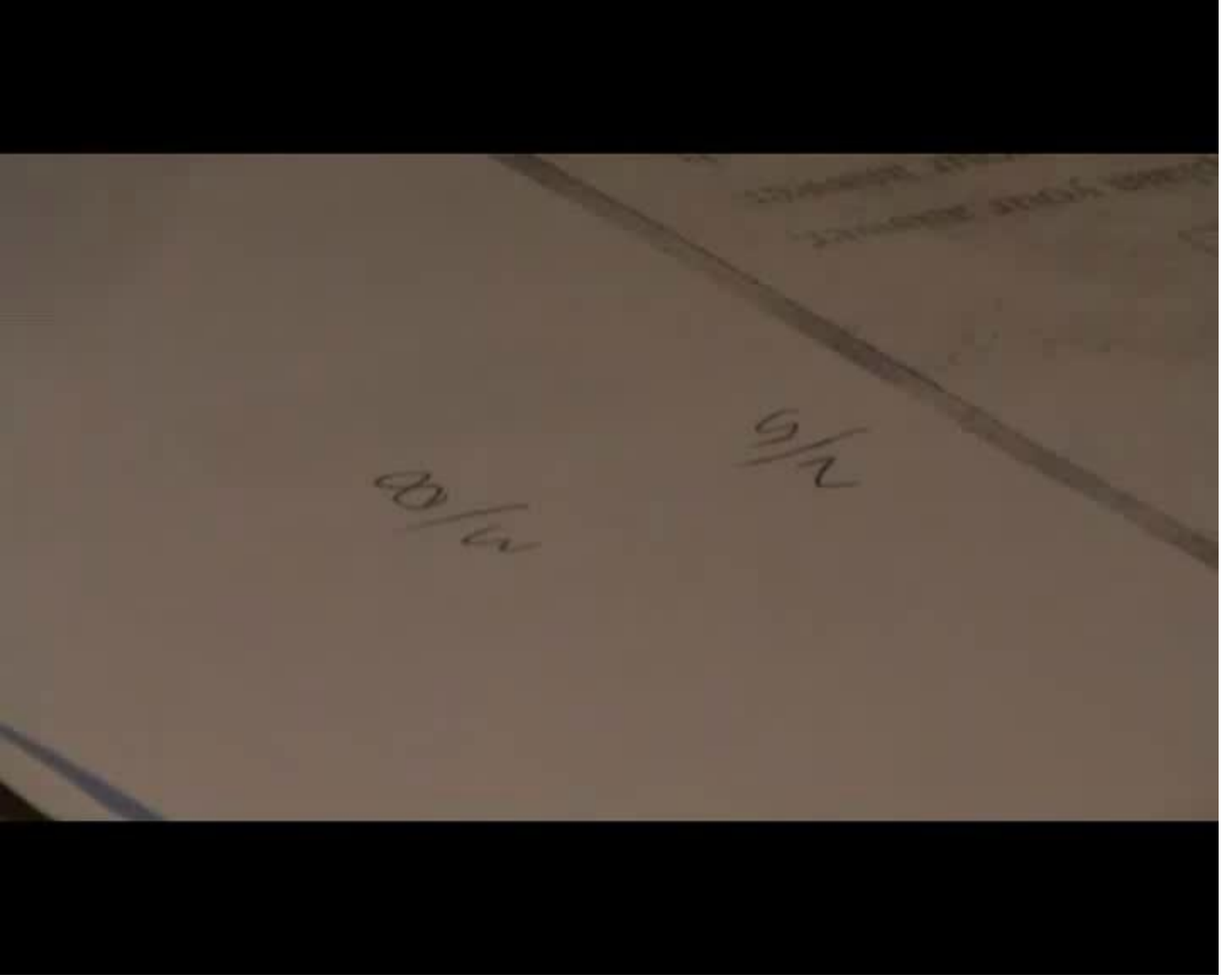
It depends which way you look at it ... If you have two halves it’s three quarters

Evidence of reflection

- Making a connection to the earlier problem
- Self-correcting in response to social cue
- Still thinking about the earlier problem
- Seeking feedback/approval from the teacher

What encouraged reflection?

- Task characteristics
  - simple but related to a concept that was only tentatively understood
- Teacher actions
  - chose the task, provided materials, acknowledged the child's connection of thirds with the diagram she'd produced earlier,
- Teacher knowledge/beliefs
  - how children learn generally and about fractions in particular BUT not everything.
  - young children can learn about fractions
  - conceptual understanding is important.



Year 5,
10-11 years old

Which mix is the most orangey?

Mix A	Mix B	Mix C	Mix D
2 cups concentrate 3 cups cold water	1 cup concentrate 4 cups cold water	4 cups concentrate 8 cups cold water	3 cups concentrate 5 cups cold water

Evidence of reflection

- Thinking hard about the problem
- Modified his pizza dividing strategy
- Recognised that his pizza was rough
- Drew upon existing knowledge of fractions e.g. “the smaller the number the larger the pieces”.

What encouraged reflection?

- Task characteristics
  - challenging but accessible
- Teacher actions
  - chose the task, asked questions, didn't provide a procedure or endorse a procedure to solve it
- Teacher knowledge/beliefs
  - how children learn generally and about fractions
  - Pushing students to learn and use algorithms before they have the necessary conceptual foundations is not helpful
  - Children can learn about fractions
  - Maths is about sense-making

Which mix is the least orangey?

Wouldn't be mix D... these 2 were the biggest ...(A & D)

Out of B and C ...

In a fight, ... for example, that would be ... 2 on 1 (C), that would be 4 on 1 (B) ... that would be the least (B)

Mix A	Mix B	Mix C	Mix D
2 cups concentrate 3 cups cold water	1 cup concentrate 4 cups cold water	4 cups concentrate 8 cups cold water	3 cups concentrate 5 cups cold water

Evidence of reflection

- Thinking hard about the problem
- Built on previous solution
- Returned to familiar ideas about ratio in the context of a fight rather than comparing fractions

What encouraged reflection?

- Task characteristics
  - challenging but accessible
- Teacher actions
  - chose the task, allowed thinking time, didn't prescribe a solution method
- Teacher knowledge/beliefs
  - something about how children learn generally and about learning fractions in particular
  - Children can learn about fractions
  - Mathematical reasoning that is meaningful to the student is valuable.

Year 7, 12-13 years old
Andrew is the teacher

The lesson began with $2\ ^{2}/_{3} \times 1\ ^{1}/_{4}$ written on the whiteboard.

- Andrew: What's the answer?
- Paul: $3\ ^{2}/_{12}$
- Andrew: (to whole class) Check it on your calculator.

After a few seconds

- Andrew: No? I suppose I could let you find your own solution but we'd be here till Christmas.
- Christine: Turn the wholes into fractions.

  $2\ ^{2}/_{3} = {}^{8}/_{3}$, and $1\ ^{1}/_{4} = {}^{5}/_{4}$
- Andrew: How did you do it?

Christine struggles to articulate her thinking but manages to explain her answer well.

Stacey giggled at one point in Christine's explanation.

- Andrew: One rule we have in this class; no one laughs at someone who's making an effort.

Andrew models Christine's solution with fraction tiles on the overhead projector.

- Andrew: What about 1 $^{2}/_{3}$?
- Paul: 1 $^{2}/_{3}$ = $^{6}/_{3}$
- Andrew: How did you get it?
- Paul: One whole is three thirds, and two thirds is five thirds. Oh! $^{5}/_{3}$
- Andrew: What about 1 ¾?
- Susan: $^{6}/_{4}$
- Ellie: 1 $^{3}/_{4}$ = $^{7}/_{4}$. You just add the two numbers.
- Ryan: Does it work all the time?
- Mary: Wouldn't it only work if you had one?

- Andrew: Give me an example that doesn't work.

Pauses, and then writes $7\ ^{9}/_{10}$ on the board.

- Joseph: $7\ ^{9}/_{10} = {}^{79}/_{10}$
- Andrew: How did you do it?
- Joseph: (comes to the front and writes on the board) $7\ ^{9}/_{10} = {}^{10}/_{10} + {}^{10}/_{10} + {}^{10}/_{10} + {}^{10}/_{10} + {}^{10}/_{10} + {}^{10}/_{10} + {}^{10}/_{10} + {}^{9}/_{10}$
- Drew: You put 7 and 9 together to make 79.
- Andrew: Will it work all the time? What about 1 ¼?
- Jason: Wholes × bottom + top
- Andrew: Will it work all the time? What about $2\ ^{2}/_{3}$? (Writes it on the board)
- Jason: $^{8}/_{3}$

Andrew writes his answer on the board.

- Andrew: $4\ ^{3}/_{5}$? (Writes it on the board)
- Carly: $4\ ^{3}/_{5} = {}^{23}/_{5}$

Evidence of reflection

- Volunteering solutions without being prompted e.g.

  Andrew: No? I suppose I could let you find your own solution but we'd be here till Christmas.

  Christine: Turn the wholes into fractions.

  $2\ {}^{2}/_{3} = {}^{8}/_{3}$, and $1\ {}^{1}/_{4} = {}^{5}/_{4}$

- Self-correcting e.g.

  Paul: $1\ {}^{2}/_{3} = {}^{6}/_{3}$

  Andrew: How did you get it?

  Paul: One whole is three thirds, and two thirds is five thirds. Oh! ${}^{5}/_{3}$

Evidence of reflection

- Trying to generalise e.g.

  Ellie: $1\ {}^{3}/_{4} = {}^{7}/_{4}$. You just add the two numbers.

  And later,

  Jason: Wholes × bottom + top

What encouraged reflection?

- Task characteristics
  - Apparently routine
- Teacher actions
  - chose the initial task, and subsequent problems that would elicit false generalisations
  - Accepted contributions that were not asked for
  - Asked questions
  - Required explanations
  - Responded the same way whether the answer was right or wrong
  - Let students “carry” the conversation

What encouraged reflection?

- Teacher knowledge/beliefs
  - How children learn generally and about fractions in particular
  - Had access to appropriate examples
  - Able to think on his feet and adjust the
  - Children can learn about fractions
  - Maths is about sense-making
  - The teacher has a responsibility to facilitate and students' construction of mathematical knowledge
  - Teacher has a responsibility to establish an orderly learning and mathematics focussed classroom environment
  - Students can learn from one another

- Example: teaching decimal place value to a group of Year 8 students

Evidence of reflection

- Attending to each other and the teacher
- Responding to each other's contributions
- Making arguments
- Defending viewpoints
- Acknowledging uncertainty
- Asking questions

What encouraged reflection?

- Task characteristics
- Teacher actions
  - chose the initial ‘task’, had major control of the direction of the discussion, asked questions, provided explanations, built on prior knowledge, re-iterated student thinking, respected the learners, let students “carry” the conversation, included all without pushing, praised appropriate ways of working, provided materials, made connections among representations explicit, explicit about own thinking, didn’t assume anything, focussed on important mathematical ideas

What encouraged reflection?

- Teacher knowledge/beliefs
  - Able to think on his feet and adjust the lesson
  - Maths is about sense-making
  - Simple mathematical ideas are in fact profound
  - Depth of understanding is much more important than getting through the curriculum
  - Understanding can only be inferred from multiple sources of evidence
  - The teacher has responsibility to facilitate and students' learning
  - Teacher has responsibility for the classroom environment
  - Students can learn from one another

Reflective learners …

- Think hard about the problem
- Critique and modify strategies as they consider and/or try them
- Self-correct
- Take responsibility for judging the solution to be satisfactory
- Look for feedback
- Make connections to the earlier problems
- Recognise the limitations of strategies
- Draw upon existing knowledge

Reflective learners …

- Build on previous solutions
- Rely on familiar ideas
- Volunteer solutions
- Make generalisations
- Attend to each other as well as the teacher
- Respond to each other's contributions
- Make arguments
- Defend viewpoints
- Acknowledge their own uncertainty
- Ask questions

Tasks that encourage reflective learning

- The fraction tasks
  - Challenging but accessible
    - Perhaps simple but related to a concept that is only tentatively understood
    - Offering multiple solution strategies
  - Purposefully chosen to elicit particular responses
- Place value task
  - Provided access to an in-depth discussion of fundamental ideas
- What the teacher does with the task

Teacher actions

- Choose appropriate tasks, purposively
- Provide appropriate materials
- Provide wait / thinking time
- Refrain from commenting
- Provide / model vocabulary
- Prompt evaluation of solutions
- Acknowledged connections students make
- Recognise opportunities to address important mathematics
- Ask questions
- Refrain from providing, prescribing of endorsing a procedure or approach
- Accept and address unsolicited contributions
- Require explanations

Teacher actions

- Respond the same way to correct and incorrect thinking
- Let students “carry” the conversation
- Provided explanations
- Provide links to prior knowledge
- Re-iterate student thinking
- Respect learners
- Include all without pushing
- Praise appropriate ways of working
- Made connections among representations explicit
- Make own thinking explicit
- Don’t assume anything in terms of understanding
- Focus on important mathematical ideas

Teacher knowledge/beliefs

- Teachers need to know
  - The mathematics they teach
    - What's important about it and why, where it leads, what it depends upon
  - How to teach it
    - tasks that are appropriate for particular purposes and learners and how to use them
  - About their learners
    - Cognitive and affective characteristics
  - About the curriculum

Know

- The mathematics they teach
  - What's important about it and why, where it leads, what it depends upon
- How to teach it
  - tasks that are appropriate for particular purposes and learners and how to use them
- About their learners
  - Cognitive and affective characteristics
- About the curriculum

Believe

- What mathematics is, what it means to learn, know, do, teach mathematics and why this is important
- What the teachers' role is, what characterises a 'good' task, how students should engage with tasks
- What do learners know and need to know, what can they do, how they learn, what motivates them
- What's most important, what is before, after

Important teacher beliefs for nurturing reflective learners

- Maths is about sense-making
  - Simple mathematical ideas are in fact profound
  - Depth of understanding is much more important than getting through the curriculum
- Mathematics is inherently interesting

Important teacher beliefs for nurturing reflective learners

- Understanding can only be inferred from multiple sources of evidence
- All students can learn mathematics
  - They can reason and want to learn,
  - They deserve respect
  - Students can learn from one another

Important teacher beliefs for nurturing reflective learners

- The teacher has a responsibility actively to facilitate and guide students' construction of mathematical knowledge
- Teacher has a responsibility to establish an orderly learning and mathematics focussed classroom environment

Grade 11, 16-17 years old
{S} is the student

- {K} And you said you disengaged too... Because it was boring?
- {S} Because it was just dreadful. ‘Cause like all through Primary School and grade 7 I was a very bright kid. Like, even from a young age I had extraordinarily high IQ. And so I was just getting it all done.
- {K} Yes.
- {S} And then I got bored. And the teacher [...] my work. He told me that my work was probably incorrect because I had completed it so quickly, and I [...]. I was like “Nup!” And so I, it got to the stage where I was so frustrated that I just wagged all the maths classes. ... There was just no point in going.

Thank you