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Reflections of a reflective teacher

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What is reflective thinking?

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"Dewey (1933) suggests that reflective thinking is an active, persistent and careful consideration of a belief or supposed form of knowledge and the further conclusions to which that knowledge leads." (p.46)

"In short, reflection creates meaning." (p.49)

McDonald, J. & Dominguez, L. (2009) Reflective writing. *The Science Teacher*, March, 46-9.

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Metacognition

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- Metacognition is concerned with *thinking about thinking*, as Flavell suggested:
  - "Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective."
- Means different things to different people, at least to an extent

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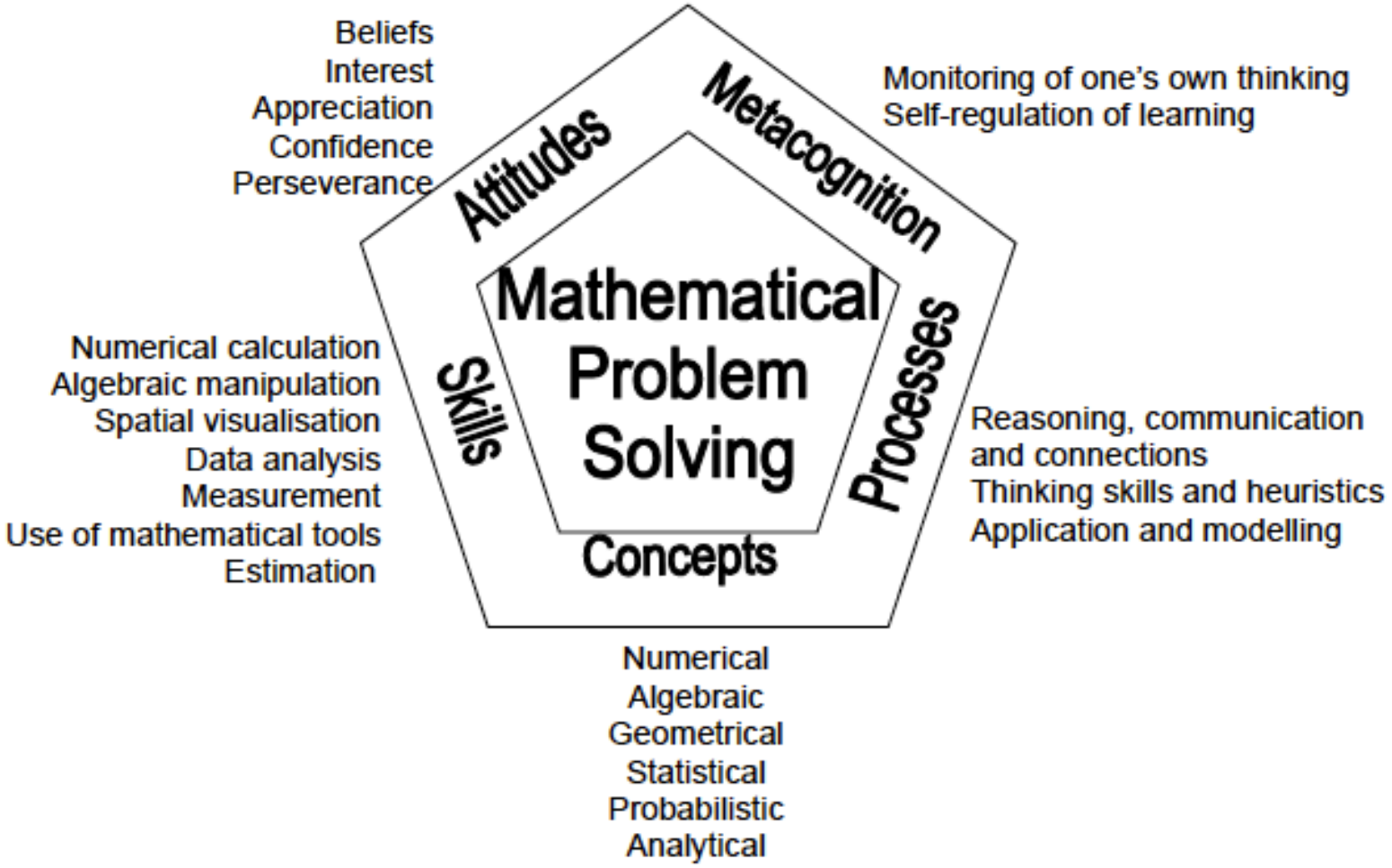
Singapore syllabus



Metacognition in problem solving

- Schoenfeld's work (with undergraduates)
  - What are you doing?
  - Why are you doing that?
  - How will it help you?
- Evidence that good problem solvers have developed metacognition
  - but is that why they are good problem solvers?

Jenni Wilson's questions

- Students asked to indicate whether they:
  - Think about what you already know
  - Ask yourself a question about the problem
  - Think about what problem is asking you to do
  - Make a plan to work it out
  - Try to remember if you have ever done a problem like this before
  - Guess the answer because it's like a problem you've done before
  - Think about what you will do next
  - Go back and check your work
  - Think about a different way to solve the problem
  - Change the way you are working

Wilson, J. (1998) *Metacognition within mathematics: A new and practical multi-method approach*, MERGA

Metacognition and learning

- Much of the work on metacognition has been connected to problem solving
- But it's possible to think about *learning* as well as about *problem solving*
  - ➢That is a focus of this presentation
- Part of the enterprise of developing teachers as 'reflective practitioners'
  - ➢And myself as a reflective practitioner too

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Thinking about learning

- Three learning environments:
  - Whole class
  - Pairs (or small groups)
  - Individually
- These provide different opportunities for reflective thinking by students

Richard Skemp on understanding

- ‘Understanding’ mathematics
- Instrumental understanding
  - Knowing what to do
  - Such as following a procedure
- Relational understanding
  - Knowing why
  - Connecting the pieces together

Skemp, R. (1976) Relational and instrumental understanding, *Mathematics Teaching*, 77, 20-26.

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Perspectives on maths

- Abstract first
  - Then ‘applications’ later
- Procedure first
  - Learning what to *do*
- Intention to make sense

Wheatley, G. R. (1992) The role of reflection in mathematics learning. *Educational Studies in Mathematics*, 23(5), 529-541.

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My classes

- My focus here is on prospective primary teachers
  - First year of university
- Including some students doing a unit intended for those with limited mathematical backgrounds
- To ensure that they developed a sound understanding of at least the mathematics they will teach

Three assessment tasks involving reflection

- Reflective journal
  - (Private) weekly mathematics journal kept
  - Reflecting on how some maths was learned
- Learning with technology
  - Analysing and describing how technology was used for learning
- Problem solving
  - Solving a problem personally and then seeing how others addressed it

Themes

- Several themes emerge from these sorts of reflective tasks
- These are of value to both
  - learners (reflecting about their learning
  - and teacher (reflecting about his teaching)
- What follows is a selection of these themes
  - Regarded by me as likely to be relevant to primary teachers and their students

Histories and emotions

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- Several students described their (unhappy) histories as students of maths
- Terror
- Feelings, including fears
- Beliefs about mathematics and themselves
- Systematic avoidance of mathematics
- Catharsis of writing about it?

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Assessment

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- Assessment in the past has focused on the ‘right answer’, regardless of how it is obtained
- At least in the minds of the students
- So reflections often focused on how to get the right answers rather than on how to make sense of them

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Rules of thumb

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- It is clear that for some students, mathematics comprises a large collection of ‘rules’, such as:
  - Two minuses make a plus
  - Change the side and change the sign
  - Invert and multiply (to divide by fractions)
  - Move the decimal point (when multiplying or dividing by powers of ten)
- Rules are remembered and applied even if not understood (or appropriate)

When are we ever gonna use this?

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- A common response to difficulties
- A serious question or a defense mechanism?
- Maths is portrayed as ‘useful’, perhaps emphasising and provoking this sort of response
  - ➢See Wheatley’s ‘abstract first’ perspective

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The one right way

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- Maths is sometimes described as a verb
- But there seem to be two meanings for this
  - Active mathematical thinking
  - Using mathematical procedures (e.g. ‘doing’ fractions)
- Learning maths for some students involves repeatedly practising the ‘correct’ procedure

Mental arithmetic

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- Many students seemed to use standard algorithms to calculate mentally
  - That is, imagined the paper and pencil version in their heads
- Partitioning of numbers and the role of place value was a revelation
  - Especially non-standard partitioning

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Meanings of operations

- Many students did not realise that operations had meanings
  - They tend to rely on the arithmetic signs
  - Which perhaps accounts in part for the difficulty of ‘word problems’
- E.g. difference and complementary addition as well as take away for subtraction
  - E.g., what is 3 – (-2)?

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It's simple …

- I have realised what a poor practice it is to say something is 'easy' or 'simple'
- This is quite intimidating (for students who don't yet understand something)
- Many student reflections referred to 'simple' ways of doing things
  - Such as 'invert and multiply'
  - These tended to be learned as rules and procedures … the 'simple' way

Mathematical experimentation

- When reflecting on use of technology, it is clear that many students equate ‘explanation’ with teaching
- Even when the technology is meant to involve active experimentation
- Virtual manipulatives are a good example of this

Visual learners

- Many students claimed that they were ‘visual learners’
  - ➢Not always clear what they meant
- Yet many of these did not seem to use virtual manipulatives well
- How well do they interact with visual representations?
- What sort of help do they need?

HOTmaths

- The online learning system, *HOTmaths*, was very popular in student reflections
  - Widgets were used in class
  - These tended to be visual in nature
- However, it seems that some students confuse knowing what to do with understanding the mathematics
  - E.g. the Scorchers in *HOTmaths* focusing on speed and accuracy

NLVM and *Illuminations*

- The *National Library of Virtual Manipulatives* was a popular choice for student technology reflections
  - E.g., Pan balance
  - E.g., Factor trees
- *Illuminations* was also popular
  - E.g., mean, median and mode

Learning from manipulatives

- Virtual manipulatives may be interpreted differently by those who understand the mathematics and those who don't
  - E.g., the teacher and the student
- Students may need direction and guidance to reflect on what they see and do
  - … to exploit the possibilities for learning

Calculators

- It was clear that many students still equate using a calculator with getting an answer
  - ➢Which is not the most powerful way of using a calculator
  - ➢Although consistent with word, ‘calculator’
- It is clear to me that I need to emphasise the learning role more carefully
  - ➢As it does not happen naturally

The calculator as a virtual manipulative

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- Many possible manipulations with a modern calculator. E.g.,
  - Counting with the calculator
  - Multiplying and dividing by 10
  - Fractions proper and improper
  - Fractions and decimals
- Students do not seem to experiment in these ways unless directed

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GeoGebra

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- Potentially very useful for learning
- However, few student reflections related to *GeoGebra*, the dynamic geometry system, exploited its capabilities for learning
- Students seem reluctant to experiment, although this is a major purpose of the software for educational use
  - ➢More experience & confidence needed first?

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How do I learn?

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- My students do not seem accustomed to thinking about this
- Many regard themselves as genetically inferior
- Memory is often associated with mathematics
- Students focus on what happens in class as the main (only) locus of learning
- Teacher's role is to explain/show/tell

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Problem solving reflection

- Students were given a task to address
- After solving it, they were asked to get others to do so, and compare solutions
- Many students expect that there is only one way to solve problems, so had trouble comparing two solutions
- Reluctance to think back and reflect on better solutions or different solutions

Implications for primary school?

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- Discuss how learning happens directly in class
- What can students do to help their learning? Use of discussion
- Metacognitive questions: What are you doing? Why? How will it help?
- How else could you do this problem?
- *Why* is it so?

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An intention to make sense

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- Overall, reflective thinking provides an opportunity for both the teacher and the student to ***make sense*** of what is happening
- With teaching mathematics
- With learning mathematics

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Thanks for listening!

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